— Easy things first

$$dE = -p\,dV + T\,dS + \mu\,dN$$

$$F = E - TS \qquad \underline{H = E + pV}$$

$$G = E - TS + pV$$

$$\mathcal{G} = E - TS - \mu N$$

Extensivity (local interaction) : $\boxed{E = \mu N - pV + TS}$

$$G = \mu N$$

$$\mathcal{G} = -pV$$

Probability cumulant expansion

LN5 : ~~law of~~ rule of large #'s

Stirling

Boltzmann : $f_1 = \left\langle \sum_{i=1}^{N} \delta(\vec{p} - \vec{p}_i)\,\delta(\vec{q} - \vec{q}_i) \right\rangle$

$$\int d^3\vec{q}\, f_1(\vec{q}) = N$$

$$\frac{df_1}{dt} = \left.\frac{df_1}{dt}\right|_{coll} = -\frac{f_1^0 g}{\tau_x}$$

$$f_1' = f_1^0(1+g)$$

$$f_1(\vec{p},\vec{q},t) = N\,\rho_1(\vec{p},\vec{q},t)$$

$$\therefore \int d^3\vec{p}\,d^3\vec{q}\; f_1(\vec{p},\vec{q},t) = N$$

$$\therefore \int d^3\vec{p}\; \underbrace{f_1(\vec{p},\vec{q},t)}_{} = n$$

normalized to n in the momentum space

locally

$$Z = \sum_{\mu_s} e^{-\beta E(\mu_s)} \Rightarrow \int \frac{d\vec{p}\,d\vec{q}}{N!\;h^{3N}}$$

(p, q)

if identical and exchanging scattering.

$$P(\mu_s) = \frac{e^{-\beta E(\mu_s)}}{Z}$$

CN 7

$$Z = \sum_{E} \Omega(E_s)\, e^{-\beta E(\mu_s)}$$

$k_B \log \Omega = S$

$$= \sum_{E} e^{-\beta(E - TS(E))} \qquad \Omega = e^{\frac{S}{k_B}}$$

$Z \Rightarrow \dfrac{Z(\beta)}{Z(\gamma)}$ is the characteristic function

$$\Rightarrow \boxed{\left.\frac{\partial^n \log Z}{\partial(-\beta)^n}\right|_x = \langle E^n \rangle_c}$$

$$\langle E^2 \rangle_c = k_B T^2 C_x$$

$$F = -k_B T \log Z \qquad = \left.\frac{\partial \langle E \rangle_c}{\partial(-\beta)}\right|_x$$

Grand canonical

$$\mathcal{Q} = \sum_{\mu_s} e^{-\beta(E-\mu N)}$$

$$\mathcal{G} = \cancel{PV}\; E - TS - \mu N = -k_B T \log \mathcal{Q}$$

$$\langle N^n \rangle_c = \frac{1}{\beta^n} \frac{\partial^n \log \mathcal{Q}}{\partial \mu^n} \qquad (= PV)$$

Gibbs $\quad \mathcal{Z} = \sum e^{-\beta(E - Jx)}$

$$\langle x^n \rangle_c = \frac{1}{\beta^n} \frac{\partial^n \log \mathcal{Z}}{\partial J^n}$$

$$G = -k_B T \log \mathcal{Z} \quad (= \mu N)$$

$$\langle H^n \rangle_c = \frac{\partial^n \log \mathcal{Z}}{\partial(-\beta)^n}$$

Classical ideal gas

$$Z = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$$ derive

$N! \approx N^N e^{-N}$

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

$n = \frac{e^{\beta\mu}}{\lambda^3} \rightarrow n\lambda^3 = e^{\beta\mu}$

ideal gas classical. trivial

Interacting particles

$$Q = 1 + e^{\beta\mu}\frac{V}{\lambda^3}$$

$$Q = \sum_{N=0} e^{\beta\mu N} Z(N,T,V)$$

how to derive

$$N = \frac{\partial \log Q}{\partial(\beta\mu)}$$

$$\mathcal{G} = -PV = -k_B T \log Q$$

$$\frac{e^{\beta\mu}\frac{V}{\lambda^3}}{1+e^{\beta\mu}\frac{V}{\lambda^3}} = N$$

small

$N = 0, 1, 2 \rightarrow$ will give you result!

small

$$(P + an^2)(V - bN) \approx N k_B T$$

$$a = \frac{u_0 \Omega}{2} \qquad b = \frac{\Omega}{2}$$

Isotherms

C-M

Critical behaviors

"Symmetry breaking"

LN 10 → Scattering → correlation ftn
→ all info ∈ in CM!

Quantum → Equipartition theorem helps!!
qualitatively!

Know your ~~distribution ftn~~
~~occupancation~~ #! !

$$n_p = \frac{1}{e^{\beta\varepsilon} - 1}$$

$$n_{BE} = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1} = \frac{1}{z^{-1}e^{\beta\varepsilon} - 1}$$

$$n_{FD} = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1} = \frac{1}{z^{-1}e^{\beta\varepsilon} + 1}$$

- Non-interacting only in this class.
- But quantum statistics play the role of interaction!
- Dispersion relation and dimensionality is important!
- Equipartition theorem still rules!
- DOS must be mastered!

if #'s are fixed.

$$N = \int d\varepsilon\, D(\varepsilon)\, n(\varepsilon)$$

$$E = \int d\varepsilon\, D(\varepsilon)\, \varepsilon\, n(\varepsilon)$$

$$P = \frac{\alpha}{d} \frac{E}{V}$$

Classical limit, Quantum limit

$$z = e^{\beta\mu} \to -\infty \text{ in CM.}$$

$$\to 1, \infty \text{ in Q.}$$

classical limit

$$P = n + B_2 n^2$$

$B_2 > 0$ fermion

< 0 boson

Q limit

Fermi gas

$$\int_{-\infty}^{\infty} H(\varepsilon) f(\varepsilon)\, d\varepsilon \approx \int_{-\infty}^{\mu} H(\varepsilon)\, d\varepsilon + \frac{\pi^2}{6} H'(\mu)\, (k_B T)^2$$

S. expansion

Bose gas

$$g_m(z) = \sum_{n=1}^{\infty} \frac{z^m}{n^m} = \frac{1}{(m-1)!}\int_0^\infty dx \frac{x^{m-1}}{z^{-1}e^x - 1}$$

$$n = \frac{1}{\lambda^3} g_{\frac{3}{2}}(z) + \frac{n(0)}{V}$$

$$\frac{E}{V} = \frac{3 k_B T}{2\lambda^3} g_{\frac{5}{2}}(z)$$

$$P = \frac{k_B T}{\lambda^3} g_{\frac{5}{2}}(z)$$

BEC $\boxed{n = \frac{1}{\lambda^3} g_{\frac{3}{2}}(1)}$

LN 15. "Core Results"

Understand power laws

isotherms $\frac{b_0}{2}(T-T_c)m^2 + \frac{b}{4}m^4$

Ising model, MFT, Landau ~~form~~ free energy

must be expert!

$$H = -J\sum_{\langle i,j\rangle} \sigma_i \sigma_j - h\mu_B \sum_i \sigma_i$$

$\langle \sigma_i \rangle = \tanh \beta \mu$

Must

MFT $H_i = -J \sum_{j=n.n.} \langle \sigma_j \rangle \sigma_i$

$(h_{tot} = h + \frac{J}{\mu} \sum_j \langle \sigma_j \rangle$

phase transition

Curie Weiss law