Notes for Lecture 4

SHO, Waves

4.1 SHO examples

Before we start on waves, two important examples of the SHO are worth our attention. One is a pendulum (simple or physical), and the other is a torsion pendulum. We start with the latter.

4.1.1 Torsion pendulum

Let us take the torsion pendulum, first, since it is easier. As Figure T14-18 shows, the idea is that you have a thin wire of some sort (e.g., metal) on which a mass with the rotational inertia I is hanging. This mass and the wire are welded together, or firmly joined in some other way, so that when the mass is twisted, the wire itself gets twisted. The wire resists this, exerting a **restoring torque** of Hooke's law form, but in the angular coordinate, as in $\tau = -\kappa\theta$, where τ (tau) is torque, κ (kappa) is torsion coefficient, and angle θ is small

$$|\theta| \ll 1 \qquad \text{small angle (rad), e.g., 0.1 rad} = 6^\circ \tag{4.1}$$

Note that we use radian for the unit of angle, unless otherwise stated, in this course. The rotational form of Newton's second law then reads

$$I\ddot{\theta} = -\kappa\theta \qquad \text{Newton's 2nd law; } \kappa = K \text{ of the textbook} \tag{4.2}$$

This equation can be re-written as

$$\ddot{\theta} = -\omega^2\theta \qquad \omega \equiv \sqrt{\kappa/I} \tag{4.3}$$

Comparing this with Eq. 2.11, the fundamental EOM for SHO, we realize that this EOM is just another SHO EOM, with θ replacing x. Thus, the resulting motion is

$$\theta = A\cos(\omega t + \phi) \tag{4.4}$$

As we shall see shortly, this equation also describes the motion for a simple pendulum and a physical pendulum.

Which angle?

Rotational SHM, like the one that we just identified for a pendulum, can be very confusing, since there are two angles involved. One is the real space angle, θ, and the other is the phase $\omega t + \phi$ (which is the angle in the so-called the "phase space"). **Do not mix them up!** Note that in previous lectures we used the notation $\dot{\theta} = \omega$, which was fine for the mass on spring problem, *but not for pendulum problems!*

If all this is confusing, I do not blame you. However, you owe it to yourself not to get confused from now on. Follow this guideline. If a problem involves an oscillation (SHM) in angle, θ, *do not even think about the UCM* that is related to the SHM in a way described in Section 3.3!

Indeed, the UCM-to-SHM connection discussed in Section 3.3 is a nice piece of math, but unfortunately it has a potential to be misunderstood greatly at this point.

4.1.2 Simple pendulum, physical pendulum

Figure 4.1 defines these two important pendulums. A simple pendulum is a pendulum consisting of a point mass at distance l from the pivot point. A physical pendulum is a rigid body that hangs from a pivot point.

Assuming that the gravitational field is constant, e.g., near the surface of the Earth, the center of gravity[1] for the physical pendulum is then identical with its

[1] A physical pendulum in a non-constant gravitational field is complicated, since the center of gravity, at which the net force is exerted, will in general change as the pendulum swings.

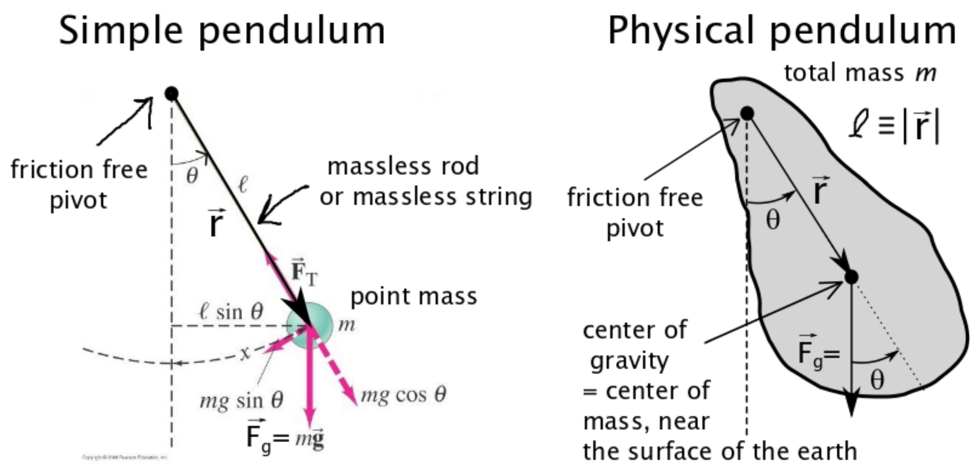


Figure 4.1: The schematics of the simple pendulum and the physical pendulum.

center of mass. We assume that the center of mass is at length l from the pivot point.

In both cases, the equation of motion in the rotational form is useful.

$$I\ddot{\theta} = \tau = -mgl\sin\theta \tag{4.5}$$

Here, we already used the fact that the torque[2] $\vec{\tau} = \vec{r} \times \vec{F}_g$ is given by, in both cases,[3] $\tau = -mgl\sin\theta$, since the angle between the vector $\vec{r}$, whose magnitude is l, and $\vec{F}_g$, whose magnitude is mg, is θ. And, the negative sign means that the direction of the torque is opposite to that of θ (positive as shown in the diagrams), meaning a **restoring torque**.

For small angle, $|\theta| \ll 1$ (Eq. 4.1), we get

$$\sin\theta \approx \theta \qquad |\theta| \ll 1 \tag{4.6}$$

using which the above Newton's equation of motion becomes

$$I\ddot{\theta} = -mgl\theta \tag{4.7}$$

This is, again, a SHO EOM, with

$$\omega = \sqrt{\frac{mgl}{I}} \qquad \text{physical (or simple; see below) pendulum} \tag{4.8}$$

[2]This is the net torque. In the simple pendulum case, the tension force $\vec{F}_T$ does not give rise to any torque on point mass m, since $\vec{F}_T \parallel \vec{r}$. In the physical pendulum case, any normal force that the physical pendulum experiences at the pivot is zero for the same reason.

[3]Note that here the symbol τ means the z component of $\vec{\tau}$, assuming the pendulum oscillation occurs in the xy palane.

It goes without saying that for the simple pendulum case, we can simplify this equation further, by using $I = ml^2$.

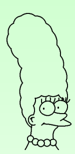
$$\omega = \sqrt{\frac{g}{l}} \qquad \text{physical pendulum} \tag{4.9}$$

Almost any pendulum around us is an example of a physical pendulum: a dining table lamp that is bumped by your head, or a big piece of meat hanging in a butcher shop.

4.2 Wave – what is it?

Waves are everywhere. But, what are waves? This is generally *not* an easy thing. There is a certain mystery about the wave phenomena, except classical mechanical waves. In fact, we do not understand waves such as light waves very much. Einstein famously quipped "for the rest of my life, I will contemplate on what light is." Imagine this coming from Einstein!

So much about what we don't understand. Let us consider what we *do* understand – mechanical waves. The following defining characteristics of mechanical waves must be noted.



Mechanical wave

1. A wave is a collective phenomenon. It involves many particles, not just one particle.
2. A wave moves, propagating energy, but each constituent particle stays around a fixed position. The disturbance of each constituent particle is part of the wave.

Examine Figure T15-1 with the above points in mind. First, note that each small segment of a string can be considered a particle. So, a string consists of many particles, confirming point 1 above. Second, the wave propagation velocity and the velocity of each particle in the string are quite different. Indeed for this example of a string wave, the average velocity of each particle is zero, while the wave itself has a definitely finite velocity, even when averaged over time. This confirms point 2 above.

As we shall see next, a fundamental form of local disturbance of a particle is the SHM. In this case, we obtain a sinusoidal wave.