Due Nov. 12, Tuesday

This homework covers Lectures 9, 10, and 11, which will *not* be included in the midterm, to be held on November 5 during our normal class hours. The midterm will cover all materials from Lecture 1 through Lecture 8, and Homework 1 through Homework 4. While this homework is important, please dedicate yourself to this homework after the midterm, rather than before.

Problem 1 (20 points) Problem T7.2, Variational principle and SHO.

Problem 2 (20 points) Problem T7.5, Variational principle and perturbation.

Problem 3 (30 points) Problem T7.14, Yukawa potential, variational principle.

Problem 4 (30 points) **Continuity equation, probability current.**

(a) Consider a one dimensional flow of *classical mechanical* particles. Let the density of particles be $\rho(x,t)$ and the velocity of the flow be $v(x,t)$. Assuming that particles are conserved *locally*, i.e. they do not appear out of nowhere and they do not disappear into nowhere, show that

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0 \qquad\qquad \text{where } j = \rho v$$

Hint: consider a very small segment of x and account for particles coming in and going out during time dt.

(b) Show that in three dimensions

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0 \qquad\qquad \text{where } \vec{j} = \rho \vec{v}$$

Hint: same hint as in the previous part, except that you need to consider a small volume element and use the divergence theorem.

(c) In quantum mechanics, we can define an analogous quantity, j (or $\vec{j}$), but *only through* the continuity equation, not the other way around. Consider the one dimensional Schrödinger equation

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t)$$

Find the expression for $\frac{\partial \rho}{\partial t}$ where $\rho \equiv \Psi^* \Psi$ is now the probability density. Your expression must turn into the continuity equation of part (a). Show that the *probability current* can be defined as

$$j = \frac{i\hbar}{2m}\left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x}\right)$$

(d) Take the WKB wave function, Eq. 10.9. Calculate the probability density and the probabilty current for *each* term of that wave function (i.e. consider one of A, B to be zero), in the classical region ($E > V(x)$). Show that they correspond to a constant probabilty current with the probability density $\propto 1/p(x)$. Are these what you expected? Explain.

Problem 5 (20 points) Problem T8.3, Tunneling, WKB approximation.

Problem 6 (extra credit; 30 points) Problem T8.10, Tunneling, WKB approximation.

Problem 7 (20 points) Problem T8.7, Bohr-Sommerfeld quantization rule, WKB approximation.