Due Oct. 30, Wednesday

Problem 1 (30 points) Consider a particle in three dimensions. Consider three possible potential energies $V(\vec{x})$ as below. In each case, (1) indicate which of the following nine symmetries exist for the Hamiltonian of the system: translational symmetries along three axes, rotational symmetries around three axes, parity symmetries (reflection symmetries) for three axes, (2) group symmetry operators into compatible ones (here the word "group" is used as an everyday word, not denoting a mathematical group; officially, we never mention mathematical group in this class[1]) and (3) find the *most general form* of the energy eigenstate as determined by each group of compatible symmetry operators.

(a) $V(\vec{x}) = 0$ (free particle) (For part (2), there are many groups—14 groups—of compatible symmetry operators. If you identify 10 groups correctly, then you will be given full points for part (2).)

(b) $V(\vec{x}) = V(r)$ (central potential)

(c) $V(\vec{x}) = V(z)$

Problem 2 (30 points; Linear Stark effect) A uniform electric field is applied to a Hydrogen-like atom. We shall consider the effect of the electric field on $2s$ and $2p$ orbitals. Excluding the spin degeneracy, we have 4 orbitals to consider. The three $2p$ orbitals can be represented either as p_x, p_y, p_z orbitals (Eq. 6.23) or as spherical harmonics, whichever is more convenient for you. The perturbing Hamiltonian is given by

$$\hat{H}_1 = e\hat{z}E_0$$

where the uniform electric field is taken to be along the z direction: $\vec{E} = E_0\vec{e}_3$. Assume that the effect of the electric field dominates over the fine structure effect, which you can ignore. That is, the unperturbed Hamiltonian is given by Eq. 6.12.

(a) Construct the 4×4 matrix of $\hat{H}_1$ in the Hilbert subspace spanned by $2s$ and $2p$ states.

(b) By carrying out the degenerate perturbation theory, i.e. by diagonalizing the matrix you just obtained, obtain the first order correction in energy and the *good* zeroth order energy eigenstates.

[1] However, if you are really interested, then this is how you can obtain the mathematical group. Include all symmetry operations—for instance, if rotation is considered, include not just rotations around special axes such as x, y, or z, but all possible combined rotations involving any angular amounts. Add the identity operation, if it is not already included. Such a set will form a group, mathematically speaking.

(c) Let us assume that there is one electron to occupy $2sp$ orbitals (so we are considering a Li atom). Find the induced electric dipole moment

$$\vec{p} \equiv \left\langle -e\hat{\vec{r}} \right\rangle$$

and the polarizability α_p in

$$\vec{p} = \alpha_p \vec{E}.$$

(d) Find the condition for E_0 in terms of its order of magnitude (in unit of V/m for example), which (1) satisfies our assumption that the Stark effect dominates over the fine structure effect, and (2) ensures that the Stark effect is small compared with the unperturbed energy level difference between $1s$ and $2sp$.

[Hint: for the evaluation of (off-diagonal) matrix elements, you would need to use Tables T4.3 and T4.7.]

Problem 3 (10 points) Problem T6.16, basic commutators.

Problem 4 (20 points) Problem T6.18, fine structure splitting of the Balmer line.

Problem 5 (20 points) Problem T6.21, $2sp$ states, weak Zeeman.

Problem 6 (30 points) Problem T6.23, $2sp$ states, strong Zeeman.

Problem 7 (30 points; extra credit) Problem T6.31, van der Waals interaction, simplified.

Please start reviewing all materials that we covered in this class so far.
