Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

1 problem, 15 minutes.

Problem 1 Consider a spin 1/2 particle, whose Hamiltonian is given by

$$\hat{H}(t) = \omega_0 \left(\frac{t}{T}\right)^2 \hat{S}_z$$

where ω_0, T are constants for a frequency scale and a time scale, respectively, t is time, and $\hat{S}_z$ is the operator for the spin component along the z direction.

(a) Which of the quantities, $\hat{H}, \hat{S}_x, \hat{S}_y, \hat{S}_z$, are conserved, and which are not? Briefly explain why.

(b) Initially, $|\Psi(t=0)\rangle = |S_x = \hbar/2\rangle$. Find the S_z representation of $|\Psi(t=0)\rangle$, i.e. find a_0 and b_0 in $|\Psi(t=0)\rangle \doteq \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$, where $a_0 \equiv \langle\uparrow|\Psi(t=0)\rangle$, $b_0 \equiv \langle\downarrow|\Psi(t=0)\rangle$, $\uparrow$ means $S_z = \hbar/2$, and $\downarrow$ means $S_z = -\hbar/2$.

(c) Find the solution $|\Psi(t)\rangle \doteq \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$ in the S_z representation, i.e., express $a(t)$ and $b(t)$ as functions of $t, \omega_0, T, \hbar$. Assume the same initial condition as in the previous part.

Note that Pauli matrices, $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, are the S_z representations of $2\hat{S}_x/\hbar$, $2\hat{S}_y/\hbar$, $2\hat{S}_z/\hbar$, respectively, for a spin 1/2 particle.

Your name: