Due Oct. 23, Thursday.

Problem 1 (10 points + 10 points extra credit) Consider the projectile motion problem with air resistance $-mk\vec{v}$, as we solved it in class (LN 3). We consider the optimum throw angle that maximizes the horizontal range of the motion for a fixed initial speed v_0. Recall that the optimum throw angle is 45 degrees, if air resistance did not exist.

(a) (10 points extra credit) Based on Eqs. 3.24 and 3.29 (or 3.28), provide a *mathematically rigorous* proof that, when $k > 0$, the optimum throw angle is less than 45 degrees. Discussions in pages 12 and 13 can be used as starting points. What is required here is a mathematical rigor, which is hidden in those discussions.

(b) (10 points) In the limit of $\alpha \stackrel{def}{=} kv_{y,0}/g \ll 1$, obtain the expression for the optimum throw angle up to the leading order correction due to perturbation by air resistance. Eq. 3.31 (or the equation above it) may be helpful to use.

Problem 2 (10 points) A particle of mass m can move on a *rough* horizontal table and is attached to a fixed point on the table by a light inextensible string of length l. The resistance force exerted on the particle is $-mk\vec{v}$, where $\vec{v}$ is the velocity of the particle. Initially the string is taut and the particle is projected horizontally, at right angle to the string, with speed v_0. Find the angle turned through by the string before the string comes to rest. Find also the tension in the string at time t.

Problem 3 (20 points) A charged particle of mass m and charge $q > 0$ is moving under the combined influence of a uniform electric field $E_0\hat{y}$ and a uniform magnetic field $B_0\hat{z}$. Initially the particle is at the origin and is moving with velocity $v_0\hat{x}$. The signs of E_0, B_0, and v_0 are left unspecified. Show that the trajectory of the particle is given by

$$\begin{aligned} x &= a\omega_c t - b\sin(\omega_c t), \\ y &= b(1-\cos(\omega_c t)), \\ z &= 0. \end{aligned}$$

Here, ω_c is the cyclotron frequency qB_0/m (note that here we define ω_c as the absolute value of the angular velocity; in the lecture note 4, ω_c was defined as the angular velocity, and was negative for $q > 0$). As you derive the above solution, you must find a and b as functions of v_0, E_0 and B_0.

Problem 4 (15 points) Consider a compass. Let the magnetic moment of the compass needle be $\vec{M}$, whose magnitude is constant, and the magnetic field in which the compass is immersed be a constant vector, $\vec{B}$. Assume that the compass

needle is free to rotate, i.e. it rotates without any friction, in the horizontal plane. Also, for simplicity, we will assume that the $\vec{B}$ field has no vertical component.
The potential energy is given by

$$U = -\vec{M} \cdot \vec{B}$$

Let the moment of inertia of the compass needle be I.

(a) Draw a diagram showing the stable equilibrium position of the needle. Mark the magnetic north and south of the needle, as well as those for the $\vec{B}$ field.

(b) Prove that the equilibrium is indeed a stable equilibrium.

(c) Find the angular frequency ω of the small angle oscillation around the equilibrium point, in terms of M, B and I.

(d) Consider oscillation around the equilibrium point, and consider an arbitrary time t and a small time interval dt around it. During the time interval dt, does the field $\vec{B}$ do any work on the compass needle?

Problem 5 (5 points) Prove that

$$\frac{1}{\tau_n} \int_{t_0}^{t_0+\tau_n} dt \sin^2(\omega t - \delta) = \frac{1}{2}$$

$$\frac{1}{\tau_n} \int_{t_0}^{t_0+\tau_n} dt \cos^2(\omega t - \delta) = \frac{1}{2}$$

where $\tau_n = n\tau/2$ (n = non-zero integer), $\tau \overset{def}{=} 2\pi/\omega$ (period), and t_0, δ are any real constants.