Exam

1 crib sheet allowed.
US letter size
(front & back)

- Newton's laws
- Perturbation : the leading order
- Work - Energy theorem : { • useful when / • dK
- SHO w or w/o damping and driving
- $L = T - U$

$x_c(t)$

- Newton's laws
- Perturbation : the leading order correction
- Work - Energy theorem : $\begin{cases} \cdot \text{ useful when time is } \underline{\text{not}} \text{ asked for.} \\ \cdot \quad dK = \vec{F} \cdot d\vec{r}, \quad \vec{F} \cdot d\vec{r} : \text{the wo} \end{cases}$

  on the par
  by <u>all for</u>
  on it
- SHO w or w/o damping and <u>driving force</u>.

  $x_c(t) \quad x_p(t)$
- $L = T - U$

aws

tion : the leading order correction

nergy theorem $\left\{ \begin{array}{l} \cdot \text{ useful when time is } \underline{\text{not}} \text{ asked for.} \\ \cdot \quad dK = \vec{F} \cdot d\vec{r} \,, \end{array} \right.$ $\vec{F} \cdot d\vec{r}$: the work done on the particle by all forces acting on it $\parallel \vec{F}$

or w/o damping and driving force.

$x_c(t) \quad x_p(t)$

$- U$

$$\vec{F} = m\vec{a}$$

$$\vec{F} \cdot d\vec{r} = m \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \, d\vec{v} \cdot \left(\frac{d\vec{r}}{dt}\right)$$

$$= m \, \vec{v} \cdot d\vec{v}$$

$$= d\left(\frac{1}{2} m \vec{v} \cdot \vec{v}\right)$$

$$\because \; d(\vec{v} \cdot \vec{v}) = d\vec{v} \cdot \vec{v} + \vec{v} \cdot d\vec{v} = 2\,\vec{v} \cdot d\vec{v}$$

Consequences of W-E theorem

① If all forces are conservative

then E = conserved.

$$= K + U$$

② If $\exists$ non-conservative forces, then

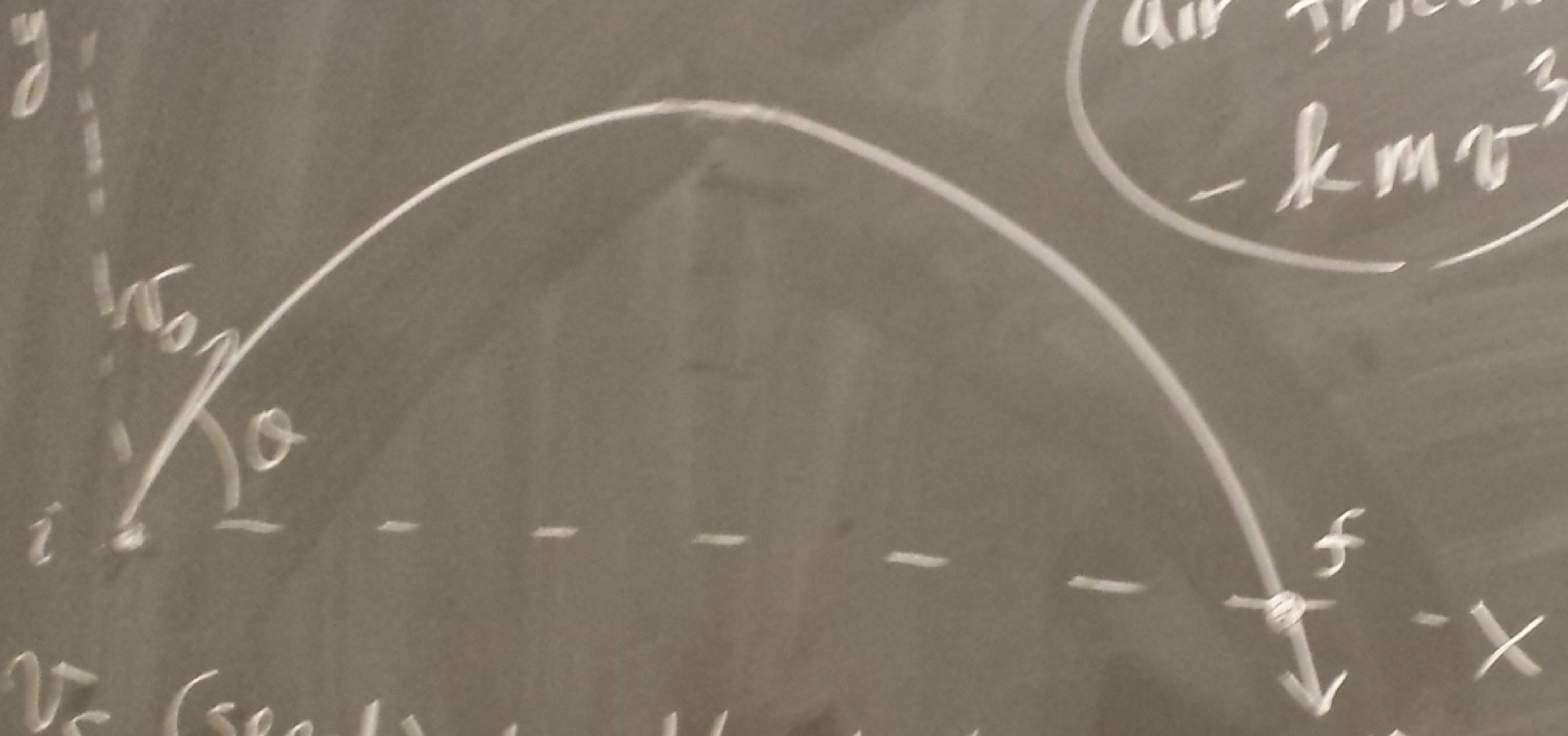
$$dE = \vec{F}_{n.c.} \cdot d\vec{r}$$

$\vec{F}\cdot d\vec{r}$

Last Year's Midterm 4

$kv^3 \sim g$

air friction
$-kmv^3$



v_f (speed) to the leading correction by air resistance?

$v_f = ?$

$\Delta K = \int$

$\|$

$\frac{1}{2} m (v_f$

$\vec{F}_{net}$:

$\therefore \Delta K$

$kv^3 \sim g$

$$\Delta K = \int \vec{F}_{net} \cdot d\vec{r} = \Delta W_{net}$$

$$\parallel$$

$$\frac{1}{2} m \left(v_f^2 - v_0^2 \right)$$

$\vec{F}_{net}$: $\left(\begin{array}{l} \text{gravity} : \Delta W = 0 \\ \text{air friction} : \Delta W < 0 \end{array} \right.$ $\left(\because \Delta W = -\Delta U \right)$

$$\therefore \Delta K = \int \vec{F}_{\text{air friction}} \cdot d\vec{r}$$

$$= -k \int m v^2 \vec{v} \cdot d\vec{r}$$

$O(1) + \boxed{O(k) + \cancel{O(k^2) + \cdots}}$

$\therefore$ We can evaluate $\int m v^2 \vec{v} \cdot d\vec{r}$ using the 0^{th} order solution without air resistance to evalute ΔK to the first order.

$$\int m v^4\, dt = 2\int_{\text{initial}}^{\text{top}} m v^4\, dt = 2m\int_0^{\frac{v_0 S_0}{g}} dt \left\{ v_0^4 C_0^4 + 2 v_0^2 C_0^2 (v_0 S_0 - gt)^2 + (v_0 S_0 - gt)^4 \right\}$$

$$= 2m \left\{ v_0^4 C_0^4 \frac{v_0 S_0}{g} + \frac{2 v_0^2 C_0^2}{3} \frac{(v_0 S_0 - gt)^3}{(-g)}\Bigg|_0^{\frac{v_0 S_0}{g}} + \frac{(v_0 S_0 - gt)^5}{5(-g)}\Bigg|_0^{\frac{v_0 S_0}{g}} \right\}$$

For the up motion

$$v_x = v_0 C_\theta$$

$$v_y = v_0 S_0 - gt$$

$$v^2 = v_0^2 C_\theta^2 + (v_0 S_0 - gt)^2$$

$$v^4 = v_0^4 C_\theta^4 + 2 v_0^2 C_\theta^2 (v_0 S_0 - gt)^2 + (v_0 S_0 - gt)^4$$

$$\frac{2 v_0}{g}$$

$$\frac{1}{2} m$$

$$v_f^2 =$$

$$v_f =$$

$$v_0^4 c_\theta^4 + 2 v_0^2 c_\theta^2 (v_0 s_\theta - gt)^2$$

$$+ (v_0 s_\theta - gt)^4 \}$$

$$\frac{2v_0^2 c_\theta^2}{3} (v_0 s_\theta - gt)^3 \cdot \left(-\frac{1}{g}\right) \Big|_0^{\frac{v_0 s_\theta}{g}}$$

$$\frac{(v_0 s_\theta - g t)^5}{5\,(-g)} \Big|_0^{\frac{v_0 s_\theta}{g}} \}$$

$$\frac{2 v_0^5}{g} \left\{ c_\theta^4 s_\theta + \frac{2}{3} c_\theta^2 s_\theta^3 + \frac{1}{5} s_\theta^5 \right\}$$

$$\frac{1}{2} m \left(v_f^2 - v_0^2 \right) = -\frac{2 k v_0^5 m}{g} \left\{ \text{''} \right\}$$

$$v_f^2 = v_0^2 \left[1 - \frac{4 k v_0^3}{g} \left\{ \text{''} \right\} \right]$$

$$v_f \approx v_0 \left[1 - \frac{2 k v_0^3}{g} \left\{ \text{''} \right\} \right]$$

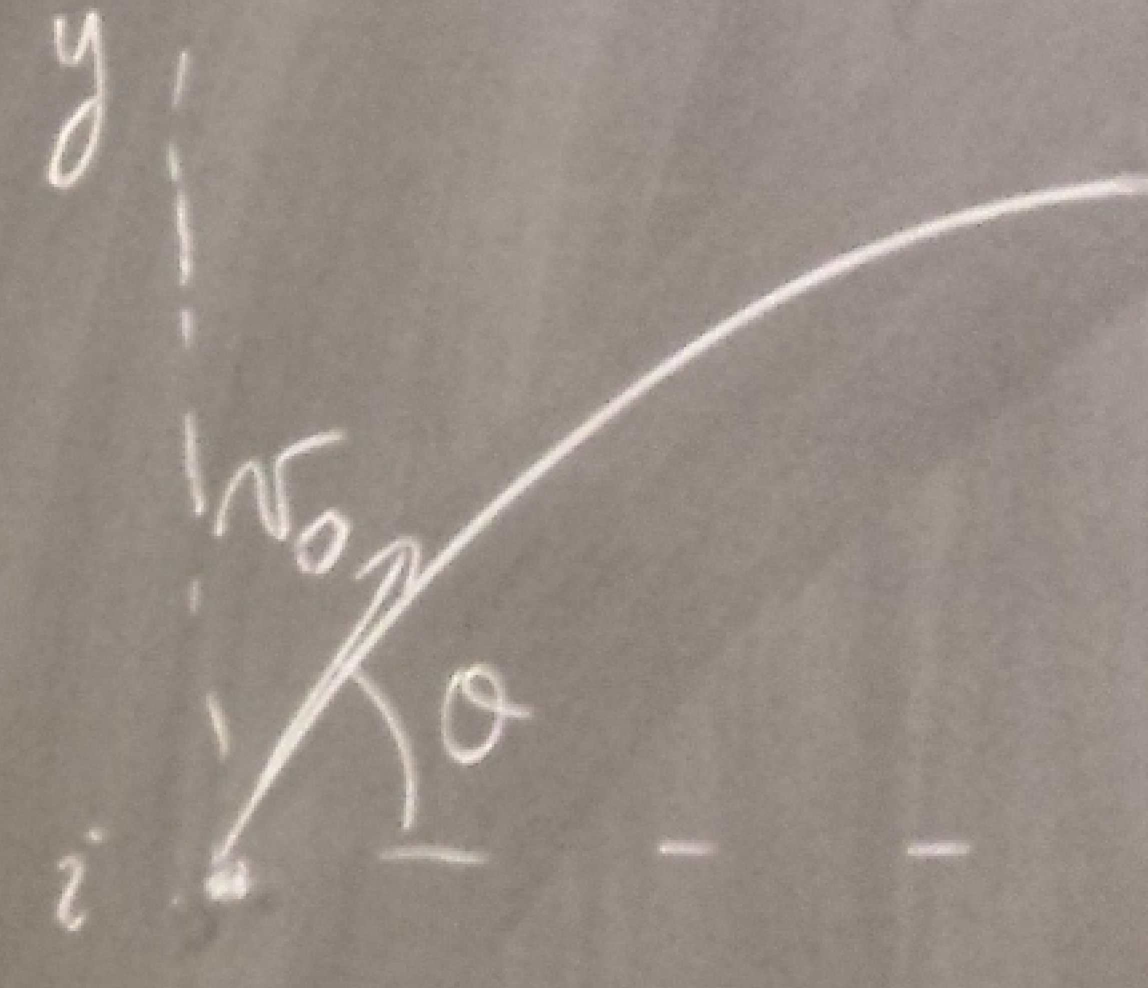
If v is known up to $O(k)$
why is v^2 not known up to $O(k^2)$?

$$v = \boxed{v^{(0)} + a k} + b k^2 + \cdots$$

$$v^2 = \left(v^{(0)} + ak + bk^2 + \cdots\right)^2$$

$$= (v^{(0)})^2 + 2 a v^{(0)} k + (a^2 + 2 b v^{(0)}) k^2 + \cdots$$

Last Year's Mi



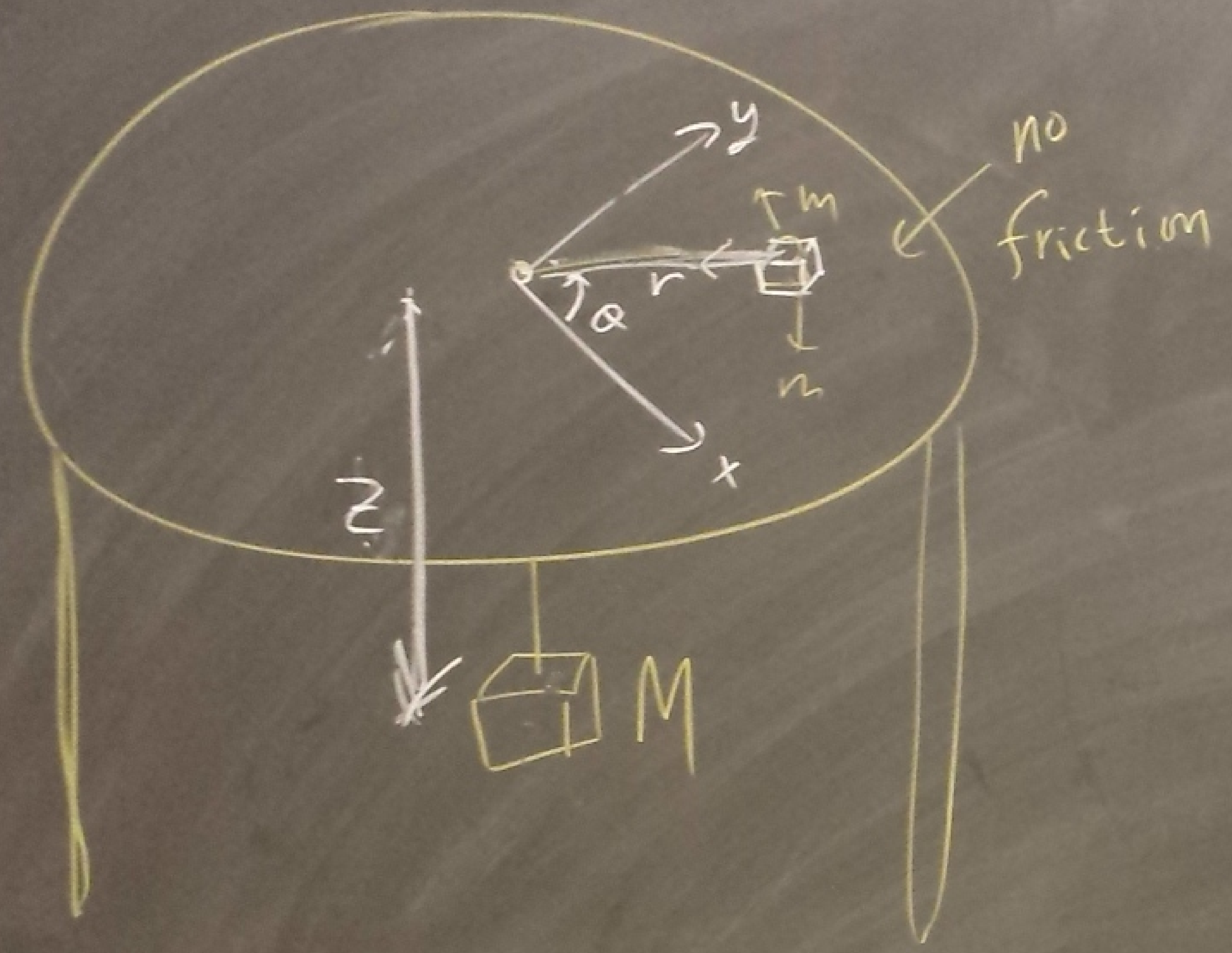
v_f (speed) to t

correcti

Problem 6

$z + r = \ell$

constant



Generalized coordinates

r, θ

Kinetic Energy

$$K = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2} M \dot{r}^2$$

Potential Energy

$$U = -Mgz = -Mg(\ell - r) = Mgr - Mg\ell$$

L

L

$$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2} M\dot{r}^2 - Mgr$$

$$L = K - U$$

$$\dot{\theta}^2) + \frac{1}{2} M \dot{r}^2$$

$$-Mg(l-r) = Mgr - Mgl$$

Angular momentum is conserved

why? $\vec{r}_m \parallel \vec{F}_m$ = tension

$\vec{r}_M \parallel \vec{F}_M$ = gravity, tension

no torque on m

no torque on M

Angular

$(I\vec{\omega}$

$E = K +$

$\frac{1}{2}$

$\frac{1}{2}(m$

<u>Angular momentum</u> $= mr^2\dot{\theta} = \underline{\underline{\text{const.}}} = L$

$(I\vec{\omega})$

$$E = K + U = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{1}{2}M\dot{r}^2 + Mgr = \underline{\text{const.}}$$

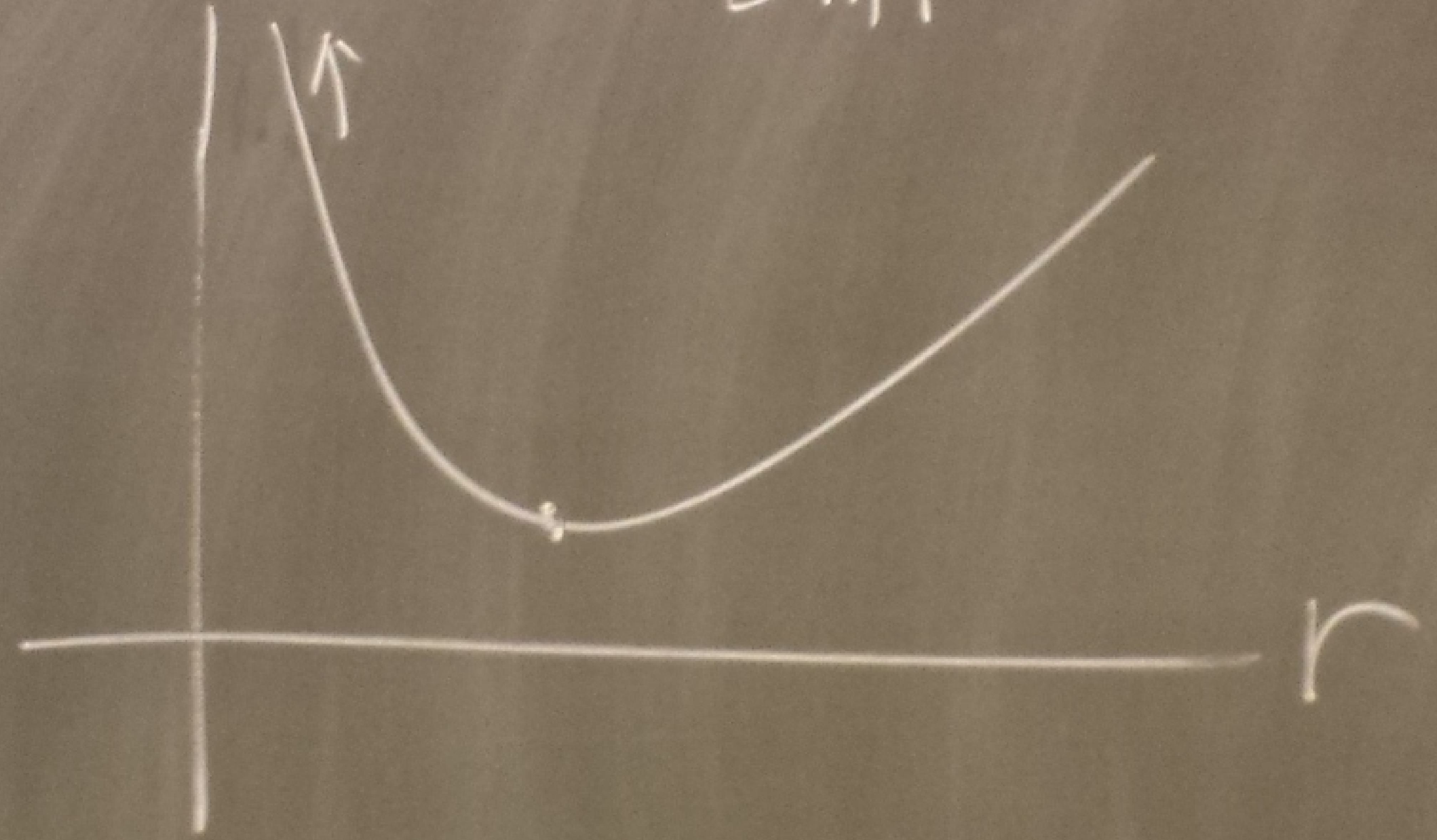
$$\frac{1}{2}mr^2\dot{\theta}^2 = \frac{1}{2}mr^2\frac{L^2}{(mr^2)^2}$$

$$\frac{1}{2}(m+M)\dot{r}^2 + \frac{L^2}{2mr^2} + Mgr = \text{const.}$$

L

$$U_{eff}(r) = \frac{L^2}{2mr^2} + Mgr$$

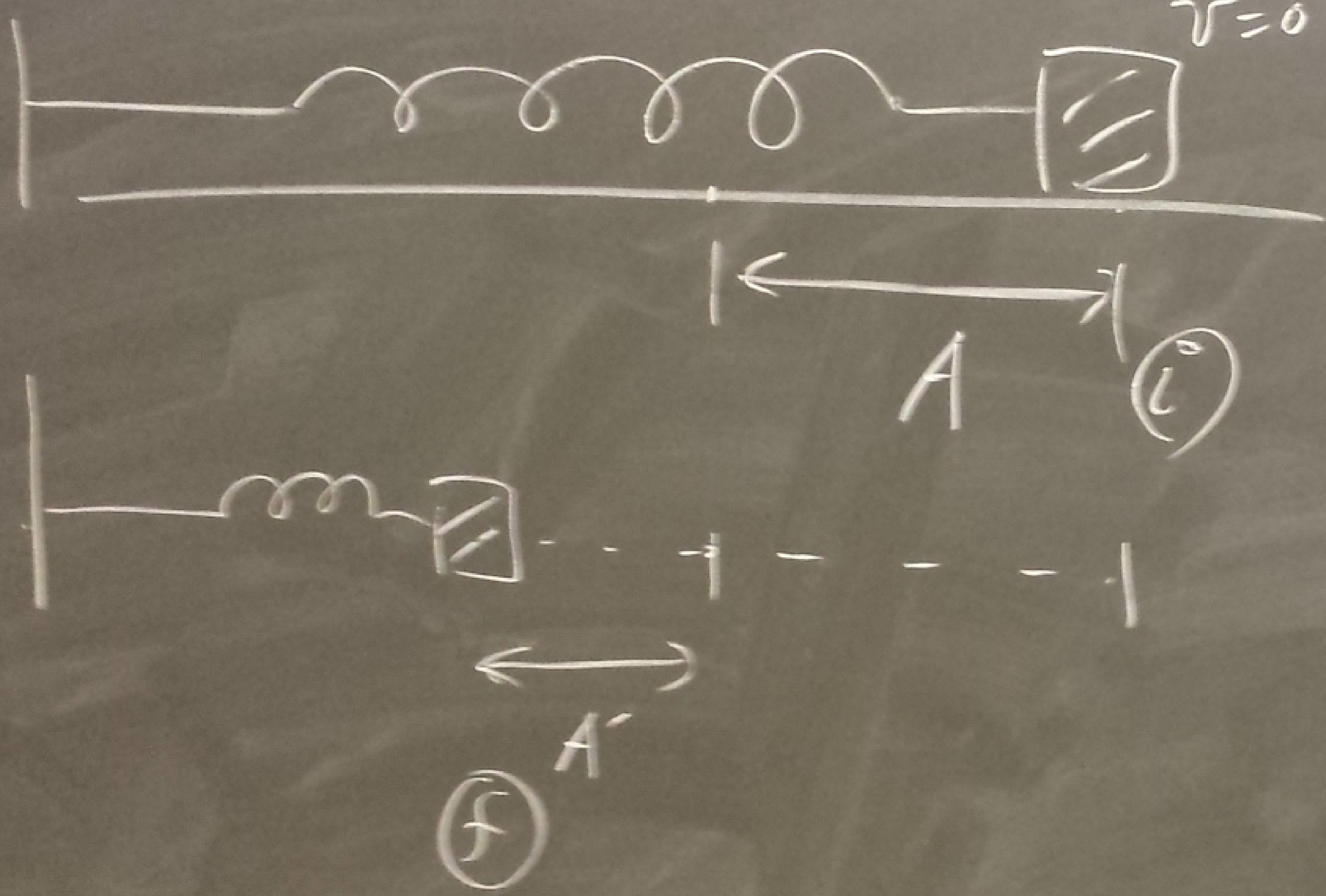
$gr = \underline{\text{const.}}$



Problem 3

Kinetic friction

$\mu_k mg$



initial point

turning point

$x = -A'$ $\quad A' < A$

Energy poi

$\Delta E =$

$\| \; i \to f$

$\frac{1}{2} k (A'^2 -$

$(A' - A$

$A' -$

Energy point of view

$$\Delta E = \text{Work done by friction}$$

$$\| \; i \to f \qquad\qquad \|$$

$$\frac{1}{2} k \left(A'^2 - A^2\right) = -\mu_k mg \left(A + A'\right)$$

$$(A'-A)(A'+A)$$

$$\boxed{A' - A = -\frac{\mu_k mg \cdot 2}{k}}$$

$$m\ddot{x} = -kx + mg\mu_k$$

$$x - \frac{mg\mu_k}{k} = \xi$$

time between turning points

$$= \frac{\pi}{\omega} = \pi \sqrt{\frac{m}{k}}$$

An

$$E =$$