Due Oct. 13, Thursday.

Problem 1 (10 points) Consider the projectile motion problem with air resistance $-mk\vec{v}$, as we solved it in class. We consider the optimum throw angle that maximizes the horizontal range of the motion, for a fixed initial speed v_0. Recall that the optimum throw angle is 45 degrees, if air resistance did not exist.

(a) Based on the first equation ($x = x_0 + \cdots$.) of section 3.4[1], provide rigorous arguments as to why the optimal throw angle is less than 45 degrees when air resistance exists ($k > 0$). The discussions in pages 9 and 10 of LN 3 may be helpful, but they should be made more rigorous. A simple argument such as "less time, less air resistance" is of vague general validity, but would not be rigorous at all. The result of LN 3, pages 7-9, regarding the time of flight T, can be used as a (small) part of the argument.

(b) In the limit of $\alpha \stackrel{def}{=} kv_{y,0}/g \ll 1$, obtain the expression for the optimum throw angle up to the leading order correction due to the air resistance.

Problem 2 (10 points) Consider Example T2.12. Let us solve this example without using the potential concept at all. Use free body diagrams only, to answer the following questions.

(a) Show that an equilibrium can exist only if $2m_1 > m_2$. Also, find $x_{1,0}$ and $x_{2,0}$, the values of x_1 and x_2 at equilibrium, in terms of m_1, m_2, b (total length of the string) and d (the half width of the setup – see the figure in the textbook).

(b) Now, consider displacing x_1 slightly away from the equilibrium value. Show that the equation of motion reduces to the simple harmonic motion, thereby proving that the equilibrium point is a stable one. Find the angular frequency ω of the motion.

Problem 3 (10 points) 2-53 of the textbook.

Problem 4 (5 points) Reconsider the problem 4 of the previous homework. Instead of using the trick $a = dv/dt = v(dv/dy)$ (which may be elusive at times?), use the work-energy theorem to do the problem. You do not need to re-do all the steps. At an appropriate point, you can say "the rest is the same."

Problem 5 (10 points) Consider a compass. Let the magnetic moment of the compass needle be $\vec{M}$, whose magnitude is constant, and the magnetic field in which the compass is immersed be a constant vector, $\vec{B}$. Assume that the compass

[1] Any section number, equation number, or figure number, not preceded by letter "T" means an entry in my lecture note, not in the textbook. That is, section T2.1 means a section in the textbook, while section 3.4 means one in the lecture note.

needle is free to rotate, i.e. rotate without any friction, in the horizontal plane and (for simplicity) that the $\vec{B}$ field has no vertical component.
The potential energy is given by

$$U = -\vec{M} \cdot \vec{B}$$

Let the moment of inertia of the compass needle be I.

(a) Draw a diagram showing the stable equilibrium position of the needle. Mark the magnetic north and south of the needle, as well as those for the $\vec{B}$ field.

(b) Prove that the equilibrium is indeed a stable equilibrium.

(c) Find the angular frequency ω of the small angle oscillation around the equilibrium point, in terms of M, B and I.

(d) Consider oscillation around the equilibrium point, and consider an arbitrary time t and a small time interval dt around it. During the time interval dt, does the field $\vec{B}$ do any work on the compass needle?

(e) (Extra credit, 5 points) As we learned in LN 4, the Lorentz force does not do any work, as the power $P = \vec{F} \cdot \vec{v} = 0$. Considering this fact and your answer in the previous part, determine which of the following statements must be true. Provide some corroborating argument for your answer – for example, information that you may acquire independently, e.g. by google search on reputable sources, could be helpful. (1) Everything is fine. The magnetic field never does any work, really. (2) The magnetic field does work in a situation like this. Which means that the Lorentz force is not the only force. There must be some other force that we have not learned yet. (3) The magnetic field does work in a situation like this. While this is in contradiction to the Lorentz force doing no work, it does not mean much. The microscopic origin of the magnetism in solids is not classical mechanics (e.g. Lorentz force) at all, despite unfortunate misconceptions sometimes taught.

Problem 6 (5 points) Prove that

$$\frac{1}{\tau_n} \int_{t_0}^{t_0+\tau_n} dt \sin^2(\omega t - \delta) = \frac{1}{2}$$

$$\frac{1}{\tau_n} \int_{t_0}^{t_0+\tau_n} dt \cos^2(\omega t - \delta) = \frac{1}{2}$$

where $\tau_n = n\tau/2$ ($n =$ non-zero integer), $\tau \stackrel{def}{=} 2\pi/\omega$ (period), and t_0, δ are any real constants.