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1 problem, 15 minutes.

Problem 1 Consider a Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_1$, where $\hat{H}_0$ is the unperturbed Hamiltonian and $\hat{H}_1$ is a small perturbation ($O(\delta)$). Let us assume that

$$\hat{H}_0 \left| n^{(0)} \right\rangle = E_n^{(0)} \left| n^{(0)} \right\rangle,$$

where $n = 0, 1, 2, 3, \ldots$ in the order of increasing energy. Let us suppose that, the first two excited states ($n = 1$ and $n = 2$) are highly interesting to us. So, we like to describe the leading order effects of the perturbation on these two states. *Assume* that all other states have zeroth order energy values that are distinct from both $E_1^{(0)}$ and $E_2^{(0)}$. Answer the following questions.

(a) $E_1^{(0)} = E_2^{(0)}$. Which theory should we use to find the perturbation correction to $E_1^{(0)}$ and $E_2^{(0)}$: degenerate theory or non-degenerate theory? No explanation necessary.

(b) $E_1^{(0)} \approx E_2^{(0)}$, by which we mean that $|E_1^{(0)} - E_2^{(0)}| = O(\delta)$. Which theory should we use to find the perturbation correction to $E_1^{(0)}$ and $E_2^{(0)}$: degenerate theory or non-degenerate theory? No explanation necessary.

(c) $|E_1^{(0)} - E_2^{(0)}| \gg O(\delta)$. Which theory should we use to find the perturbation correction to $E_1^{(0)}$ and $E_2^{(0)}$: degenerate theory or non-degenerate theory? No explanation necessary.

(d) For case (a), find the leading corrections to the energy value $E_1^{(0)} = E_2^{(0)}$. Assume that $\Delta \equiv \left\langle 1^{(0)} \left| \hat{H}_1 \right| 2^{(0)} \right\rangle > 0$. Derivation *is* necessary.

(e) Continuing the previous part, find the eigenstate for the *lower* energy state of the two states under consideration. Note that we assumed $\Delta > 0$. Derivation *is* necessary.

Your name: