Notes for Lecture 6

Waves

Let us probe a bit more deeply into the mystery of waves.

6.1 Conventions

The quantity k (wavenumber or wave vector) could in principle associated with any period function, but that would make it confusing. So, by convention, we use k only for sinusoidal waves.

As we shall see later, a sinusoidal wave can be a *standing* wave, as well as a *travelling* wave. The equation that we used in the last lecture, $D(x,t) = A\sin(kx - \omega t + \phi)$, is a *travelling* sinusoidal/harmonic wave, for an obvious reason.

The wave velocity equation

$$v = \frac{\omega}{k} \tag{5.15}$$

is then applicable **only to travelling sinusoidal waves**[1].

[1] Note that one can still write a general travelling wave in a mathematical form as $g(x \pm vt)$. For a general wave pulse, k (wavenumber) and ω (frequency) may not make sense, but v still makes sense if it is travelling.

6.2 What is intrinsic to the medium?

In the previous class, we learned that

$$\omega = vk \tag{6.1}$$

This equation is just another form of Eq. 5.15 (multiply it by k on both sides).

A question – which of the three quantities appearing above is intrinsic to the medium through which the wave propagates? The answer is v. Why so? Let us consider the banana slug observation wave example that we discussed in a previous class.

The set-up for the bf banana slug observation wave is this. A group of people form a straight line. Each person is able to see only the back of the person in front of her/him. The person at the beginning of the row is able to see a tree in front of her/him, and on that tree a banana slug is crawling. This first person's head follows the lateral movement of the banana slug (but not the vertical motion). As this person's head moves, the person behind copies the motion, which is copied by the next person and so on.

So, this example is quite analogous to the wave on string. Except the shaking of string is replaced by the banana slug crawling, the tension on the string is replaced by the human interaction. It is clear that the "banana slug wave" will propagate at a velocity that depends on how fast human reacts to signal (a few hundred ms) and the spacing between people in the row. Note that this velocity is determined by the properties of the row of people alone, and is independent of the banana slug, assuming that the banana slug is moving slow enough.

6.3 Wave velocity examples

Coming back to the wave on string, the velocity of the wave is given by

$$v = \sqrt{\frac{F_T}{\mu}} \tag{6.2}$$

where μ is the linear density of mass (mass per unit length), F_T is the tension on string.

This formula can be derived using a "long wave length" approximation. This approximation amounts to assuming that the wave amplitude is much smaller than the wave length. Or, $|v'| \ll |v|$, where v' is the velocity at which each segment moves up or down, and v is velocity at which the wave propagates. Note that for a travelling sinusoidal wave, $v = \lambda f$ and $|v'|_{max} = A\omega = A2\pi f$. Therefore, $|v'/v| \leq 2\pi A/\lambda$, which will become very small in the long wave length limit.

In the long wave length limit, the tension on string is essentially unchanged from the tension on string in equilibrium. Using this, a simple derivation for v can be made, as shown during the lecture and as also shown in the textbook (from a slightly different point of view). However, you are advised to take the setting up of the wave equation (Section T15-5) as the best method to prove the wave velocity. So, an advanced student, who is comfortable with multi-variable calculus, is strongly encouraged to read T15-5 and understand it. Understanding the derivation of the wave equation will not be required for this class. In the next lecture, we will **mention and use the wave equation** and discuss its properties (such as superposition), without bothering to derive it.

The above expression for wave velocity, Eq. 6.2, describes the wave velocity for a particular transverse wave.

It is interesting to note that many wave velocities have the expression $\sqrt{E/I}$, where E stands for the elastic property and I stands for the inertial property. You will note that this is also qualitatively true even for the expressions for ω's that we derived for typical SHO problems (Eqs. 2.5, 4.3, 4.8, 4.9)! This is not surprising at all, given Eq. 5.15. Another name for the "elastic tendency" is the "resilience" or the "restoring tendency" – i.e., the elastic property is closely connected to Hooke's law for small amplitudes.

The following two are other common examples for the wave velocity, both for longitudinal waves.

$$v = \sqrt{\frac{B}{\rho}} \qquad \text{sound wave in gas/liquid} \tag{6.3}$$

$$v = \sqrt{\frac{Y}{\rho}} \qquad \text{sound wave solid, longitudinal mode} \tag{6.4}$$

Here, ρ is the volume mass density, i.e., mass per unit volume. B is the bulk modulus, defined by $-B\Delta V/V \equiv \Delta P$. ΔP is a small change of the pressure of the system, and ΔV is a small change of the volume of the system, from V. Since an increase in pressure results in a decrease of volume, there is a minus sign in the defining equation for B so that B is positive. Note that B has the same physical dimension as P. Y is Young's modulus, $Y \equiv (F_{applied}/A)/(\Delta l/l)$, where $F_{applied}$ is the applied force on a solid of surface area A (and so $F_{applied}/A$ is the pressure in this case), l is the length of the solid parallel to $F_{applied}$, and Δl is the change of l in the direction of $F_{applied}$.

It is important to note that both B and Y measure the resilience or the strength of the system in the sense that they measure the tendency for the system to resist the external pressure. The different definitions B and Y are required for gas/liquid and solid, since the property of a solid is not isotropic, i.e., its property may depend on the direction. Also, a solid can support a shear wave as well – a transverse sound wave.

Finally, let us note that the light is a wave, and it propagates at the speed of light, c, in vacuum. We do not yet know what elastic property and what inertial property, if any[2], of vacuum give rise to this fundamental constant of Nature. However, for now, it would seem to be the best practice not to get lost in thoughts about this continuing perplexing mystery. It would seem to be the best student practice just to accept c as a fundamental constant.

[2]You might ask how can a vacuum have any property? In advanced courses, you will learn the following. In the modern view of physics, the vacuum is far from nothing. It has a very rich structure!