Name:

20 minutes.

A ball is dropped from rest. Let the initial height be h_1. There is air resistance given by $-mkv$. In the downward motion, the equation of motion can be written as

$$\dot{v} = g - kv,$$

with the y axis pointing down. The ball hits the ground at speed v_b and then bounces back up with *perfect elasticity* (no speed change on bouncing). In the upward motion, the equation of motion is given by

$$\dot{v} = -g - kv,$$

with the y axis pointing up. The maximum height in the upward motion is h_2.

(a) Find v_b as a function of $v_1 \equiv \sqrt{2gh_1}$ and $v_\infty \equiv g/k$ to the first order perturbation in air resistance. [Hint: Since time is not our concern, we can make use of $\dot{v} = \frac{dv}{dy}v = \frac{1}{2}\frac{dv^2}{dy}$ to turn the equation of motion into $d(v^2) = 2gdy - 2kvdy$. Solve *this* equation perturbatively, treating the second term on the right hand side as a perturbation.]

(b) Let $v_2 \equiv \sqrt{2gh_2}$. Find $\frac{v_2^2}{v_1^2} = \frac{h_2}{h_1}$ to the first order perturbation in air resistance, and then show how measuring h_2, given h_1, can determine the value of k.

The binomial expansion $(1+\delta)^x \approx 1 + x\delta$ for $|\delta| \ll 1$ may be useful throughout this problem.