Due Nov. 21, Thursday

Problem 1 (30 points) Let us consider a perturbation potential

$$\hat{H}_1(t) = \hat{V}\,\theta(t)$$

where $\theta(t)$ is the Heaviside step function. This $\theta(t)$ simply means that the perturbation is turned on at $t = 0$. Therefore, this problem simply corresponds to Eq. 13.12 with $\omega = 0$ with the change $\hat{V} + \hat{V}^\dagger \to \hat{V}$ (which is now Hermitian). Now, Eq. 13.16 is no longer applicable, while Eq. 13.15 is still applicable if the change $\hat{V} + \hat{V}^\dagger \to \hat{V}$ is taken into account, in addition to putting $\omega \to 0$.

(a) Show that

$$C_1(t) \approx \frac{V_{1,0}}{\hbar\omega_N} \sin\left(\frac{\omega_N t}{2}\right)\left(-2ie^{\frac{i\omega_N t}{2}}\right)$$

(b) Find the transition probability $P_{0\to1}(t)$.

(c) Consider an electron spin interacting with a perturbing uniform magnetic field which is pointing in the x direction. Assume that no orbital angular momentum is involved. Find the exact solutions for $C_1(t)$ and $P_{0\to1}(t)$ compare them with the above perturbative solutions. $\hat{H}_0 \doteq \mu_B B_0 \sigma_z$, $\hat{H}_1 \doteq \mu_B B_1 \sigma_x \theta(t)$, and at $t = 0$ the spin is z down.

(d) Show that the time-energy uncertain principle $\Delta E \Delta t \sim \hbar$ is valid in each case, and discuss the proper meaning of this principle in each case.

Problem 2 (30 points) Rabi oscillation and comparison to perturbation. Problem T9.7.

Problem 3 (10 points + 20 extra credit points) Prove Eq. 13.28, assuming that Eq. 13.27 is true. If you prove Eq. 13.27, in addition, then you will get additional 20 points of extra credit.

Problem 4 (30 points) Prove points 1, 2, and 3 at top of page 5 of LN 13. For point 3, prove it for $j = 0, 1, 2$. Note that for point 3, j means the order of the perturbation for $|C_n|^2$, *not* for C_n. Note that proving points 2 (for $n = 0$) and 3 are not trivial.

Problem 5 (20 points) Problem T9.18.