TISE = time independent Schrödinger equation. SS = stationary state. 1D = one dimension.

$$\hat{H}\,|\Psi(t)\rangle = i\hbar\frac{d}{dt}\,|\Psi(t)\rangle \qquad \text{Schrödinger equation} \qquad (1)$$

$$H(\hat{x},\hat{p},t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \qquad \text{Schrödinger eq., } x\text{-repr, spinless particle in 1D} \qquad (2)$$

$$\hat{H}\,|\psi\rangle = E\,|\psi\rangle \qquad \text{TISE: } \frac{\partial \hat{H}}{\partial t} = 0,\ \text{SS, } |\Psi(t)\rangle \equiv \exp(-iEt/\hbar)\,|\psi\rangle \qquad (3)$$

$$\hat{\mathcal{T}}(\Delta x) = \exp\left(-i\frac{\Delta x\,\hat{p}}{\hbar}\right) \qquad \text{translation operator in 1D} \qquad (4)$$

$$\hat{\mathcal{R}}(\Delta\theta) = \exp\left(-i\frac{\Delta\theta\,\hat{L}_\theta}{\hbar}\right) \qquad \text{rotation operator} \qquad (5)$$

$$\left[\hat{x},\hat{p}\right] = i\hbar \qquad \text{also for other canonical conjugate pairs} \qquad (6)$$

$$\left[\hat{\theta},\hat{L}_\theta\right] = i\hbar \qquad \text{also for other canonical conjugate pairs} \qquad (7)$$

$$\left[\hat{L}_j,\hat{L}_k\right] = i\hbar\epsilon_{jkl}\hat{L}_l \qquad \text{similarly, if all } L\to S \text{ or if all } L\to J \qquad (8)$$

$$\left[\hat{L}_j,\hat{p}_k\right] = i\hbar\epsilon_{jkl}\hat{p}_l \qquad j,k,l = \text{any of } 1,2,3 \text{ (or } x,y,z) \qquad (9)$$

$$\left[\hat{L}_j,\hat{x}_k\right] = i\hbar\epsilon_{jkl}\hat{x}_l \qquad \epsilon_{jkl} = \text{Levi-Civita symbol} \qquad (10)$$

$$\left[\hat{L}_j,\hat{L}^2\right] = 0 \qquad \text{similarly, if all } L\to S \text{ or if all } L\to J \qquad (11)$$

$$\left[\hat{L}_j,\hat{p}^2\right] = 0 = \left[\hat{L}_j,\hat{r}^2\right] \qquad \hat{p}^2 \equiv \hat{p}_x^2+\hat{p}_y^2+\hat{p}_z^2,\quad \hat{r}^2 \equiv \hat{x}^2+\hat{y}^2+\hat{z}^2 \qquad (12)$$

$$\left[\hat{L}_j,\hat{S}_k\right] = 0 \qquad \hat{L} \text{ and } \hat{S} \text{ live in orthogonal Hilbert spaces} \qquad (13)$$

$$\hat{H} = -\hat{\vec{\mu}}\cdot\hat{\vec{B}} \qquad \text{for magnetic moment } \vec{\mu} \text{ in a } \vec{B} \text{ field} \qquad (14)$$

$$\hat{\vec{\mu}} = -\frac{\mu_B}{\hbar}(2\hat{\vec{S}}+\hat{\vec{L}}) \qquad \text{for electron } (g_{spin}\approx 2),\ \mu_B = \frac{e\hbar}{2m_e} = \text{Bohr magneton} \qquad (15)$$

$$\left[\hat{A},f(\hat{B})\right] = 0 \ \text{ if }\ \left[\hat{A},\hat{B}\right] = 0 \qquad \text{for any analytic function } f \qquad (16)$$

$$f(\hat{B})\,|B\rangle = f(B)\,|B\rangle \ \text{ if }\ \hat{B}\,|B\rangle = B\,|B\rangle \qquad \text{for any analytic function } f \qquad (17)$$

$$\langle\hat{T}\rangle = \frac{n}{2}\langle\hat{V}\rangle \qquad \text{Virial theorem for } \hat{V}\propto\hat{x}^n,\hat{r}^n,\ \text{SS} \qquad (18)$$

$$\int_{-\infty}^{\infty} dx\,e^{i(k-k')x} = 2\pi\delta(k-k') \qquad \delta(k) = \text{Dirac delta function} \qquad (19)$$

$$\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \text{Pauli matrices, spin 1/2, } \hat{S}_j \doteq \frac{\hbar}{2}\sigma_j \qquad (20)$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}+\hat{a}^\dagger\right) \qquad \text{1D simple harmonic oscillator} \qquad (21)$$

$$\hat{p} = \frac{1}{i}\sqrt{\frac{m\hbar\omega}{2}}\left(\hat{a}-\hat{a}^\dagger\right) \qquad \text{one-dimensional (1D) SHO} \qquad (22)$$

$$\hat{a}\,|n\rangle = \sqrt{n}\,|n-1\rangle,\quad \hat{a}^\dagger\,|n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad \text{1D SHO, “ladder” operators} \qquad (23)$$