Due May 21, Thursday

Problem 1 (10 points) Consider a monochromatic plane wave with a complex wave vector propagating in a linear medium. And consider a very general case when the permittivity (due to bound charges) and the conductivity (due to free carriers) are complex (however, we leave the magnetic permeability μ as real). Let us define

$$\tilde{\varepsilon}(\vec{k},\omega) = \tilde{\epsilon}(\vec{k},\omega) + i\frac{\tilde{\sigma}(\vec{k},\omega)}{\omega}.$$

Note that here we also generalized the permittivity (ϵ) and the conductivity (σ) to have the $\vec{k}$ (the real part of the complex wave vector) dependence as well. In all our discussions so far, we have assumed a long wave length limit and so we did not bother to include the $\vec{k}$ dependence, but in fact we must in general.

Let us assume that $\tilde{\varepsilon}(\vec{k},\omega) = 0$. Show by using this condition and Maxwell's equations that a purely electric *longitudinal* solution exists. Do this by finding a solution that satisfies all of the following conditions:

$$\begin{aligned} \vec{B} &= 0, \\ \vec{k}\times\vec{E} &= 0, \\ \vec{k}\cdot\vec{E} &\neq 0, \\ \rho &= \mathrm{Re}\left(i\epsilon_0\tilde{\vec{k}}\cdot\tilde{\vec{E}}\right). \end{aligned}$$

For this reason, $\tilde{\varepsilon}(\vec{k},\omega) = 0$ is referred to as the longitudinal dispersion relation.

Problem 2 (10 points) A plasma oscillation that exists in the bulk, or the surface, of a material is a good example for the kind of solution that is discussed in the previous problem. Here, let us consider a very simple way to figure out the plasma frequency in the $\vec{k}\to 0$ limit for the bulk plasma oscillation in a metal.

Consider a metal, which can be composed of two parts, a crystal lattice of positive ions and free electrons. Positive ions are fixed in space and, in total, carry the equal and opposite charges of those of free electrons. Let us say that this metal occupies the infinite volume between $z = 0$ and $z = L$ with the x (and y) going from $-\infty$ to ∞. In equilibrium, the charge density due to the free electrons is uniform. Also, the charge density due to the ions can be considered to be uniform, if we assume that we are not considering a short wave vector, whose wavelength is on the order of the lattice constant. Now, let us consider perturbing the system by displacing the whole body of free electrons by a minute distance δ along the z direction, while leaving the ions fixed in space. If you now let it go, show that (1) the subsequent motion is a simple harmonic oscillation with angular frequency given precisely by ω_p of Homework 4.7, (2) there is no magnetic field, but there is an electric field, and (3) $\tilde{\varepsilon} = 0$ at $\omega = \omega_p$. For (3), you may use the results of Homework 4.7.

Problem 3 (10 points) Problem 10.11 (Potentials and fields)

Problem 4 (10 points) Problem 10.18 (Potentials)

Problem 5 (10 points) Problem 10.20 (Fields of a moving charge)

Problem 6 (10 points) Problem 11.3 (Radiative resistance)

Problem 7 (10 points) Problem 11.4 (Rotating dipole)