Diffraction

Single slit (rectangular): $D\sin\theta = m\lambda$ (destructive), $m = \pm1, \pm2, \ldots$ This equation with $m = 0$ is a constructive interference condition (giving the brightest fringe, by far). Other constructive interferences: $D\sin\theta \approx \pm\left(m+\frac{1}{2}\right)\lambda$ with $m = 1, 2, 3, \ldots$ Thus, not only the central bright fringe the most intense, it is also twice wider. The intensity is given by $I = I_0\frac{\sin^2(\beta/2)}{(\beta/2)^2}$, where $\beta = kD\sin\theta$ where $k = 2\pi/\lambda$. β: the phase difference for lights emanating from the two end points of the slit. $D\sin\theta = m\lambda$ means $\beta = 2m\pi$, and $D\sin\theta \approx \pm\left(m+\frac{1}{2}\right)\lambda$ means $\beta \approx \pm(2m+1)\pi$.

Double slit (very narrow slits): $d\sin\theta = m\lambda$ (constructive). $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$ (destructive). $m =$ any integer. With $\delta \equiv kd\sin\theta$ (the phase difference), these conditions read $\delta = 2m\pi$ and $\delta = (2m+1)\pi$, respectively. $I = I_0\cos^2(\delta/2)$.

Double slit (rectangular, finite width D): $I = I_0\cos^2\left(\frac{\delta}{2}\right)\frac{\sin^2(\beta/2)}{(\beta/2)^2}$. The (slowly varying) single slit pattern, modulated by the (fast changing) double slit pattern.

Diffraction grating, or N slits: $d \equiv$ groove/slit spacing. Principal maxima given by $d\sin\theta = m\lambda$ ($m = 0, \pm1, \pm2, \ldots$). Minima given by $d\sin\theta = \left(m+\frac{n}{N}\right)\lambda$ where $n = 1, \ldots, N-1$ for *each* value of m. So, for each principal maximum fringe, $\exists\ N-1$ minima next to it. Approximately midway between any two adjacent pair of these minima, $\exists$ a low intensity local maximum. All of these derive from one equation: $I = I_R\cdot\frac{\sin^2\frac{N\delta}{2}}{\sin^2\frac{\delta}{2}}$, where $\delta \equiv kd\sin\theta$, and $I_0 = I_R N^2$ is the peak intensity for *any* principal maximum. I_R is independent of N. I_0 scales as N^2. Peak width scales as $1/N$.

X-ray diffraction: A crystal provides an infinite number of different diffraction gratings simultaneously. One way to decompose the crystal into a set of lattice planes corresponds to one diffraction grating. $2d\sin\phi = m\lambda$ ($m = 0, \pm1, \pm2, \ldots$) is the principal maximum condition (**Bragg diffraction**), if d is the spacing between lattice planes. The diffraction arises from specular reflections off of lattice planes (a little like for thin film). ϕ is measured w.r.t. the lattice plane, *not* its normal.

Single slit (circular, diameter D): Airy pattern. The first minimum occurs at $D\sin\theta = 1.22\lambda$, instead of $D\sin\theta = \lambda$ (for a rectangular slit; see above).

Overarching assumption for all of the above: l (the slit/grating to screen distance) is much greater than any slit/grating dimension (D, d, Nd, crystal size).

Resolution: For a circular single slit, the angular resolution due to diffraction is given by $\theta = 1.22\lambda/D$ (Rayleigh criterion; assuming small θ). This applies to any circle-shaped optical element. Or, qualitatively speaking ($\theta \sim \lambda/D$), to any thing size D that bends/reflects/emits light. For lateral resolution, multiply θ by the relevant distance. A diffraction grating can resolve $\Delta\lambda = \lambda/(N|m|)$ for $m = \pm1, \pm2, \ldots$.

Polarization

Linearly polarized light: can be generated and detected by a conducting rod (or an array of such rods/long-molecules, as in a Polaroid sheet). The electric field,

$\vec{E}$, oscillates in one particular direction. If a linearly polarized light passes through an ideal linear polarizer, $I = I_0 \cos^2\theta$, where θ is the angle between the incoming polarization axis and the polarization axis of the polarizer (perp. to rods).

Brewster angle: If the reflection angle plus the refraction angle is 90 degrees, then the reflected light is linearly polarized 100 % parallel to the surface/interface. $\theta_B = \tan^{-1}(n_2/n_1)$, for light going from medium 1 to medium 2.

Fluids

Archimedes principle: the buoyant force is equal to the weight of the displaced fluid. **Pascal's principle:** pressure applied to a confined fluid is applied at any point of the fluid, and in any direction. These two principles can be explained within the kinetic theory of fluid ("simple analysis of pressure" problem and lecture note). $P = F/A$. Pressure in a fluid is a scalar, an **intrinsic property of a fluid within a very short length scale (mean free path)**; it is determined by its intrinsic properties (density, temperature, etc) alone. External agents (like a gravitational field) apply pressure on, and add to the pressure of, a fluid, simply because they modify these properties of the fluid.

Hydrostatics: $dP = -\rho g dy$ (fluid in a gravitational field; compressible or not; ρ is the mass density, M/V). $P = P_0 - \rho g y$, if incompressible. **Hydrodynamics (Bernoulli eq):** $P + \frac{1}{2}\rho v^2 + \rho g y$ is conserved, for a laminar non-viscous flow of an incompressible fluid. **In all these eqs., P is the equilibrium (i.e. static) pressure**. Torricelli's theorem ($v = \sqrt{2gh}$) follows from Bernoulli eq., for incompressible fluid drawn from a large jug with a spigot at depth h (assuming constant ambient pressure).

One atm of pressure: 1.013×10^5 Pa = 1013 mbar = 14.7 psi = 760 Torr = 760 mm Hg $\approx$ 10 m H_2O. Gauge pressure: usually, ΔP over ambient pressure ($\approx$ 1 atm).

Continuity: (for uniform laminar flow (i.e., non-viscous flow)) $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$, where A_1, A_2 are the cross sections. If incompressible, then $A_1 v_1 = A_2 v_2$.

Statics

Equilibrium conditions: zero net force and zero net torque. If zero net force, then the net torque is independent of the reference point (nice!). Torque: $\vec{\tau} = \vec{r} \times \vec{F}$.

Do not assume that you know force unless you really know it. Solve for forces using free body diagrams. Know the nature of tensions, pulleys, springs, etc.

Center of gravity: same as the center of mass, near the surface of the earth, or where the gravitational field is constant. Generally, *not* a well-defined concept, but the concept arises when one *desires* to describe the net torque due to gravity on an extended object, within a simple one-particle picture.

THE Principle in 5A,5B: $\vec{F} = m\vec{a}$ (reads: we learn $\vec{F}$ *from* $\vec{a}$).