Due Nov. 17, Thursday.

Problem 1 (20 points) Consider a cylindrical shell with uniform density and infinite length. The cylinder parallel to the z direction. Mass exists between radii a and b ($a < b$) while no mass exists for $\rho < a$ and $\rho > b$, where $\rho = \sqrt{x^2 + y^2}$. Let the mass per unit length along the cylinder be A.

(a) Find the field $\vec{g}$ due to the cylinder at any position (including inside the cylinder). [Use the Gauss law.]

(b) Find the potential Φ due to the cylinder. In the cylindrical coordinate system (ρ, ϕ, z), sketch Φ as a function of ρ starting from 0 to a value larger than b.

(c) Now consider the situation where there are two of these cylinders separated by distance $2D$ with $D > b$. Find the equilibrium point and discuss its stability with respect to the displacement in the xy plane mathematically and physically.

Problem 2 (10 points) Imagine drilling a straight hole through the Earth. This hole may or may not pass through the center of the Earth, so assume that it does not in general. For this problem, ignore the Earth spin and the size of the hole relative to the size of the Earth. Also, consider the mass density of the Earth as uniform. Drop a particle into the hole. What kind of motion will the particle experience, assuming that there is no other force than the gravity of the Earth acting on the particle? Express the time taken for the particle to appear on the other end of the hole, in terms of G, M_E, R_E, where M_E and R_E are the earth mass and the earth radius, respectively. How long is that time in minutes (two sig-figs)?

Problem 3 (5 points) Assume that the Earth's orbit around the Sun is circular. Suddenly, the Sun's mass decreases by half, and the Earth's mass increases two-fold (with the speed unchanged). What is the new orbit of the Earth? Will the Earth escape the Sun? [Hint: Start by considering the relations between E, T, and U for a circular motion.]

Problem 4 (10 points) 8-16 of the textbook. Assume the Kepler problem $U = -k/r$.

Problem 5 (15 points) A spacecraft is parked in a circular orbit 600 km above Earth's surface. We want to use a Hohmann transfer to send the spacecraft to the Moon's orbit. What are the total Δv and the transfer time required?

Problem 6 (20 points) **Collision.** A circular disk of total mass M and radius R is mounted horizontally. It is mounted in such a way that it is free to rotate horizontally around a vertical pole that fits tightly in a small hole at the center of the disk. The rotational inertia for the disk is $\frac{1}{2}MR^2$ to a good approximation, assuming that the central hole is small. A monkey of mass m runs and lands on the edge of the disk with a horizontal velocity. Ignore the size of the monkey relative to the radius R.

(a) The event of the monkey landing on the disk is a collision. Is the total angular momentum conserved during this collision? Is the total kinetic energy conserved (i.e. is it an elastic collision)? Is the total linear momentum conserved? For each answer, briefly explain why (in terms of torque/work/force or any Newtonian concepts).

(b) Assume that the monkey's initial velocity is $v_r\hat{r} + v_\theta\hat{\theta}$, where r, θ define the polar coordinate system of the disk. What is the angular velocity of the subsequent motion?

(c) In this subsequent motion, sketch the trajectory of the center of mass of the disk+monkey system. Identify the external force that is responsible for this motion of the center of mass.

(d) Now, let us assume that the disk is mounted vertically. The surface gravity is g, downward. The monkey falls down vertically and lands on the disk. For this collision, answer the same set of questions of (a). As usual, assume that the time of collision is very short.

(e) In this vertical setup, assume that the monkey's speed just before it lands on the disk is v. Let the horizontal distance of the monkey from the center of the disk prior to the collision be b (so this is just like the impact parameter). Find the minimum value of v, as a function of m, M, R and b, for which in the subsequent motion the sign of $\dot{\theta}$ does not change (i.e. the motion is a rotation instead of an oscillation).