Homework 1

Phys 155,   Win 2012.
Due Jan 19, Thursday.
NOTE: 3 and 4 are postponed to homework #2.

1. [10 pts] Consider an infinite potential well problem in one dimension: a particle with mass m is found inside a potential energy, $V(x)$, defined as:

$$V(x) = 0 \quad \text{if} \quad |x| < a$$
$$V(x) = \infty \quad \text{if} \quad |x| > a$$

Use the semi-classical Bohr-Sommerfeld quantization rule to find the quantized energy levels for this particle. Compare your result with the fully quantum mechanical result (your modern physics book or quantum mechanics book should have it; you can use Google if you do not have any other reference).

2. [20 pts] A useful way to *estimate* the energy of a quantum mechanical system is to use the Heisenberg uncertainty principle on top of what you know from classical mechanics. In each of the problems below, use

$$\Delta x \, \Delta p \sim \frac{\hbar}{2}$$

to estimate the *ground* state energy. Here, you can use the virial theorem ($\langle T \rangle = \frac{n}{2}\langle V \rangle$ if $V \propto r^n$ where T is the kinetic energy, V the potential energy, and $\langle \rangle$ means average over one period of motion), but *not* the Bohr-Sommerfeld quantization condition. [This problem is an estimation problem and so your answer is to be ambiguous up to a multiplicative factor of $O(1)$.]

   a. A hydrogen-like problem with nuclear charge Ze, electron charge $-e$, electron mass m^*, and the dielectric constant ε. Find the ground state energy expression and find its numerical value (in eV) for $Z = 1$, $m^* = 0.1\, m_e$ (m_e is the bare electron mass), and $\varepsilon = 10$ (as relevant for an electron bound to an impurity in silicon).
   b. A simple harmonic oscillator with mass m and spring constant k. Find the expression for the ground state energy.

3. [20 pts] Consider two dimensional lattices.
   a. For a square lattice with lattice constant a, show that the distance d between the $(h\ k)$ set of lattice lines is given by

$$d = \frac{a}{\sqrt{h^2 + k^2}}$$

b. For a rectangular lattice with lattice constants a and b, which are different from each other, find a similar expression for d between the $(h\ k)$ set of lattice lines in terms of a, b, h, k.

4. [10 pts] Consider the planes with Miller indices (100) and (110): the lattice is bcc, and the indices refer to the conventional cubic cell with lattice constant a. What are the Miller indices of these planes when referred to the primitive axes defined by the three primitive vectors $\vec{a} = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z})$, $\vec{b} = \frac{1}{2}a(-\hat{x} + \hat{y} + \hat{z})$, $\vec{c} = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z})$?

5. [20 pts] The *packing fraction* is the maximum proportion of the available volume that can be filled with hard spheres for a given structure. Show that it is $\frac{\pi}{6} = 0.524$ for a simple cubic (sc) structure, $\frac{\sqrt{3}}{8}\pi = 0.680$ for a body centered cubic (bcc) structure, and $\frac{\sqrt{2}}{6}\pi = 0.740$ for a face centered cubic (fcc) structure. Lastly, without doing any further calculation, but using one of these results, find the packing fraction for the ideal hexagonal close packed (hcp) structure ($c = \sqrt{\frac{8}{3}}a$).