This is an open-book take-home exam.

1. This exam format has been chosen to be able to test students on advanced concepts without imposing time pressure or any other pressure and, in addition, to accommodate different schedules of students.
2. My rough estimate is that it will take anywhere from 5 hours to 10 hours for a student to finish this exam, assuming that the student has been following the course materials. This rough time estimation is for your information only. It is recommended that you review materials, and then take this exam in a reasonable amount of time, while reviewing some more as necessary.
3. *Here is how to submit your work.* Your work must be submitted *in person* to me at my office (ISB 249) between 9 AM and 5 PM on Wednesday (June 10) and Thursday (June 11), unless you made some other special arrangement with me beforehand. Do not leave your work in my mailbox—if you do so, your work may be *completely disregarded and discarded.* No work will be received after 5 PM on June 11.
4. If you have any questions about this exam, you can ask me by email (gweon@ucsc.edu), and I may post my answer on the course forum.
5. In all other ways, however, this exam is to be regarded just as any in-class exam.
   - You should not communicate with any other person in any manner regarding this exam.
   - The solutions to this exam should be worked out by you and you alone.

Good luck!

Consider an infinitely long solenoid of radius a and the number of turns per unit length n. Let us define the long axis of the solenoid as the z axis. Inside the solenoid, a small charged particle, with charge q and mass m, is moving with its velocity perpendicular to the z axis. Let the speed of the particle be v. In this problem, we will assume that no matter what motion the charged particle goes through, the solenoid current is given by a constant value I. Also, we will assume that the charged particle never bumps into the solenoid wall. Assume $v \ll c$ except in parts (k) and (l). (All parts are equally important, with each part worth 10 points. So, the maximum possible score is 130 points, where 20 points is to be considered as extra credit scores, attributed roughly to two of the last three parts.)

(a) What is the magnetic field due to the solenoid? You must derive it explicitly, explaining all details (how you determine the direction and the magnitude of $\vec{B}$) both for inside and outside the solenoid, even though you may well remember the answer. (If you use the symmetry argument and the pseudo-vector nature of $\vec{B}$ for figuring out the direction, then you may get some extra credit.)

(b) Find the solution for the motion of the charged particle, using Newton's law. Do not consider the effect of the radiation energy loss on the motion of the particle in this or any other part of this problem, except in part (m).

(c) Find the $\vec{E}$ field and the $\vec{B}$ field due to the solenoid in the reference frame of the charged particle (hint: Faraday's law may be helpful. Also, the time-reversal symmetry argument can be used for figuring out the direction of $\vec{E}$ (this may be worth extra credit)).

(d) Explain the motion of the particle from the point of view of someone living in the reference frame of the charge particle (you must discuss any real forces and any fictitious force(s) that might give (fictitious) gravitational effect).

(e) Find the $\vec{E}$ field and the $\vec{B}$ field due to the moving charged particle, for everywhere inside the solenoid. Here, you can, of course, use a textbook formula. But, be sure to make non-relativistic approximation, while making sure to keep the leading order for the radiation term.

(f) From your expression for $\vec{E}$ (and $\vec{B}$), show where you can identify the wavelength λ of the radiation field and find the value of λ.

(g) From $\vec{E}$ and $\vec{B}$, find the Poynting vector $\vec{S}$ due to the charged particle. Find it only for inside the solenoid.

(h) Assume that $a \gg \lambda \gg d$, where d is the length scale of the orbital motion of the charged particle. Find the Poynting vector $\vec{S}$ inside and near the wall of the solenoid using an appropriate textbook formula, based on this approximation. Assume that when averaged over a long time the position of the charged particle is at the center of the solenoid.

(i) Show that the answer that you got for the previous two parts agree if you apply the same set of approximations made in part (h) and $ad \gg \lambda^2$.

(j) Find the rate at which the energy radiated from the charged particle is absorbed on the inside wall of the solenoid. Assume that the wall is perfectly absorbing. Make the same set of assumptions as in the previous part.

(k) Revising part (b), find the solution of the motion of the charged particle, assuming a relativistic case.

(l) Revising part (c), find the $\vec{E}$ field and the $\vec{B}$ field due to the solenoid in the instantaneous inertial reference frame of the charged particle, assuming a relativistic case. (Using Faraday's law with relativistic length contraction effect and considering the time dilation effect for the solenoid current is all you need. Or, you can use Lorentz-transformation equations for $\vec{E}$ and $\vec{B}$.)

(m) Discuss what your answer to part (j) implies in terms of the motion of the charged particle.