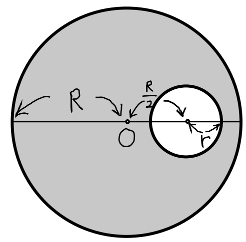
Problems for your study (especially the first two problems). NOT due.

Problem 1 Consider a disk with a small hole cut out from it. It is a thin disk, whose thickness is negligible. The radius of the disk is R and the radius of the small hole cut out is r. The center of the cut-out hole is at distance $R/2$ from the center of the disk. We assume $r = R/3$.



(a) Find the position of the center of mass.

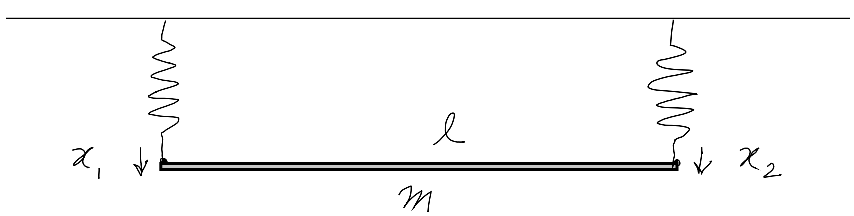
(b) Use the parallel axis theorem to calculate the inertia tensor around the center of the disk. For the inertia tensor of a thin disk around its center of mass, do the integration yourself without looking up any table. Find and use the principal axes of rotation so that the inertia tensor is diagonal.

(c) Use the parallel axis theorem to calculate the inertia tensor around the center of mass of the disk. Find and use the principal axes of rotation so that the inertia tensor is diagonal. Make use of the result of (b).

Problem 2 Consider a thin rod of length l and mass m.

(a) Calculate the inertia tensor around the center of mass. Do not look up the table, but do the required integration yourself.

(b) Assume that the rod is hanging from the ceiling by two identical springs with spring constant k. The springs are attached to the ends of the rod. Find the normal modes of this system. Assume small amplitudes for the spring oscillation so that the ends of the springs and the center of mass of the rod move only vertically.



Problem 3 When a washing machine load is out of balance, it can lead to a very unpleasant situation as the drum of the washing machine knocks about. Analyze this situation qualitatively.

Problem 4 A bicycle wheel is hanging from a string as shown in the figure. It is spinning fast. Initially, a hand is supporting the axle of the wheel, as shown, and the wheel is just spinning. The next moment, the hand is withdrawn, and the wheel is let go. Discuss the subsequent motion of the wheel from the point of view of the equation of motion of a rigid body, $d\vec{L}/dt = \vec{N}$. Assume that the wheel is spinning very fast so that the "nutation" (the wobbling of the wheel up and down) can be ignored by averaging out.

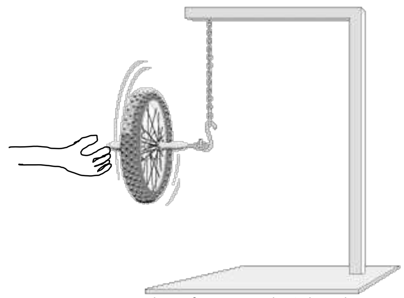


image from www.exploratorium.edu

Problem 5 A symmetric top is spinning fast around its axis of symmetry. Its center of mass is at distance l from the bottom tip. Its rotational inertia around the axis of symmetry is given by Mr^2, where r is the "radius of gyration" and M is the total mass. The bottom tip experiences a sliding friction μn, where $\vec{n}$ is the normal force, $n = |\vec{n}|$ and μ is the coefficient of friction. Assume that the top is spinning very fast so that (1) the nutation can be ignored (or averaged out), and (2) the effect of the gravity (or the normal force) and the friction is a small perturbation. Show that the effect of the gravity is to make the top precess slowly, while the effect of the friction is to make the top rise slowly. Given the initial angle θ_0 and the spin angular speed ω_3, find the time for the top to rise up to the vertical position, and the number of precessions that the top goes through during that time.