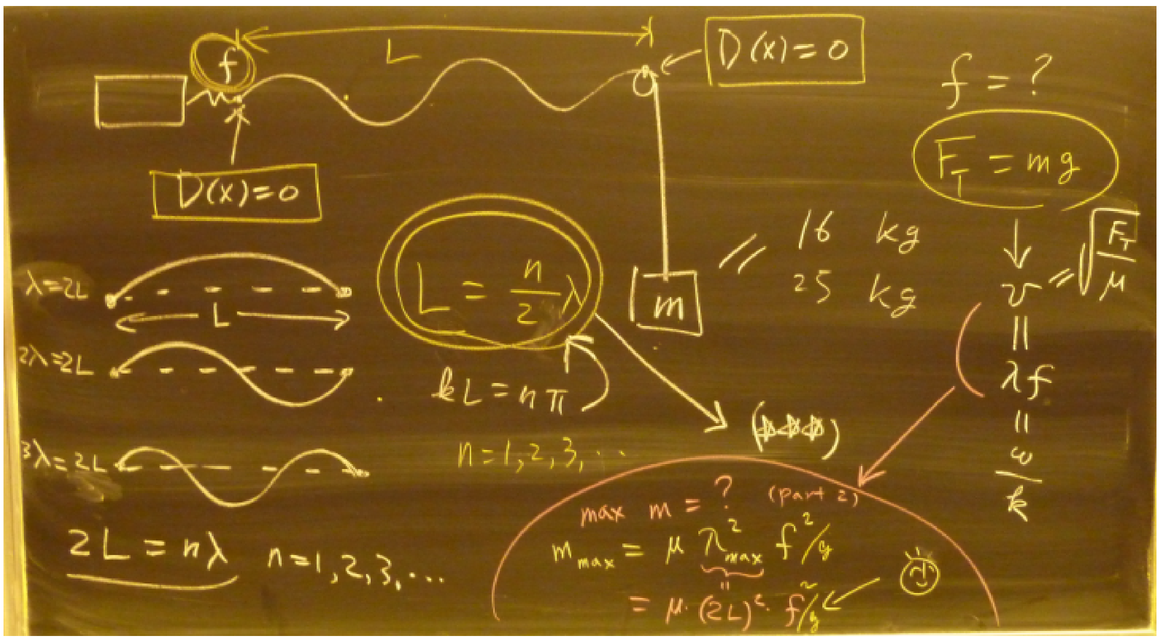
If you are pressed for time, you can leave extra credit problems of homework for future work.

The last problem of the practice exam

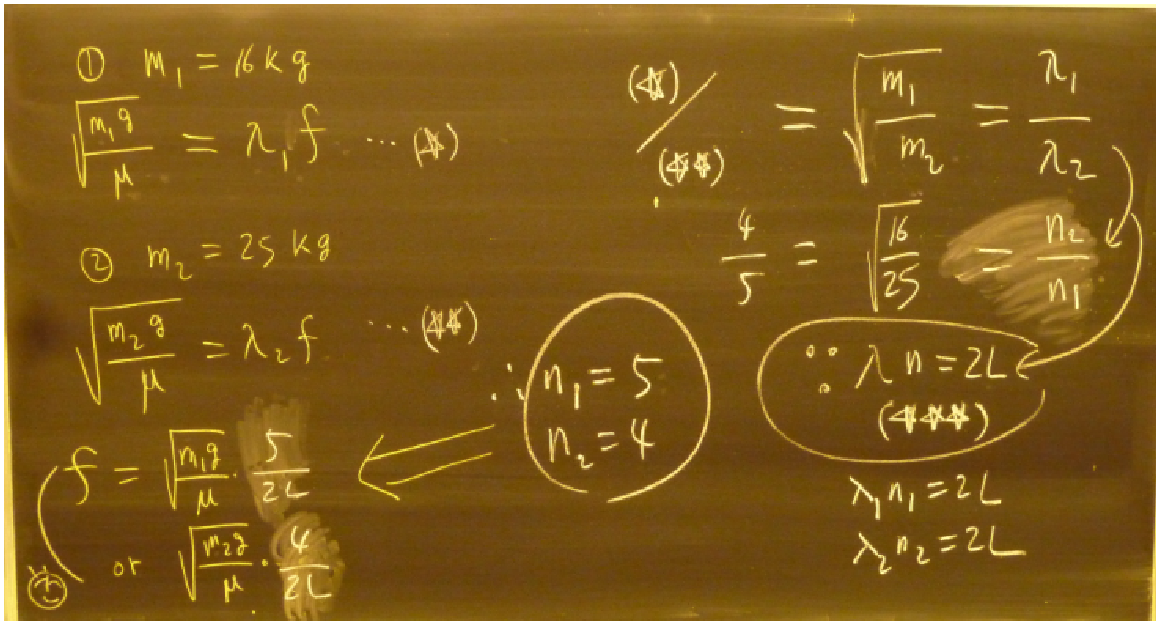
This problem was solved by noting that the wave that can be excited in the horizontal portion of the string with boundary conditions $D(x) = 0$ at both ends correspond to the guitar string problem. As shown on the first image, $2L = n\lambda$ with $n = 1, 2, 3, ...$ was deduced as the condition for the wave to exist under these boundary conditions. Then, it is also noted that $F_T = mg$ and $v = \sqrt{F_T/\mu} = \lambda f$.



Moving to the second image, the equation for F_T and v is used to write down two equations: one for m_1, λ_1 and the other for m_2, λ_2. f is constant since it is given by the machine. As shown on the right side of image, dividing these two equations, we get

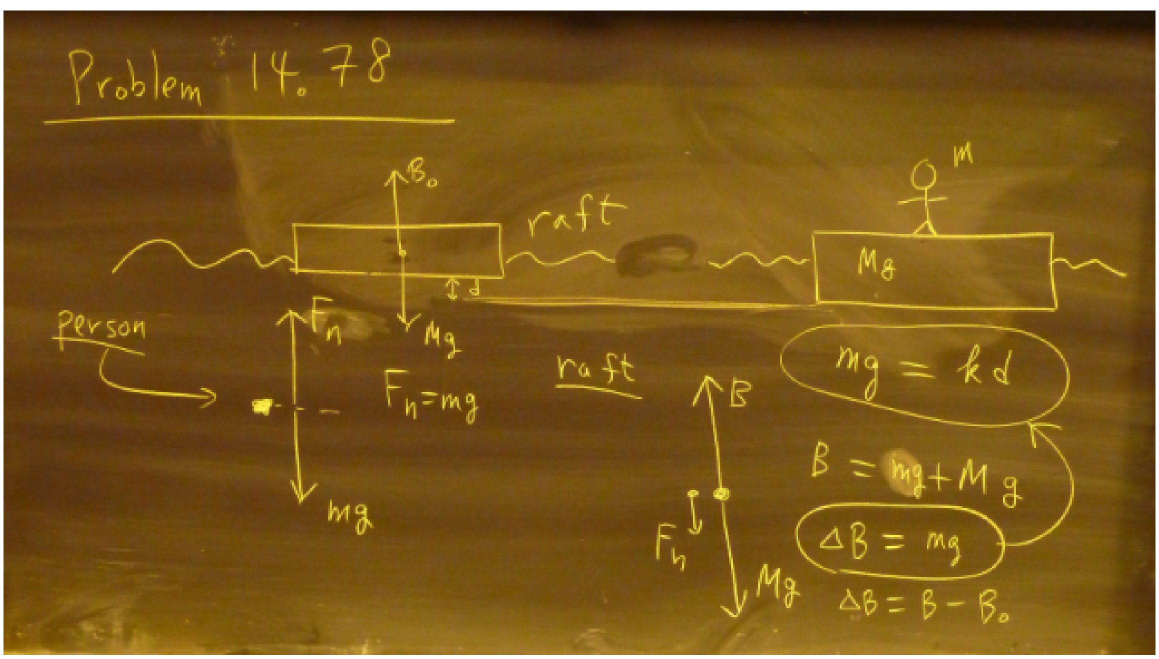
$$\frac{4}{5} = \frac{n_2}{n_1}$$

where $\lambda_1 n_1 = 2L = \lambda_2 n_2$ (see the first image). Even though this equation involves two unknowns, n_1 and n_2, it can be solved to give unique answers $n_1 = 5$ and $n_2 = 4$, since n_1, n_2 are positive integers (first image), and the question states that there is no mode between these two (so, e.g., $n_1 = 50$ and $n_2 = 40$ won't do). Using this result, we get f, using either equation that we set up for two the cases, since f is a constant value imposed by the machine. Using this result for f, we can go back to the first image, and obtain the m_{max} value (lower part with a pink line around) (thanks to R who pointed out some missing part and a typo in this part after the review session!).



Problem 14.78

For doing this problem, the knowledge of the buoyancy force is not really a requirement. The following image shows the setup and two free body diagrams.



Even without free body diagrams, it is possible to write down $mg = kd$. Since the initial state (raft without a person on it) is a stable equilibrium state, the vertical displacement by d must result in a Hooke's law force kd.

Free body diagrams show details.

First, for the person the normal force F_n that the person experiences by the raft must be equal to the person's mg. Due to the above stability argument, F_n must be kd (with the assumption that d is small enough).

Second, the free body diagram for the raft shows the connection of this force to the buoyancy force: B_0 is the buoyancy with no person on the raft, and B is the buoyancy with the person on the raft. This part is not really necessary to derive $mg = kd$: it just shows where kd part comes from – it comes from the buoyancy force.

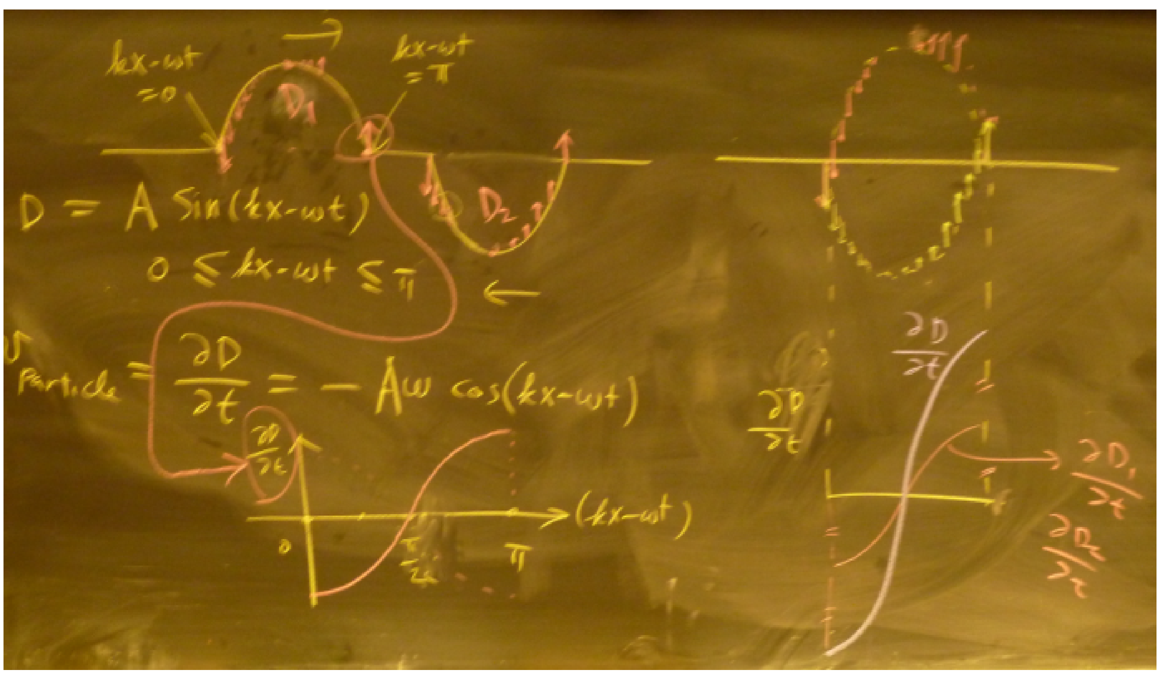
In any case, what is angular frequency at which the raft oscillates, if the person jumps off the raft? It is given by $\omega = \sqrt{k/M} = \sqrt{mg/(dM)}$, where M is the mass of the raft alone.

Practice exam problem 5

For this problem, the choices other than (c,d) are easily ruled out (or they are easily ruled out by discussions below). Note that (c) and (d) refer to the *velocity* of particles ($v_{particle} = \partial D/\partial t$) in the string.

In the image below, small vertical lines indicate the velocity of each part of the string for the counter-propagating identically shaped pulses (taken to be of a half sine pulse here).

In the left half of this image, please replace all D's with D_1's.



On the right half of this image is depicted the situation where the two pulses completely destroy each other. So, $D = D_1 + D_2 = 0$ at this time, where D_1 is the right moving positive pulse and D_2 is the left moving negative pulse. The particle velocity is given by $\frac{\partial D}{\partial t} = \frac{\partial D_1}{\partial t} + \frac{\partial D_2}{\partial t}$: so not only D follows the superposition principle superposes, but also does $\frac{\partial D}{\partial t}$. While the total displacement $D = 0$ (so the string is entirely flat), the total particle velocity at each point is exactly twice that for D_1 or D_2, since the particle velocities for D_1 and D_2 are exactly the same ($\frac{\partial D_1}{\partial t} = \frac{\partial D_2}{\partial t}$; see the figure) for every particle at the moment of the total destructive interference. Also, note that $\frac{\partial D_1}{\partial t} = \frac{\partial D_2}{\partial t} = 0$ exactly at the crest (trough) of D_1 (D_2). The total velocity is zero precisely at this point, and at that point only within the pulse. So, (c) is the answer.

Note that clearly $K = \int \frac{1}{2} dx \mu v_{particle}^2$ is positive at the instant when the two waves have a totally destructive interference. While there is no potential energy at this instant, K accounts for the total energy, which is conserved throughout the motion.