(!)

$$S = \int_{t_1}^{t_2} dt\, L \qquad \text{fix } t_1, t_2 \quad \text{allow } \delta\vec{r}_2$$

$$\delta S = \int_{t_1}^{t_2} dt\, \delta L = \int_{t_1} dt \left(\frac{\partial L}{\partial \vec{r}} \cdot \delta\vec{r} + \frac{\partial L}{\partial \dot{\vec{r}}} \cdot \delta\dot{\vec{r}} \right)$$

$$= \int_{t_1}^{t_2} dt \left[\frac{\partial L}{\partial \vec{r}} \cdot \delta\vec{r} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{r}}}\right) \cdot \delta\vec{r} \right]$$

$$+ \frac{\partial L}{\partial \dot{\vec{r}}} \cdot \delta\vec{r} \Big|_1^2$$

$= 0$ if we take a physical path.

$$\therefore\ \delta S = \frac{\partial L}{\partial \dot{\vec{r}}} \cdot \delta\vec{r}_2 \qquad \text{if } \delta\vec{r}_2 \text{ is varied for a physical path.}$$

(*) $\boxed{\therefore\ \delta S = \vec{p} \cdot \delta\vec{r}}$ ← end point.

For a physical path, $S = \int_{t_1}^{t} dt'\, L$ (**)

Consider this as a ftn of the end point $(\vec{r}_2) \equiv \vec{r}$ and the end time (t)

What we saw in (*) $= \dfrac{\partial S}{\partial \vec{r}} = \vec{p}$.

From (**), $\dfrac{dS}{dt} = L$

$$\therefore\ \frac{dS}{dt} = \frac{\partial S}{\partial t} + \frac{\partial S}{\partial \vec{r}} \cdot \frac{d\vec{r}}{dt} = \frac{\partial S}{\partial t} + \vec{p} \cdot \vec{v} = L$$

$$\therefore\ \frac{\partial S}{\partial t} = L - \vec{p} \cdot \vec{v} = -H \qquad \boxed{\therefore\ dS = -H\,dt + \vec{p} \cdot d\vec{r}}$$

In relativity (free particle, e.g.)

$$S = -mc^2 \int_1^2 \frac{d\tau}{\gamma}$$

$$= -mc^2 \int_1^2 \sqrt{dt^2 - \frac{(d\vec{r})^2}{c^2}}$$

$$\delta S = -mc^2 \int_1^2 \left(\underbrace{\frac{\partial \sqrt{\ }}{\partial (dt)} \cdot d(\delta t)}_{\text{fix space. vary time.}} + \underbrace{\frac{\partial \sqrt{\ }}{\partial (d\vec{r})} \cdot d(\delta \vec{r})}_{\text{fix time vary space}} \right)$$

We can see that this makes it possible to deal with seperate variations of t and $\vec{r}$.

So at one stroke we get all of $\frac{\partial S}{\partial t}$ and $\frac{\partial S}{\partial \vec{r}}$

end point   end point

in this case!

. . .

. . .

As in class $\underbrace{\delta S}_{\text{4-scalar}} = p^\nu \underbrace{\delta x_\nu}_{\text{4-vector}}$

$\therefore$ 4-vector!