Please provide your solutions on a separate sheet of paper provided.
Write your name down first, on that sheet!
You can keep this sheet.

15 minutes.

Consider the following equation of motion (x is a linear displacement):

$$\ddot{x} = -\omega_0^2 x - \alpha x^2$$

In the simple harmonic oscillator problem, the term $-\alpha x^2$ (and higher) is ignored. With this term thrown in, now the problem becomes more realistic. However, students of 105 have no fear of a realistic but un-integrable problem: *of course*, we can apply the perturbation theory, obtain the leading order correction and gain insights.

So, let us do just that in this quiz! To answer the following questions, you need only the basic understanding of the perturbation theory. We shall treat the term $-\alpha x^2$ as a perturbation term.

(a) Starting from the unperturbed solution (with $\theta_0 = 0$ without loss of generality)

$$x = A\cos(\omega_0 t)$$

determine the first order perturbation equation to solve.

(b) Which one of the following would be the correct perturbation parameter λ for this problem?

(1) α (2) αA^2 (3) $\alpha A^2/\omega_0^2$ (4) $\alpha A/\omega_0^2$

(c) Solve the equation set up in (a), by assuming that the solution is

$$x = A\cos(\omega_0 t) + B + C\cos(2\omega_0 t)$$

and then solving for the unknown coefficients B and C in terms of A, α, ω_0. After obtaining your answers for B and C, show that they have correct dimensions. The identity $\cos^2 z = \frac{1}{2}(1 + \cos(2z))$ may be useful.