Due Oct. 15, Tuesday

Problem 1 (20 points) Consider the following simple Hamiltonian matrix (representation in some basis)

$$H = \begin{pmatrix} E_1 & \Delta \\ \Delta^* & E_2 \end{pmatrix}$$

where Δ is a complex number. [Note: despite the simple form of the Hamiltonian, this Hamiltonian is *very* important!]

(a) Take $\left(\begin{smallmatrix} E_1 & 0 \\ 0 & E_2 \end{smallmatrix}\right)$ as representing the unperturbed Hamiltonian, and $\left(\begin{smallmatrix} 0 & \Delta \\ \Delta^* & 0 \end{smallmatrix}\right)$ as representing the perturbation. Assuming $E_1 \neq E_2$, find the perturbative solutions up to first order for eigenstates and up to second order for eigenvalues.

(b) Assuming $E_1 = E_2$, find perturbative solutions using the degenerate perturbation theory: zeroth order solutions for eigenstates and first order solutions for eigenvalues.

(c) Find the exact eigenvalues for any values of E_1 and E_2. Show that, under a suitable assumption, they lead to your answer in part ((a)).

(d) If $E_1 \neq E_2$, but $|E_1 - E_2| \ll |\Delta|$, which approach gives a better approximation: approach of part ((a)) or that of part ((b))?

Problem 2 (20 points) Consider the Larmor precession example of Section 4.2.2 of LN 4. Find the exact solution to this problem for the total Hamiltonian, $\hat{H} = \hat{H}_0 + \hat{H}_1$, and show that you retrieve the perturbative solutions for the eigenstate (up to first order) and the eigenvalues (up to second order) that we obtained in class, by expanding the exact solution in powers of perturbation parameters, of which there are two. In writing down your exact solution, it is recommended that you use symbol θ defined as $\cos\theta \equiv \frac{B_0+B_1}{B_{tot}}$ and $\sin\theta \equiv \frac{B_2}{B_{tot}}$, where B_{tot} is the magnitude of the sum of all $\vec{B}$ fields.

Problem 3 (20 points) Griffiths, Problem 6.2 + Problem 6.4(b).

Problem 4 (10 points) In the lecture note (LN 3), the general result for the $(j+1)$-th correction to the energy eigenvalue and the energy eigenstate is given in an iterative form in Eqs. 3.19 and 3.20. Note that $j \geq 0$ in these equations. Using these equations as given, prove explicitly that

$$\left\langle n^{(0)} \middle| n \right\rangle = 1$$

by showing that

$$\left\langle n^{(0)} \right| \left(\hat{H}_1 - \Delta_n\right) |n\rangle = 0,$$

where $|n\rangle = |n^{(0)}\rangle + |n^{(1)}\rangle + |n^{(2)}\rangle + ...$, as defined during class. This proof is described briefly in the lecture note—you must provide all details.

Problem 5 (20 points) Griffiths, Problem 6.5.

Problem 6 (20 points) Griffiths, Problem 6.9.

Problem 7 (20 points) Griffiths, Problem 6.30.

Problem 8 (20 points; extra credit) Griffiths, Problem 6.32.