

Your solutions must be given on separately provided paper. Good luck!

Problem 1 (50 points) A linearly polarized monochromatic plane wave is normally incident on a flat surface of a good non-magnetic ($\mu = \mu_0$) conductor, for which you can assume

$$\sigma \gg \omega\epsilon, \quad \sigma \text{ is real.}$$

We may take the coordinate system so that half of the space ($z > 0$) is occupied by the conductor and the other half ($z < 0$) is occupied by air. Assume $\vec{E} \parallel \hat{x}$.

- Find the ratio \tilde{E}_1/\tilde{E}_0 , where \tilde{E}_0 is the complex electric field for the incoming wave and \tilde{E}_1 is the complex electric field just inside the metal surface (i.e., at $z = 0^+$). You must show that $|\tilde{E}_1/\tilde{E}_0| \ll 1$.
- Calculate the energy dissipated per unit time and per unit volume anywhere inside the conductor as a function of $E_0 \equiv |\tilde{E}_0|$, z , t , ω , and any other quantity you can and must define conveniently in terms of the above quantities (for example, the skin depth d).
- Average the result of the previous part over a period in time and integrate it over a volume created by sweeping an area A on the metal surface from $z = 0$ to ∞ .
- The result of the previous part is the time-averaged power dissipated by the considered volume. Thus, it can be written as

$$R_{eff} I_{rms}^2$$

where I_{rms} is the root-mean-squares current ($I_{rms} = I_{max}/\sqrt{2}$ for a sinusoidal current) flowing along the direction of \vec{E} in the volume. Find \vec{I} , I_{rms} , and this effective resistance, R_{eff} in terms of the conductivity σ and other quantities (such as the skin depth d). For simplicity, you may assume that A is a square whose sides are along the xy directions.

Problem 2 (50 points) Consider an infinitely long solenoid of radius a and the number of turns per unit length n . There exists one charged particle with mass m and charge q at a distance b from the solenoid, with $b \gg a$. Initially, the current on the solenoid is constant. Accordingly, the charged particle is subject to no external force, initially, and is found to be completely at rest. Then, the current on the solenoid changes by dI between time t and time $t + dt$. After this brief event, however, the current on the solenoid is perfectly constant again.

- Show that the charged particle is no longer at rest after this current changing event, and find the small velocity $d\vec{v}$ of the particle in the final state. (Hint: use Faraday's law.)
- Find \vec{A} in the initial state and the final state, throughout space, and find any relation between $d\vec{v}$ and $d\vec{A}$. What is the value of $d(m\vec{v} + q\vec{A})$?
- Show that $m d\vec{v}$ is exactly compensated by the change of the momentum of the electromagnetic field. (Note that $b \gg a$ and $\int_{-\infty}^{\infty} dx (1 + x^2)^{-3/2} = 2$.)