Due May. 22, Thursday

**Problem 1** (30 points) A cylinder is separated into two compartments by a freely sliding piston. Two ideal fermi gases are placed into the two compartments, numbered 1 and 2. The particles in compartment 1 has spin 1/2 and those in compartment 2 has spin 3/2, while all particles have the same mass. Find the equilibrium density ratios of the two gases at $T = 0$ and $T \to \infty$.

**Problem 2** (30 points) Consider a finite-mass non-relativistic ideal Fermi gas at low temperature ($T \ll T_F$).

(a) Find the low temperature expansion for $\alpha$, $\kappa_T$ and $\kappa_S$. These three quantities were defined in Homework 1.2, as well as in LN 2. Similarly as in Eqs. 13.25 and 13.26, you must find your answers up to the first non-trivial temperature dependent term and then indicate the order of the next higher order term neglected. [Hint: Eqs. 13.25, 13.26, and 12.32, along with the basic thermodynamic identities, should be very helpful.]

(b) Show that, by using the results of Homework 1.2,

$$\frac{C_P}{C_V} \approx 1 + \frac{\pi^2}{3}\left(\frac{T}{T_F}\right)^2$$

and

$$\frac{C_P - C_V}{C_V} \approx \frac{\pi^2}{3}\left(\frac{T}{T_F}\right)^2.$$

What is the order of magnitude of the next correction term, $O\left(\frac{T}{T_F}\right)^3$ or $O\left(\frac{T}{T_F}\right)^4$? [This problem shows that $C_P \approx C_V$. This can be shown for the phonon contribution as well.]

**Problem 3** (30 points) Consider a single layer graphene. The electron band structure of this material is peculiar, and is given by

$$\varepsilon_\pm(\vec{k}) = \pm\hbar v|\vec{k}|,$$

sufficiently close to the so-called Dirac point (defined by $\varepsilon_\pm(\vec{k}) = 0$). The spin degenerarcy $g = 2$ and $v$ is a speed scale, $\approx c/300$. Since graphene is a two dimensional crystal, $\vec{k}$ is a two dimensional vector. The energy dispersion given above is often described as consisting of an upper Dirac cone ($\varepsilon_+$) and a lower Dirac cone ($\varepsilon_-$). We shall consider the above dispersion around one Diract point only (in a real graphene there are two distinct Dirac points per unit cell in $\vec{k}$ space, and the generalization of our results to that case is straightforward).

At $T = 0$, the lower Dirac cone is completely filled with electrons, while the upper Dirac cone is completely empty.

(a) Find the chemical potential any temperature, $\mu(T)$.

(b) Show that

$$E(T) - E(0) = 4A \int \frac{d^2\vec{k}}{(2\pi)^2} \frac{\varepsilon_+(\vec{k})}{e^{\beta\varepsilon_+(\vec{k})} + 1},$$

where $A$ is the total area of the graphene sheet.

(c) Give a closed form answer for the energy by evaluating the above integral.

(d) Calcuate the heat capacity $C_V$.

(e) Explain qualitatively the contribution of phonons to the heat capacity of graphene. The typical sound velocity in graphene is of the order of $2 \times 10^4$ m/s. Is the low temperature heat capacity dominated by the phonon contribution or the electron contribution?

**Problem 4** (30 points) Numerical estimates.

(a) Give nuermical estimates for the Fermi temperature of (i) electrons in a typical metal (this is just a reminder—it is given in LN 13), (ii) nucleons in a heavy nucleus, (iii) $He^3$ atoms in liquid $He^3$ (volume = 46.2 Å$^3$ per atom).

(b) Estimate the ratio of the electron heat capacity and the phonon heat capacity at room temperature for a typical metal.

(c) Estimate the "degenerarcy discriminant," $n\lambda^3$, for hydrogen, helium, and oxygen gases at room temperature and pressure. At what temperatures does quantum degenerarcy become important for these gases?