Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

1 problem, 15 minutes.

Problem 1 Consider the following Hamiltonian representation

$$\hat{H} \doteq E \begin{pmatrix} 1+\epsilon & 3\epsilon & 0 \\ 3\epsilon & 1+\epsilon & -\sqrt{2}\epsilon \\ 0 & -\sqrt{2}\epsilon & 3 \end{pmatrix}$$

where a basis set $\{|0^{(0)}\rangle, |1^{(0)}\rangle, |2^{(0)}\rangle\}$ is used for the representation. Here, $\epsilon > 0$ is a perturbation parameter, which is much smaller than 1. $E > 0$ is an energy scale parameter, which you can take as unit energy ($\equiv 1$), if you like.

(a) Find the first order corrections to the two identical zeroth order energy values of 1.

(b) Find the corresponding new zeroth order states $|0'^{(0)}\rangle, |1'^{(0)}\rangle$ as linear combinations of $|0^{(0)}\rangle$ and $|1^{(0)}\rangle$.

(c) Find the representation of $\hat{H}$ using the new basis set $\{|0'^{(0)}\rangle, |1'^{(0)}\rangle, |2^{(0)}\rangle\}$.

(d) Find the leading order (non-zero) perturbation correction to the third zeroth order energy value of 3. Your answer can be obtained using either matrix representation, i.e. the one given above or the one that you obtained in the previous part.

Your name: