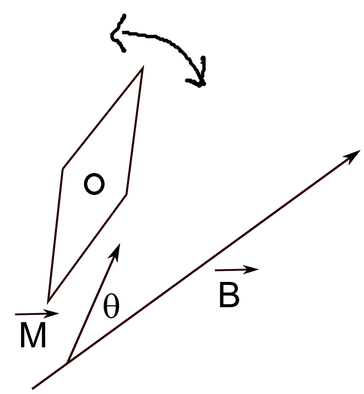
Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

15 minutes.

Consider a compass. Let the magnetic moment of the compass needle be $\vec{M}$, whose magnitude is constant, and the magnetic field in which the compass is immersed be a constant vector, $\vec{B}$. Assume that the compass needle is free to rotate, i.e. rotate without any friction, around its center (as shown), in the horizontal plane and (for simplicity) that the $\vec{B}$ field has no vertical component.

The potential energy is given by



$$U = -\vec{M} \cdot \vec{B}$$

Let the moment of inertia of the compass needle be I.

(a) List all stable and unstable equilibria for $-\pi < \theta \leq \pi$.

(b) Find the (angular) frequency ω for small oscillation around the stable equilibrium point, in terms of M, B and I. You can use, without proof, $\sin\delta \approx \delta$ and $\cos\delta \approx 1 - \frac{\delta^2}{2}$ for $|\delta| \ll 1$.

(c) For the solution of the small oscillation case of this problem,

$$\theta = A\cos(\omega t + \phi_0)$$

express $\langle T \rangle$ and $\langle U - U_{min} \rangle$ in terms of I, ω, and the (angular) amplitude A, where U_{min} is the minimum potential energy, and $\langle Q \rangle$ means the average of quantity Q over one period $2\pi/\omega$. You can use the following formulae without proof.

$$\frac{1}{\tau_n}\int_{t_0}^{t_0+\tau_n} dt\,\sin^2(\omega t+\phi_0)=\frac{1}{2}$$
$$\frac{1}{\tau_n}\int_{t_0}^{t_0+\tau_n} dt\,\cos^2(\omega t+\phi_0)=\frac{1}{2}$$

where $\tau_n = n\tau/2$ (n = non-zero integer), $\tau \stackrel{def}{=} 2\pi/\omega$ (period), and t_0, ϕ_0 are any real constants.