Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

2 problems, 30 minutes.

Problem 1 Consider the following potential energy.

$$V(x) = \begin{cases} \infty & \text{if } x < 0 \\ \frac{1}{2}kx^2 & \text{if } x > 0 \end{cases}$$

(a) Apply the Bohr-Sommerfeld quantization rule

$$\oint dx\, p = (n+\gamma)h \qquad\qquad n = 0, 1, 2, \ldots \text{ but } n \gg 1$$

to find the quantized energy level E_n for this problem in terms of n and $\hbar\omega$, where $\omega = \sqrt{k/m}$.

(b) It happens that $\gamma = 3/4$ according to the WKB approximation. Compare your result in part (a) with the exact quantum mechanical result: $E_n = \left((2n+1) + \frac{1}{2}\right)\hbar\omega$, $n = 0, 1, 2, \ldots.$

(c) Explain the meaning of the condition "$n = 0, 1, 2, \ldots$ but $n \gg 1$."

(d) Derive the exact quantum mechanical result for E_n stated in part (b), from the known result, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$, of a *full* simple harmonic oscillator ($V(x) = \frac{1}{2}kx^2$ for any x), and the boundary condition imposed at $x = 0$ for this problem.

Should this problem remain too difficult even after some time of thinking, you may take $V(x) = \frac{1}{2}kx^2$ for any x (standard SHO problem) and answer the above questions for some partial credit.

Problem 2 From the perturbation theory, we learned that the second order correction of the ground state energy is always negative or zero. Explain why this is exactly as you would have expected from the variational principle.

Your name: