Due June 15, Friday

Problem 1 (30 points) For the given Landau free energy of Eq. 16.31, calculate the magnetization per spin, the magnetic susceptibility, and the specific heat, below and above T_c, in terms of a, b_0, and c. Assuming that $a(T)$ and $c(T)$ are smooth and finite at T_c. Also, note the conditions stated in Eqs. 16.29, 16.30.

This problem is basically reproducing Lecture 16, but only using the Landau free energy. This means that the physical quantities are calculated in terms of abstract parameters a, b_0 and c, and they are calculated only for T near T_c. The calculated results must be verified to reproduce the results of Lecture 16 (Eqs. 16.17, 16.22, 16.23, 16.27), by appropriately identifying a, b_0 and c for the Ising model case.

Problem 2 (50 points) Construct a Metropolis Monte Carlo code for a two dimensional Ising magnet with the periodic boundary condition. (A partial python example is provided on the homework page of the course web site: it should be enough for part (a), and should be easily extensible for parts (b,c).) In the following, assume a ferromagnetic model, except in (b).

(a) Pick at least three temperatures, including one well above T_c, one near T_c, and one well below T_c. Start from the initial configuration where the system is magnetized up on the left half and down on the right half. Run the same number of runs (suggestion: ~ 2000 for 40×40 lattice). Compare the final configurations for different temperatures and discuss the results.

(b) Do the same for the anti-ferromagnetic model ($J < 0$). Discuss the results.

(c) Measure the magnetization and the spin correlation function

$$\Gamma_{ij} \equiv \overline{\langle \sigma_i \sigma_j \rangle} - \overline{\langle \sigma_i \rangle}\,\overline{\langle \sigma_j \rangle}$$

Examine these quantities at least for three temperatures. Try to demonstrate the existence of long range correlations near the critical point, and the spontaneous symmetry breaking below T_c.

Problem 3 (40 points) The renormalization group (RG) equation that we set up for the one dimensional Ising model, in Section 18.2.1, can be improved so that the equation applies to the anti-ferromagnetic case ($J < 0$) as well as the ferromagnetic case ($J > 0$).

(a) By constructing such an improved RG equation (hint: e.g., integrate two spins for every three spins), show that $K = J\beta = \pm\infty, 0$ are all fixed points, now.

(b) Calculate, numerically, about 20 RG iterations, starting from $K = \pm 10$, $K = \pm 5$. At each iteration, print out the value of the running constant K as well as the quantity g (Eq. 18.43), which can be compared with the exact result (which can be deduced directly from Eq. 17.25). For the initial value of g, an approximate form of the partition function for $T \approx 0$ (corresponding to the energy minimum) can be used.

(c) Do similar numerical calculations, but this time, starting from $K = \pm 0.001$, and applying the RG equation *backwards*. For the initial value of g, an approximate form of the partition function for $T \approx \infty$ (corresponding to the entropy maximum) can be used.

(d) Discuss your results in terms of how error for g propagates in case (c) or (d). Make an observation as to which scheme, (c) or (d), is more viable for calculating the free energy using the renormalization group technique.