Due Apr. 30, Thursday

Problem 1 (10 points) Problem 9.14 (Normal incidence, R and T)

Problem 2 (5 points) Problem 9.16 (Basic but important math)

Problem 3 (15 points) Problem 9.17 (Oblique incidence, s polarization)

Problem 4 (10 points) Problem 9.19 (Dissipation time, Skin depth, etc.)

Problem 5 (10 points) Problem 9.20 (Skin depth)

Problem 6 (10 points) Problem 9.21 (Energy and intensity for light in a conductor)

Problem 7 (20 points; Drude model, Plasma frequency (Plasmon)) Consider a metal or a semiconductor in which electrons carry electrical current. Consider the following simple equation (Drude model):

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\vec{E}.$$

Here, $\vec{p}$ is the *average* momentum per electron, $-e$ is the negative charge ($e > 0$) of electron, and τ is the lifetime of the electron as a current carrier, i.e., the time scale between scatterings that it goes through. First off, it must be duly noted that the above simple equation does *not* describe the motion of any individual electron at all. The above simple description is justified only if a macroscopic number of electrons are described as a whole, and this is the reason why $\vec{p}$ is the average momentum. What the above, seemingly Newtonian, equation of motion captures is the fact that inside a metal or a semiconductor, the whole random motion of electrons (whose average momentum is zero without any field) is skewed by an external field $\vec{E}$ (thus giving a finite $\vec{p}$), but this gain in momentum is reset at a time scale τ (with the energy deposited into entropy/heat). (The above equation of motion is justified only in the long wave length limit.)

(a) Suppose that $\vec{E} = E_0 \cos(kz - \omega t)\hat{x}$. By considering complex vectors $\tilde{\vec{p}}$ and $\tilde{\vec{E}}$, whose real parts are the physical quantities, show that $\tilde{\vec{p}}$ has two solution parts, the transient state solution and the steady state solution. Find a complete solution for $\tilde{\vec{p}}$, by finding both parts of the solution.

(b) Under normal circumstances, we are interested only in the steady state solution. Show that the steady state solution for $\vec{p}$ can be expressed in terms of the current density $\vec{j}$ as

$$\tilde{\vec{j}} = \tilde{\sigma}(\omega)\tilde{\vec{E}},$$

where $\tilde{\sigma}(\omega)$ is the *complex* conductivity, given by (as you must show)

$$\tilde{\sigma}(\omega) = \frac{\sigma_0}{1 - i\omega\tau},$$

where

$$\sigma_0 = \frac{ne^2\tau}{m}.$$

Here, m is the mass of an electron, and n is the number density of electrons. σ_0 is the so-called DC Drude conductivity and $\tilde{\sigma}$ is the AC Drude conductivity. These results are very widely used in explaining the conductivity of common metals and semiconductors.

(c) Starting from Eq. 9.124, and the above *complex* conductivity, show that $\tilde{k}^2$ can be written as

$$\tilde{k}^2 = \mu\omega^2\tilde{\varepsilon}(\omega),$$

where $\tilde{\varepsilon}(\omega)$ is given by

$$\tilde{\varepsilon}(\omega) = \epsilon + i\frac{\tilde{\sigma}}{\omega}.$$

(d) Show that, in the limit $\omega\tau \gg 1$, the quantity $\tilde{\varepsilon}(\omega)$ becomes real and can be written as

$$\tilde{\varepsilon}(\omega) = \epsilon\left(1 - \frac{\omega_p^2}{\omega^2}\right),$$

where ω_p is the so-called plasma frequency

$$\omega_p^2 = \frac{ne^2}{m\epsilon}.$$

The above result is valid for high ω value in general ($\omega\tau \gg 1$). In typical metals, it turns out that the high ω regime starts below ω_p—so you might say that, effectively, the permittivity can become negative!

(e) Express σ_0 in terms of ω_p^2, ϵ, and τ.