Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

1 problem, 15 minutes.

Problem 1 Consider the following Hamiltonian.

$$\hat{H} \doteq \begin{pmatrix} 0 & \Delta e^{i\omega t} \\ \Delta^* e^{-i\omega t} & \hbar\omega_0 \end{pmatrix}$$

This 2×2 matrix is the representation of the Hamiltonian operator using the two state basis, $|0\rangle$ and $|1\rangle$.

(a) The *exact* solution to this Rabi problem is known. The transition probability $P_{0\to1}(t)$ is given by

$$P_{0\to1}(t) = \frac{1}{\omega_r^2}\frac{|\Delta|^2}{\hbar^2}\sin^2(\omega_r t)$$

where

$$\omega_r = \frac{\sqrt{(\omega_0-\omega)^2 + 4\frac{|\Delta|^2}{\hbar^2}}}{2}$$

is the Rabi flopping frequency. Use the mathematical identity

$$\frac{1}{N}\frac{\sin^2(Nx)}{\sin^2 x} \to \pi\delta(x) \qquad N \to \infty$$

to find the long time limit ($t \to \infty$) of $P_{0\to1}(t)$.

(b) Find the transition rate $R_{0\to1}$ in the long time limit.

(c) Under what condition does the transition rate become the **Fermi's golden rule**? State that condition, and write down the golden rule formula that results from applying that condition. Note that $\delta(ax) = \frac{1}{|a|}\delta(x)$.

(d) Without taking the long time limit, show that the transition probability can be written as

$$P_{0\to1}(t) = A\,L(\omega)\sin^2(\omega_r t)$$

where A is a constant (that can depend on Δ), and

$$L(\omega) = \frac{1}{\pi}\frac{\Gamma}{(\omega-\omega_R)^2+\Gamma^2}$$

is the so-called **Lorentzian function** (a typical resonance profile). Find ω_R and Γ. [Note the identical nature of this this problem to the NMR problem (cf. Problem T9.20).]

Your name: