Notes for Lecture 9

Waves, Sound

9.1 Law of reflection

Continuing from the last lecture, we discussed the contents of Section T15-7, which you must read thoroughly.

Proving the law of reflection

Note that the law of reflection ($\theta_i = \theta_f$) can be proven by using the "time-reversal symmetry principle." According to this principle, if a physical process is possible, then another physical process which corresponds to the version of the first physical process, but played in reverse, just as you would play a movie backwards, is also a possible physical process, *if* no damping/heat-involving mechanism is considered. This symmetry principle holds for most microscopic laws.

9.2 Standing waves (general)

We have discussed standing waves here and there so far. Here, we will discuss them in a general fashion.

If we superpose

$$D_1(x,t) = A\sin(kx - \omega t + \phi_1) \quad (9.1)$$
$$D_2(x,t) = A\sin(kx + \omega t + \phi_2) \quad (9.2)$$

then we get a standing wave (derivation given during lecture).

$$D_s(x,t) = D_1 + D_2 = 2A\sin(kx + \tilde{\phi}_1)\cos(\omega t - \tilde{\phi}_2) \quad (9.3)$$

where new phase constants are defined as

$$\tilde{\phi}_1 = \frac{\phi_1 + \phi_2}{2} \quad (9.4)$$
$$\tilde{\phi}_2 = \frac{\phi_1 - \phi_2}{2} \quad (9.5)$$

Note that the above form of standing wave can be understood as having a **time-independent** shape $\sin(kx + \tilde{\phi}_1)$, with a **time-dependent** amplitude, given by $2A\cos(\omega t - \tilde{\phi}_2)$.

The above is the most general form of a sinusoidal standing wave. While this is not the most general form of a standing wave, as there can be a generally non-sinusoidal standing wave, it is nevertheless a very important one to keep in mind.

For a guitar string of length l, it is easy to show (cf. the last problem of the practice exam 1) that

$$n\lambda_n = 2l \qquad n = 1, 2, 3, ... \quad (9.6)$$

is the condition for the standing waves.

The same equation holds for the sound wave in an open tube. For sound waves in a half-open tube, with one end closed and the other end open, of length l, one can derive that

$$(2n - 1)\lambda_n = 4l \qquad n = 1, 2, 3, ... \quad (9.7)$$

is the condition for the standing waves.

The boundary conditions for an open tube are that $\frac{\partial D}{\partial x} = 0$ at both ends. The boundary condition for a closed tube end is that $D = 0$ at that end. The above conditions for discrete values of λ_n can be derived from these boundary conditions (suggestion: put the open end at $x = 0$ and then write down the standing wave form as $D_s(x) = A(t)\sin(kx)$, then apply the boundary condition for $D_s(l)$ or $\frac{\partial D_s}{\partial x}(l)$). Note that the frequencies corresponding to them, $f_n = v/\lambda_n$ are **natural (resonant)**

frequencies of the tube/string. Note that $v = 0$ for a standing wave, but in the expression, $f_n = v/\lambda_n$, v is the speed of a single travelling sinusoidal wave (so *not* zero!).

Note that the reason why $\Delta P = 0$ at an open end, meaning $\frac{\partial D}{\partial x} = 0$ or D = an extremum, was explained in the last section of LN 8, as was explained in words during the lecture.

Potentially very misleading diagrams

I don't know about you, but when I look at the diagrams shown in pages T434 and T435, an idea gets into my head. My brain recognizes that thick blue lines as pipes (musical instruments), and that red lines represent the wave amplitude D for the sound wave. If literally interpreted, then these diagrams suggest that D is on the order of the size of the pipe!

This is very far from true!

Example T16-7, which we went through during the lecture, shows that the displacement and the pressure variation are truly minute! It is mind-boggling how we can perceive such small variations (and how about other animals, which are better than us!). Even for really really loud sound (cf. Table T16-2) that may break the instrument, D_{max} is only about 10^{-4} m = 0.1 mm. So, one must keep in mind that D is *really small* compared to the size of the pipe, and in the longitudinal direction. To give the benefit of doubt, we must consider diagrams shown in Figures T16-11 and T16-12 as only *very crudely symbolic*, as far as the direction of D and the magnitude of D are concerned.