Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on *that* sheet!
You can keep this sheet, or you can use it as an extra sheet, if necessary.

1 problem, 15 minutes.

Problem 1 Consider a particle with spin S (so, S is one of $0, 1/2, 1, 3/2, ...$). Prove the following statement if it is true, or provide a counter example if it is not true.

$$\hat{\mathcal{R}}(360^\circ)\,|\chi\rangle \;=\; (-1)^{2S}\,|\chi\rangle\,,$$

where $|\chi\rangle$ is *any* possible spin state of the particle, and $\hat{\mathcal{R}}(\Delta\theta)$ is the operator that rotates the spin state by $\Delta\theta$ around an arbitrary axis, which you can define as the z axis.

Hint: (1) You can provide a good, complete, and short, argument if you start by examining the result of applying the rotation on each S_z eigenstate, and use the natural basis nature of those eigenstates. (2) $f(\hat{S}_z)\,|S_z\rangle = f(S_z)\,|S_z\rangle$ for *any* function f of an *operator* $\hat{S}_z$.

Your name: