Due ~~Nov. 29, Tuesday~~ Dec. 1, Thursday.
70 points total. 30 points for the extra credit (but you are advised to understand all problems).

Problem 1 (20 points) Find the center of mass for each of the following many particle systems.

(a) Three particles, two with mass m and one with mass $2m$, located at the three vertices of an equilateral triangle.

(b) A uniform density sphere with radius R, with a hollow spherical cavity with radius $R/2$. The centers of the two spherical shapes are separated by distance $R/2$.

(c) A uniform density cone, with height h.

(d) A uniform density cone with the top half (pointy part, half in terms of height) removed.

Problem 2 (10 points) Consider a general collision between two particles. Show that

$$|\vec{u}| = |\vec{v}|$$

for an elastic collision and that

$$|\vec{u}| > |\vec{v}|$$

for an inelastic collision (for which the total kinetic energy decreases). Here, $\vec{u}$ and $\vec{v}$ are defined as the relative velocities

$$\vec{u} \equiv \vec{u}_1 - \vec{u}_2$$

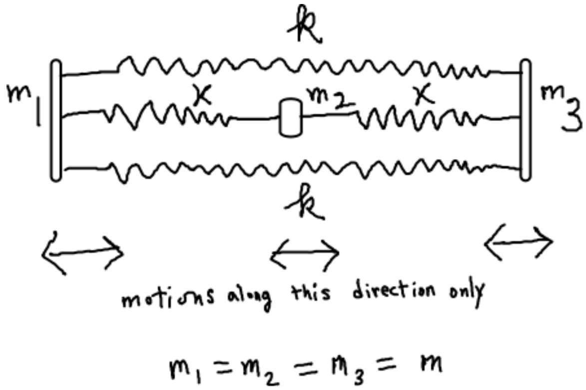
$$\vec{v} \equiv \vec{v}_1 - \vec{v}_2$$

where, as in LN 15, $\vec{u}_1$ and $\vec{u}_2$ are the velocities of m_1 and m_2 respectively in the initial state and $\vec{v}_1$ and $\vec{v}_2$ are the corresponding velocities in the final state. Note that this proof should be for general collision, not just head-on collision. However, note that by proving the above, you are also showing that the coefficient of restitution ε (LN 15), defined as $|\vec{v}|/|\vec{u}|$ for head-on collision, is strictly less than 1 for an inelastic collision, and is equal to 1 only for an elastic collision.

Problem 3 (10 points) 9-39 of the textbook.

Problem 4 (10 points) 9-46 of the textbook. Note that $\sigma(\theta)$ of the book corresponds to $d\sigma/d\Omega'$ of the LN. Also, note that the total crossection is defined as $\sigma_t \equiv \int d\sigma = \int \frac{d\sigma}{d\Omega'} d\Omega'$ for all scattered trajectories.

Problem 5 (20 points) Consider the following coupled oscillator.



Relative to the equilibrium state, depicted here, the three masses (all of equal mass m) are free to move along the horizontal direction as indicated. The displacements of the three masses relative to equilibrium can be defined as x_1, x_2 and x_3. The spring constants are k (long springs) and κ (short springs).

(a) Find all normal mode frequencies $\omega_1, \omega_2, \omega_3$.

(b) Find the $\overleftrightarrow{T}$ matrix (each column representing the eigenvector for each eigenvalue), and sketch each normal mode excitation.

(c) Find normal mode coordinates η_1, η_2, η_3 in terms of x_1, x_2, x_3.

Problem 6 (20 points) Consider the example that was studied in Lecture 16.

(a) Assume that the initial conditions are: $x_1 = x_2 = 0$ and $\dot{x}_1 = 2v > 0$ and $\dot{x}_2 = 0$. Determine D_1, D_2, ϕ_1, ϕ_2 (notation as in LN16, page 11) and, thus, find $x_1(t)$ and $x_2(t)$ at all times.

(b) We expect, for any initial condition, that the total energy of this system is the sum of the energies for the two normal modes, and that the energy of each normal mode is conserved. So, we expect these general properties to be valid for the solutions $x_1(t)$ and $x_2(t)$ of (a). By direct evaluation of the energy associated with $x_1(t)$ and $x_2(t)$, show that indeed the total energy is constant and is a sum of two parts, each of which is also constant, arising from the two normal modes.

Problem 7 (10 points) 12.6 of the textbook. [Here, you cannot use our beloved "generalized eigenvalue equation," as there are damping terms. You need to do this problem using the Newtonian method. You have two options. (1) Use the standard method of putting $x_1 = u\exp(i\omega t)$ and $x_2 = v\exp(i\omega t)$ to obtain normal modes. (2) In this "simple and symmetric" case, you can add and subtract two equations of motion to get the normal modes.]