Name:

15 minutes.

Two of Maxwell's equations for macroscopic fields in matter are given by

$$\vec{\nabla} \cdot \vec{D} = \rho_f,$$
$$\vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t},$$

where the subscript f means "free." Note: parts (a,c) are really easy.

(a) What are the other two Maxwell's equations that look just the same for macroscopic fields in matter or the microscopic fields in any situation? (You *may* look them up in your book or note, *only for this part*, should you not remember them.)

(b) By starting from $-w$, where w is the work done on free charge per unit volume $w = \rho_f(\vec{E} + \vec{v} \times \vec{B}) \cdot \vec{v}$, and using Maxwell's equations, (1) *derive the expression for* $\vec{S}$ (Poynting vector) and (2) prove that the energy density u of the field satisfies

$$\frac{\partial u}{\partial t} = \frac{\partial \vec{D}}{\partial t} \cdot \vec{E} + \frac{\partial \vec{B}}{\partial t} \cdot \vec{H}.$$

You can use the vector identity $\vec{\nabla} \cdot (\vec{f} \times \vec{g}) = \vec{g} \cdot (\vec{\nabla} \times \vec{f}) - \vec{f} \cdot (\vec{\nabla} \times \vec{g})$.

(c) The general expression for u (or rather $\frac{\partial u}{\partial t}$) that we obtained just now is enlightening in the following sense. Consider a small but macroscopic volume $d^3\vec{r}$

located at an arbitrary but fixed position. Then, the electromagnetic energy stored in this volume is given by $u\,d^3\vec{r}$ and the change of this energy is given by

$$d(ud^3\vec{r}\,) = d^3\vec{r}\;du = d^3\vec{r}\,d\vec{D}\cdot\vec{E} + d^3\vec{r}\,d\vec{B}\cdot\vec{H}.$$

This energy change can be re-interpreted as the sum of two terms, each of the form $d\vec{X}\cdot\vec{f}$, where $d\vec{X}$ is a *generalized displacement* and $\vec{f}$ is a *generalized force*. Which of $\vec{E}$ and $\vec{D}$ corresponds to the generalized fore? What is the corresponding generalized displacement experienced by volume $d^3\vec{r}$? Which of the $\vec{B}$ and $\vec{H}$ corresponds to the generalized force? What is the corresponding generalized displacement experienced by volume $d^3\vec{r}$?