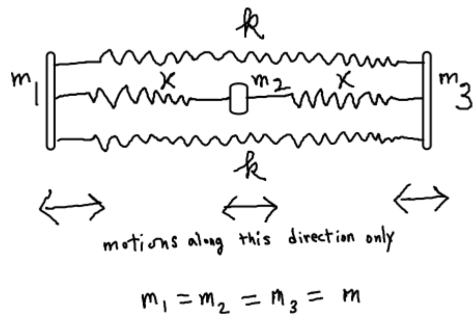
Due Dec 11, Thursday.
The perfect score for this homework is 60 points.

**Problem 1** (30 points) Consider the following coupled oscillator.



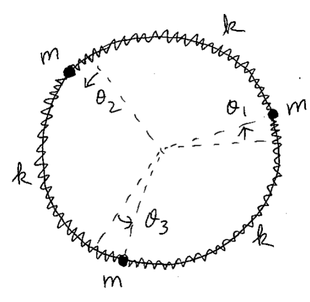

Relative to the equilibrium state, depicted here, the three masses (all of equal mass $m$) are free to move along the horizontal direction as indicated. The displacements of the three masses relative to equilibrium can be defined as $x_1, x_2$ and $x_3$. The spring constants are $k$ (long springs) and $\kappa$ (short springs).

(a) Find all normal mode frequencies $\omega_1, \omega_2, \omega_3$.

(b) Find the $\overleftrightarrow{T}$ matrix (each column representing the eigenvector for each eigenvalue), and sketch each normal mode excitation.

(c) Find normal mode coordinates $\eta_1, \eta_2, \eta_3$ in terms of $x_1, x_2, x_3$.

**Problem 2** (30 points) Consider a circular wire in which three identical masses ("beads") can slide without friction. The masses are connected by three identical springs, as shown.




For small angular displacements, $\theta_1, \theta_2, \theta_3$ of the three masses, answer the following questions. You can set the radius of the circle to be 1, for convenience.

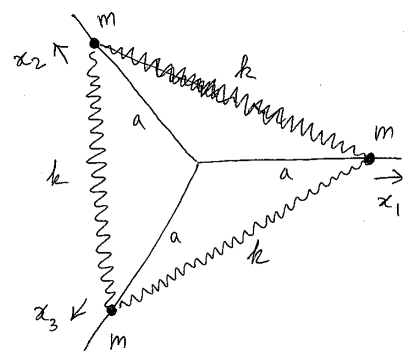
(a) Find the eigen-frequencies and the normal modes, by solving the eigenvalue equation

$$\overleftrightarrow{A}\vec{T} = \omega^2 \overleftrightarrow{M}\vec{T}$$

(b) Your answer should show that one of the eigen-frequencies is zero. Explain the physical reason why this is so.

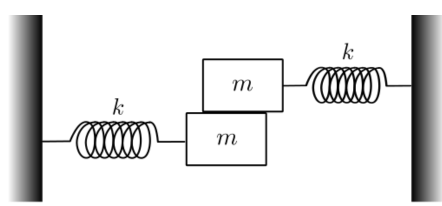
(c) Your answer should show that the modes such as $2\theta_1 = 2\theta_2 = -\theta_3$ and $\theta_1 = -\theta_2, \theta_3 = 0$ are degenerate, i.e. they have the same eigen-frequency. Explain why this is so by considering Newton's law (for one mass is sufficient) specifically for these normal modes.

**Problem 3** (30 points) Consider three identical masses ("beads"), each moving along a linear frictionless wire. The wires lie in one plane with a common end point, and are at 120 degrees with respect to each other.




The masses are connected by three identical springs, forming an equilateral triangle when in equilibrium. In equilibrium, each mass is at distance $a$ from the center. Considering small displacements from the equilibrium configuration, calculate the eigen-frequencies and the normal modes.

**Problem 4** (20 points)




Two mass on spring systems are coupled by friction, which is proportional to the relative velocity of the two masses. There is no gravity in this problem. The magnitude of the friction is given by $\alpha$ times the magnitude of the relative velocity. Solve this problem for the normal mode coordinates, $\eta_1$ and $\eta_2$, and then find the most general solution for the displacement of the two masses, $x_1$ and $x_2$ as a function of time. [You need to do this problem using the Newtonian method, not the Lagrangian method. You have two options. (1) Use the standard method of putting $x_1 = u\exp(i\omega t)$ and $x_2 = v\exp(i\omega t)$ to obtain normal modes. (2) Note the symmetry and you can probably solve this problem more quickly.]