Simple Harmonic Motion

$$\begin{aligned} \ddot{x} &= -\omega^2 x \\ x &= A\cos(\omega t + \phi) \end{aligned}$$

Examples

- $\omega = \sqrt{k/m}$ (mass on spring, horizontal or vertical; $F_{net} = -kx$; the gravity effectively disappears from the problem in the vertical spring problem if x measures the displacement relative to the new equilibrium *with* mass on spring)
- $\omega = \sqrt{\kappa/I}$ (rotational; Net torque $\tau_{net} = -\kappa\theta = I\ddot{\theta}$)
- $\omega = \sqrt{mgl/I}$ (physical pendulum; small angle; $\kappa = mgl$)
- $\omega = \sqrt{g/l}$ (simple pendulum, $I = ml^2$; small angle)

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad \text{angular frequency, frequency, period}$$

Other kinematical quantities

$$\begin{aligned} \dot{x} &= -\omega A \sin(\omega t + \phi) && \text{velocity} \\ \ddot{x} &= -\omega^2 x = -A\omega^2 \cos(\omega t + \phi) && \text{acceleration} \end{aligned}$$

So, max speed = $A\omega$, and max acceleration = $A\omega^2$.

Energy conservation

$$\begin{aligned} E &= U_{max} = K_{max} = K(t) + U(t) = \text{constant in time} \\ U(t) &= \frac{1}{2}m\omega^2 x^2 \quad \text{or} \quad \frac{1}{2}I\omega^2\theta^2 \\ K(t) &= \frac{1}{2}m\dot{x}^2 \quad \text{or} \quad \frac{1}{2}I\dot{\theta}^2 \end{aligned}$$

Wave

Any wave can be broken down into travelling sinusoidal waves. A travelling sinusoidal wave (a "plane wave") and a standing wave are our utmost concerns. A spherical wave or a circular wave is also of our concern, and is a more realistic wave from a small source in three or two dimensions. However, locally, they can be approximated as plane waves.

$$\begin{aligned} D_{\pm}(x,t) &= A\sin(kx \pm \omega t + \phi) && \text{travelling sinusoidal wave} \\ D_s(x,t) &= D_+ + D_- = 2A\sin(kx + \phi)\cos(\omega t) && \text{standing sinusoidal wave} \end{aligned}$$

The travelling sinusoidal wave has a sine shape and it is moving at the **wave speed** $v = \omega/k$ (this formula valid **only for** travelling sinusoidal wave) right ($-$) or left ($+$). The standing sinusoidal wave has a sine shape which is not moving, but experiencing a "time dependent amplitude" $2A\cos(\omega t)$: so it is "breathing," rather than moving. So, its speed[1] is zero. In either case,

$$\lambda = \frac{2\pi}{k} \qquad \text{wave length } (\lambda)\text{, wave number } (k\text{; } \textit{not } k \text{ in Hooke's law force } -kx!)$$

A **transverse wave** (e.g., a string wave) is a wave for which D and the wave velocity (or k, as wave *vector*) are perpendicular to each other, and a **longitudinal wave** (e.g., a sound wave in air/water) is a wave for which D and the wave velocity (or k, as wave *vector*) are along the same axis.

For the above sinusoidal waves (travelling or standing), the local motion, for fixed value of x, is just a SHM (just examine $D_\pm$ and D_s as a function of t)! Physically, this means that a sinusoidal wave is a coordinated SHM, where each *particle* defined by the small segment between x and $x + dx$ is going through a SHM and this motion transpires to its neighbors. For this reason, ω (or f) **is a preserved quantity** when a wave propagates/reflects/refracts/diffracts/etc. in an inhomogeneous medium. The **particle velocity** is given by $\partial D/\partial t$, and should not be confused with the **wave velocity**.

For travelling wave (sinusoidal or not), the **wave speed** is given by $v = \sqrt{F_T/\mu}$ (string wave), or $\sqrt{B/\rho}$ (sound wave in gas/liquid), etc.

Superposition principle is (assumed to be) valid throughout this course. It is best viewed as a mathematical principle. **Interference** can be thought of as a physical realization of the superposition principle: when two waves (D_1 and D_2) meet, then

$$D = D_1 + D_2 \qquad \text{fundamental for any coexisting waves}$$

Energy in a wave (Section T15-3): $E = 2\pi^2 \rho S L f^2 A^2$ (for a travelling sinusoidal wave over a length L; the integral of SHM energy). $P = \Delta E/\Delta t$, and $I = P/S$. These are only for the plane wave. A point source in three dimensions will result in a spherical wave: $I \propto r^{-2}$ since $S = 4\pi r^2$.

Reflection, boundary condition, and phase shift: If D is constrained to be zero (fixed end), then the wave reflects by changing sign (π phase shift). If D has no constraint (free end), then the reflected wave involves no sign change (no phase shift).

Sound wave: $\Delta P = -B\ \partial D/\partial x$. Other general properties of wave, as discussed above, also apply to the sound wave, of course.

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[1] This can be understood as the magnitude of the group velocity, which is out of scope for this course.