

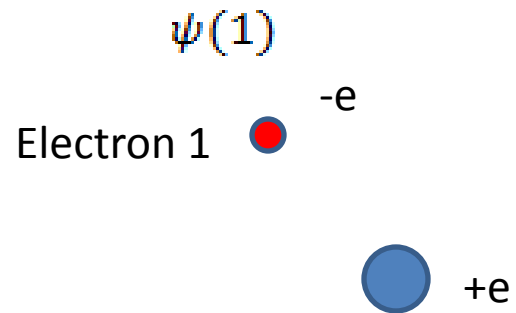
# Announcements

- Mid-term exam: 01/28, Tues, in class
  - Chapters 7 and 8
  - Open book and open lecture notes but no personal notes or electronics
  - 6 question sets
    - 1 question from each homework set
    - 1 true/false question (multiple sub questions)
    - Learning experience while taking exams
  - More conceptual, simple calculations if any.
  - Review session (235 ISB, Friday, at 3:30 pm)

# Lecture 6 Topics:

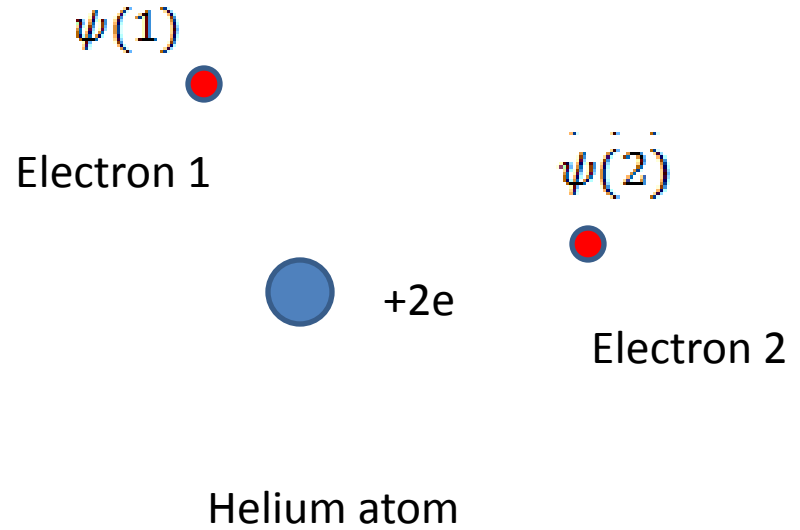
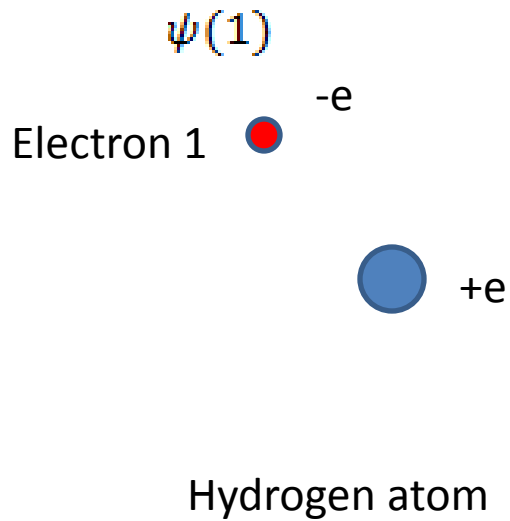
- Identical particles
- Periodic table trends
- Paschen-Back Effect
- Zeeman Effect

# Identical particles



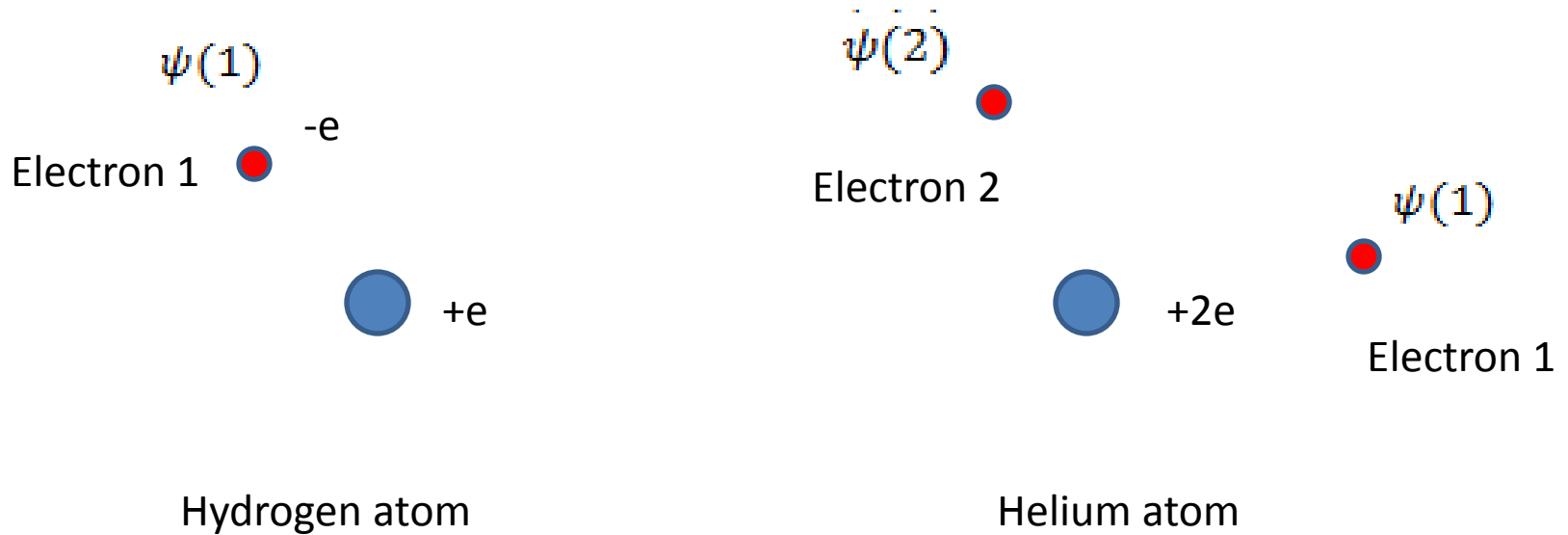
Hydrogen atom

# Identical particles



$$|\psi(1,2)\rangle = \psi(1)\psi(2)$$

# Identical particles

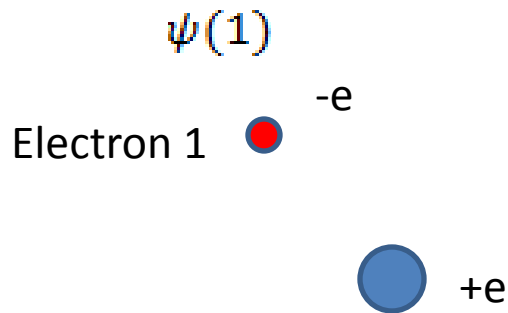


$$|\psi(1,2)\rangle = \psi(1)\psi(2)$$

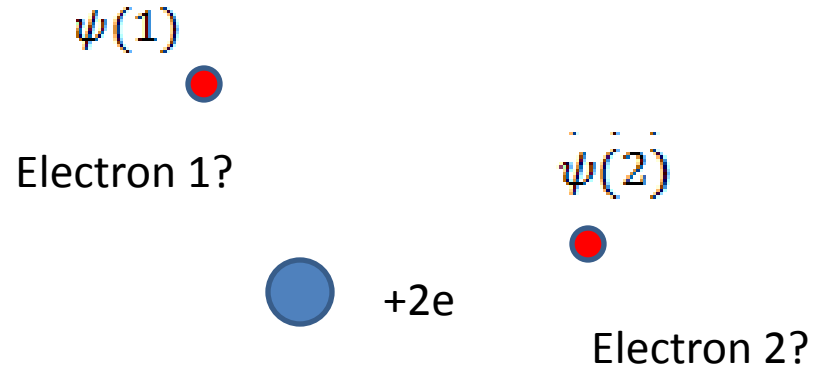
After exchanging two particles

$$|\psi(2,1)\rangle = \psi(2)\psi(1)$$

# Identical particles



Hydrogen atom



Helium atom

$$|\psi(1,2)|^2 = |\psi(2,1)|^2$$

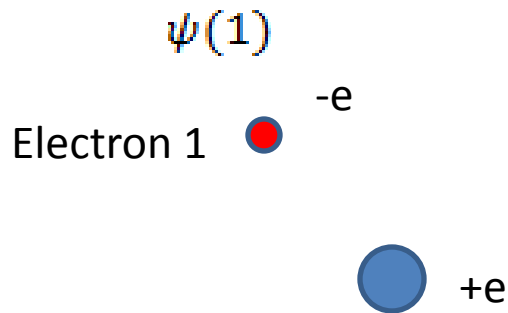
$$\begin{aligned} \psi(1,2) &= \psi(2,1) \\ \psi(1,2) &= -\psi(2,1) \end{aligned}$$

$$\psi(1,2) = \psi(1)\psi(2)$$

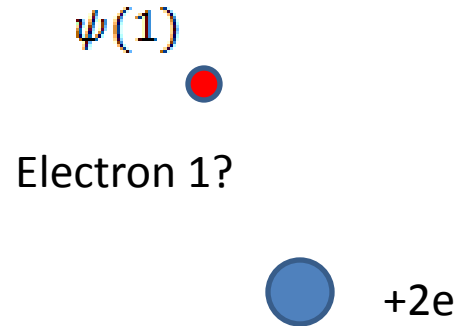
After exchanging two particles

$$\psi(2,1) = \psi(2)\psi(1)$$

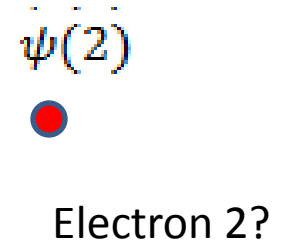
# Identical particles



Hydrogen atom



Helium atom



$$|\psi(1,2)|^2 = |\psi(2,1)|^2$$

$$\psi(1,2) = \psi(2,1)$$
$$\psi(1,2) = -\psi(2,1)$$

Symmetric under exchange operation

Anti-symmetric

When Particle 1 is in  $n$  state and Particle 2 is in  $n'$  state

$$\psi_I = \psi_n(1)\psi_{n'}(2)$$

When Particle 2 is in  $n$  state and Particle 1 is in  $n'$  state

$$\psi_{II} = \psi_n(2)\psi_{n'}(1)$$

$$|\psi(1,2)|^2 = |\psi(2,1)|^2$$

$$\psi_{Symmetric} = \frac{1}{\sqrt{2}}[\psi_n(1)\psi_{n'}(2) + \psi_n(2)\psi_{n'}(1)]$$

$$\psi_{Anti-Symmetric} = \frac{1}{\sqrt{2}}[\psi_n(1)\psi_{n'}(2) - \psi_n(2)\psi_{n'}(1)]$$

# Bosons

$\psi_{\text{Symmetric}}$  satisfies  $\psi(1,2) = \psi(2,1)$

- Systems of bosons are described by wave functions that are symmetric upon the exchange of any pair of bosons.
- Bosons are integer spin particles such as
  - Photon ( $s=1$ ) Deuteron ( $s=1$ )
  - Pion ( $s=0$ )
  - Helium nucleus (alpha particle;  $s=0$ )
- Bosons can occupy the same quantum state.

# Fermions

$\psi_{\text{Anti-Symmetric}}$  satisfies  $\psi(1,2) = -\psi(2,1)$

- Systems of fermions are described by wave functions that reverse sign upon the exchange of any pair of electrons.
- Fermions' spin numbers are half-integer:
  - Electron, proton, and neutron =  $\frac{1}{2}$
- If  $n = n'$ ,  $\psi_{\text{Anti-Symmetric}} = ???$

$$\psi_{\text{Anti-Symmetric}} = \frac{1}{\sqrt{2}} [\psi_n(1)\psi_{n'}(2) - \psi_n(2)\psi_{n'}(1)]$$

# Fermions

$\psi_{\text{Anti-Symmetric}}$  satisfies  $\psi(1,2) = -\psi(2,1)$

- Systems of fermions are described by wave functions that reverse sign upon the exchange of any pair of electrons.
- Fermions' spin numbers are half-integer:
  - Electron, proton, and neutron =  $\frac{1}{2}$

• If  $n = n'$ ,  $\psi_{\text{Anti-Symmetric}} = 0$ .

$$\psi_{\text{Anti-Symmetric}} = \frac{1}{\sqrt{2}} [\psi_n(1)\psi_{n'}(2) - \psi_n(2)\psi_{n'}(1)]$$

Two fermions of the same type cannot occupy the same quantum state in an isolated system.

→ Exclusion Principle

# Periodic Table

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1	1																	2
1	H																	He
2	3	4											5	6	7	8	9	10
	Li	Be											B	C	N	O	F	Ne
3	11	12											13	14	15	16	17	18
	Na	Mg											Al	Si	P	S	Cl	Ar
4	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	55	56	57*	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
	Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	87	88	89**	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118
	Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Uuq	Uup	Uuh	Uus	Uuo

○ Non Metals	● Noble Gases
● Alkali Metals	● Metalloids
● Alkaline Metals	● Halogens
● Transition Metals	● Other Metals
● Rare Earth Elements	

\*Lanthanides

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

\*\*Actinides

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

# Periodic Table: Two basic rules

- A system of particles is stable when its total energy is a minimum
- Only one electron can exist in any particular quantum state in an atom (exclusion principle).

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1	1 H																	2 He
Period 2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
Period 3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
Period 4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
Period 5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
Period 6	55 Cs	56 Ba	57* La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
Period 7	87 Fr	88 Ra	89** Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

\*Lanthanides

58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
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\*\*Actinides

90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr
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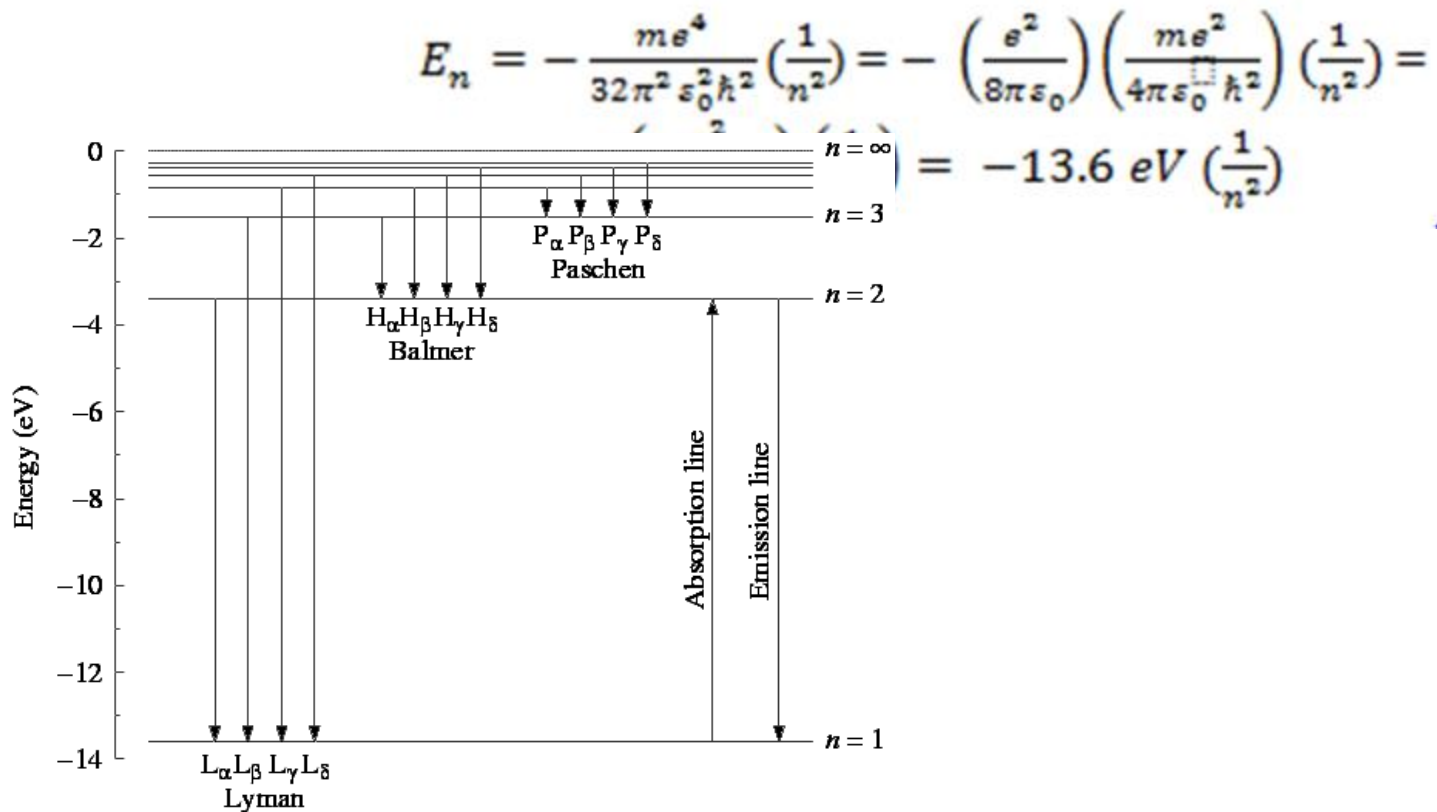
# Which quantum state has a lower energy?

- When  $n$  is lower.

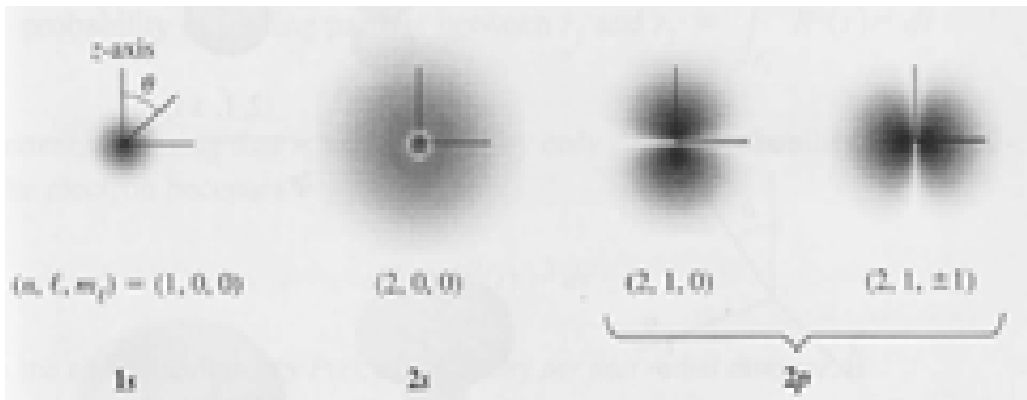
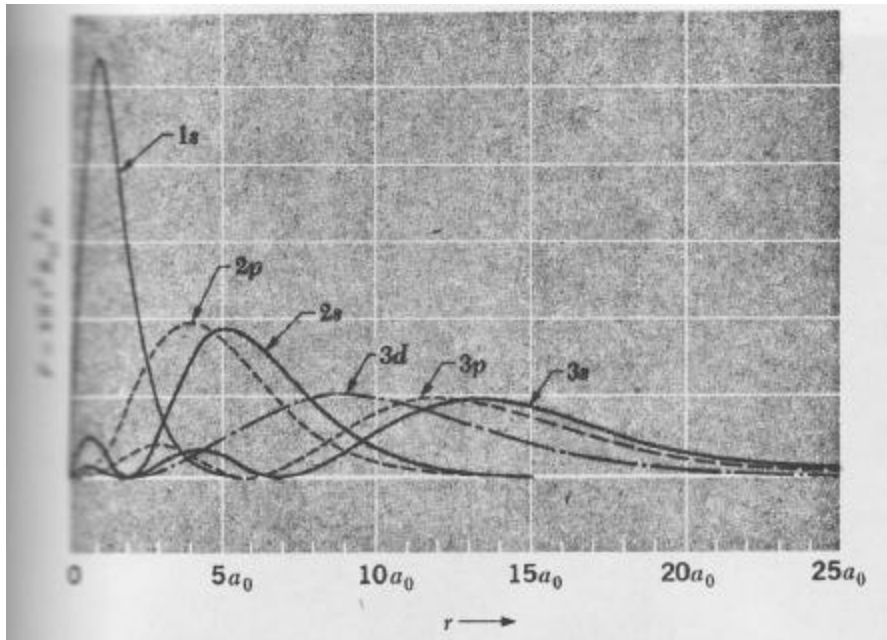
$$E_n = -\frac{m e^4}{32 \pi^2 \epsilon_0^2 \hbar^2} \left(\frac{1}{n^2}\right) = -\left(\frac{e^2}{8 \pi \epsilon_0}\right) \left(\frac{m e^2}{4 \pi \epsilon_0 \hbar^2}\right) \left(\frac{1}{n^2}\right) =$$
$$-\left(\frac{e^2}{8 \pi \epsilon_0 a_0}\right) \left(\frac{1}{n^2}\right) = -13.6 \text{ eV} \left(\frac{1}{n^2}\right)$$

# Which quantum state has a lower energy?

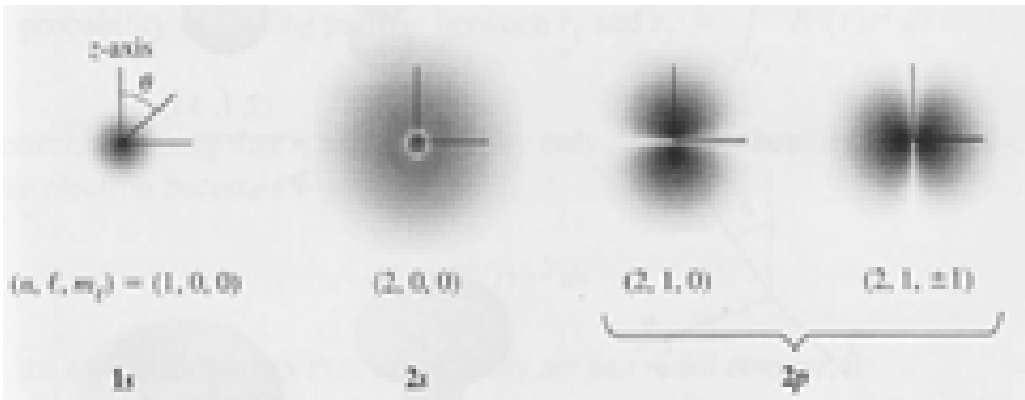
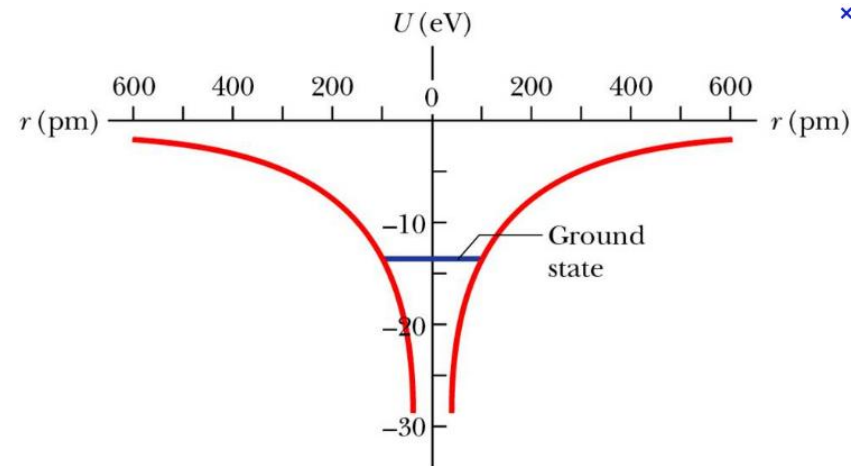
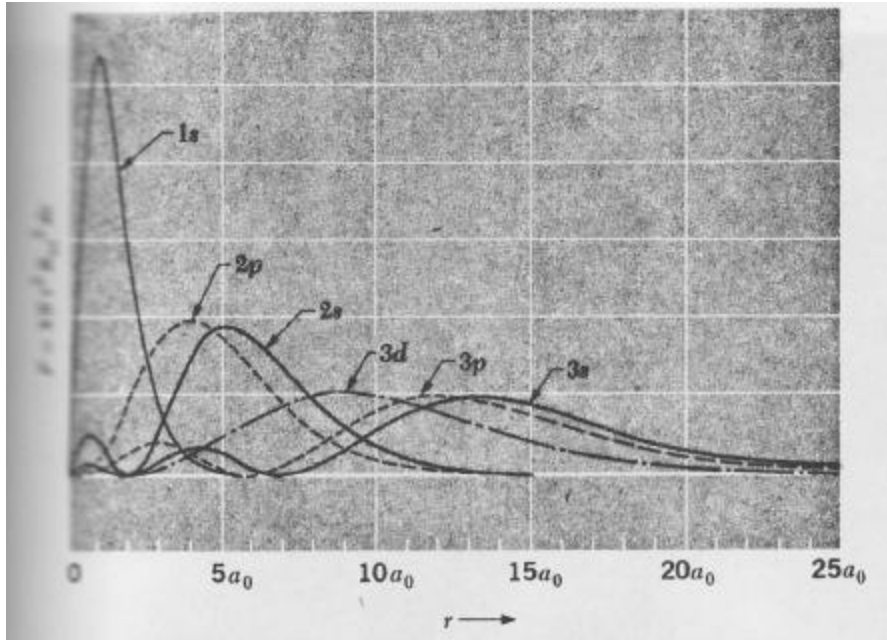
- When  $n$  is lower.



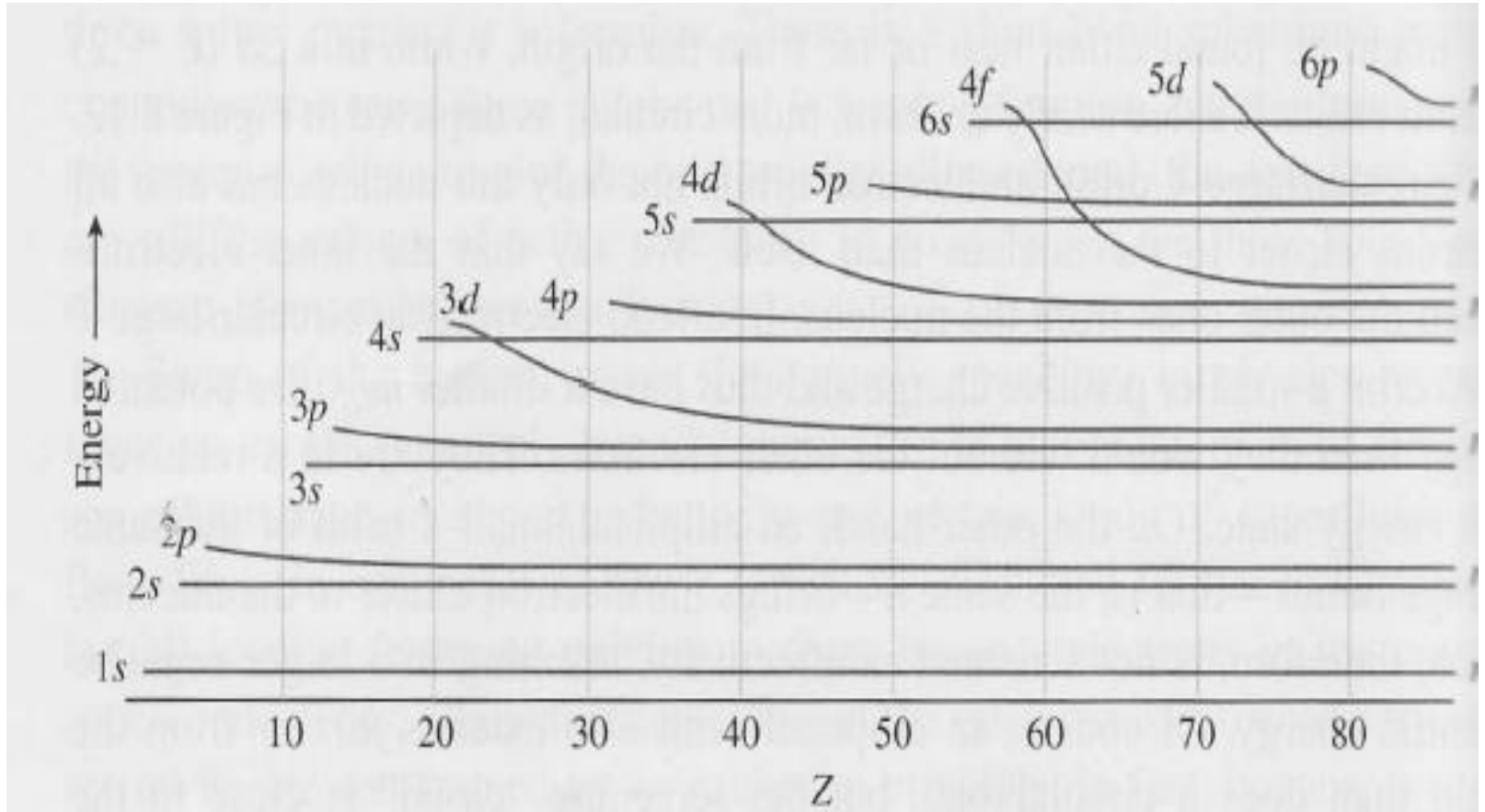
With a given  $n$ , when  $l$  is lower

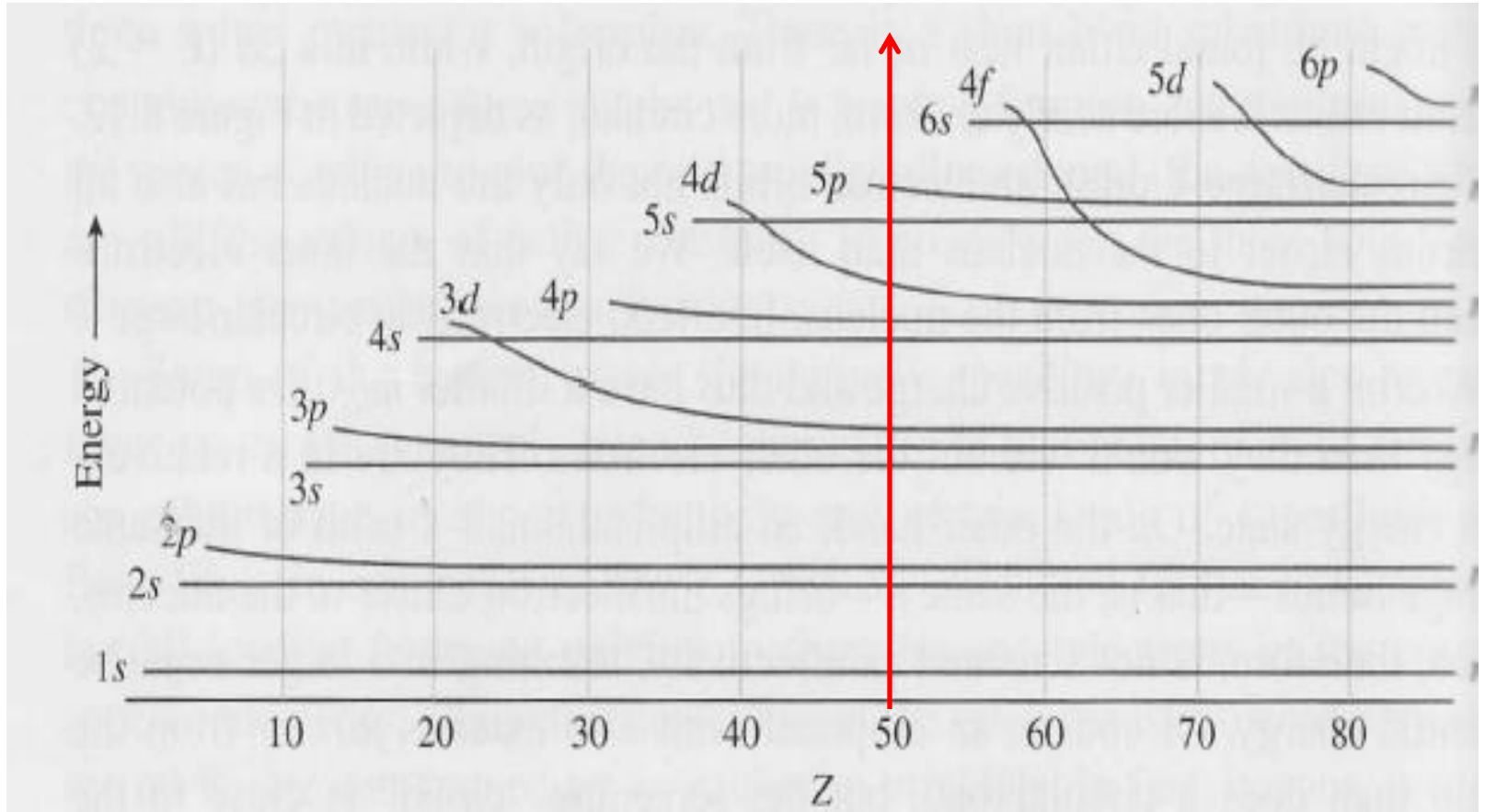


With a given  $n$ , when  $l$  is lower



$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{-e^2}{r}$$





With a given  $l$ , parallel spin arrangements lower energy.

wave function where electron 1 occupies  $n$  state and electron occupies  $n'$  state:

$$\psi(1,2) = \psi_{n n'}(1,2) \uparrow\uparrow$$

If we exchange electron 1 and electron 2, a wave function becomes

$$\psi(2,1) = \psi_{n n'}(2,1) \uparrow\uparrow$$

According to the exclusion principle,

$$\psi(1,2) = -\psi(2,1)$$

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If  $n = n'$ ?

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If  $n = n'$ ?

$$\psi_{n n}(1,2) = -\psi_{n n}(2,1) = 0$$

$$\psi_{n n'}(1,2) \uparrow\uparrow = -\psi_{n n'}(2,1) \uparrow\uparrow$$

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→ Lower Coulomb Interaction (by decreasing repulsive interaction between two electrons)

# P orbital spin arrangements

	$p_x$	$p_y$	$p_z$
Lower E	↑		
Higher E			

# P orbital spin arrangements

	$p_x$	$p_y$	$p_z$
Lower E	↑		
	↑	↑	
Higher E			

# P orbital spin arrangements

4

	$p_x$	$p_y$	$p_z$
Lower E	↑		
	↑	↑	
	↑	↑	↑
	↑↓	↑	↑
Higher E	↑↓	↑↓	↑
	↑↓	↑↓	↑↓

□

# Periodic Table

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# Periodic Table: Two basic rules

- A system of particles is stable when its total energy is a minimum
- Only one electron can exist in any particular quantum state in an atom (exclusion principle).
- Extending from energy levels conceived from the hydrogen atom solutions

$$H\psi_{n,l,m_l} = E_n\psi_{n,l,m_l}$$

$$L^2\psi_{n,l,m_l} = l(l+1)\hbar^2\psi_{n,l,m_l}$$

$$L_z\psi_{n,l,m_l} = m_l\hbar\psi_{n,l,m_l}$$

- Lower Energy can be obtained
  - When  $n$  is lower.
  - With a given  $n$ , when  $l$  is lower.
  - With a given  $l$ , parallel spin arrangements lower energy.

# What we know so far:

- Number of electrons in each atom
- Electrons should be in one of the orbitals determined by  $n$ ,  $l$ , and  $m_l$ 
  - $n$  limits what types of  $l$  orbitals an electron can occupy.
  - Each  $l$  orbital has  $2l+1$  possible  $m_l$  states:
    - $s$  orbital ( $l=0$ ) = 1
    - $p$  orbital ( $l=1$ ) =  $2 \times 1 + 1 = 3$
    - $d$  orbital ( $l=2$ ) =  $2 \times 2 + 1 = 5$
    - $f$  orbital ( $l=3$ ) =  $2 \times 3 + 1 = 7$
- Due to electron's spin where  $\frac{1}{2}$  and  $-\frac{1}{2}$  are possible, each  $m_l$  orbital can have two additional possible states.
- As a result, in each  $n$ , there are  $2n^2$  possible energy states.

**TABLE 8.2** Subshell ordering and capacity

Subshell $n\ell$	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s	4f	5d	6p	7s	5f	6d
$n + \ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell + 1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10

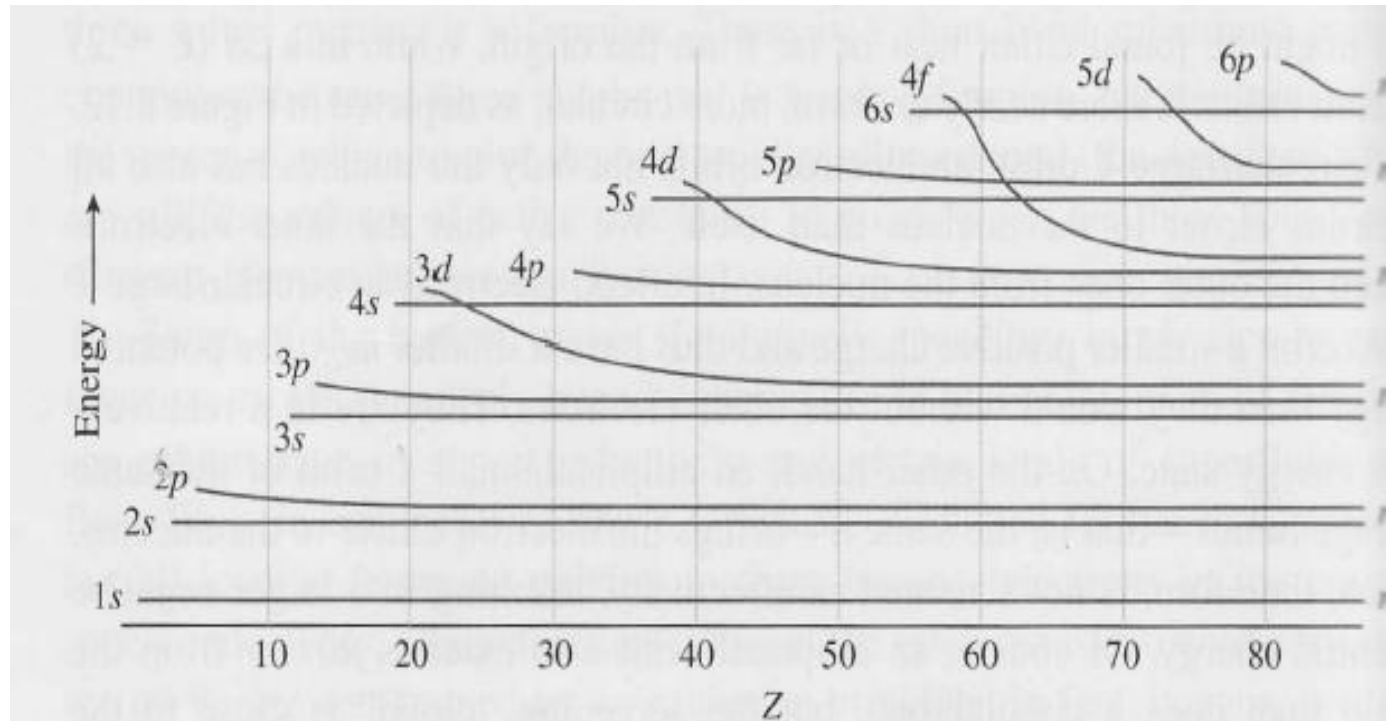


TABLE 8.2 Subshell ordering and capacity

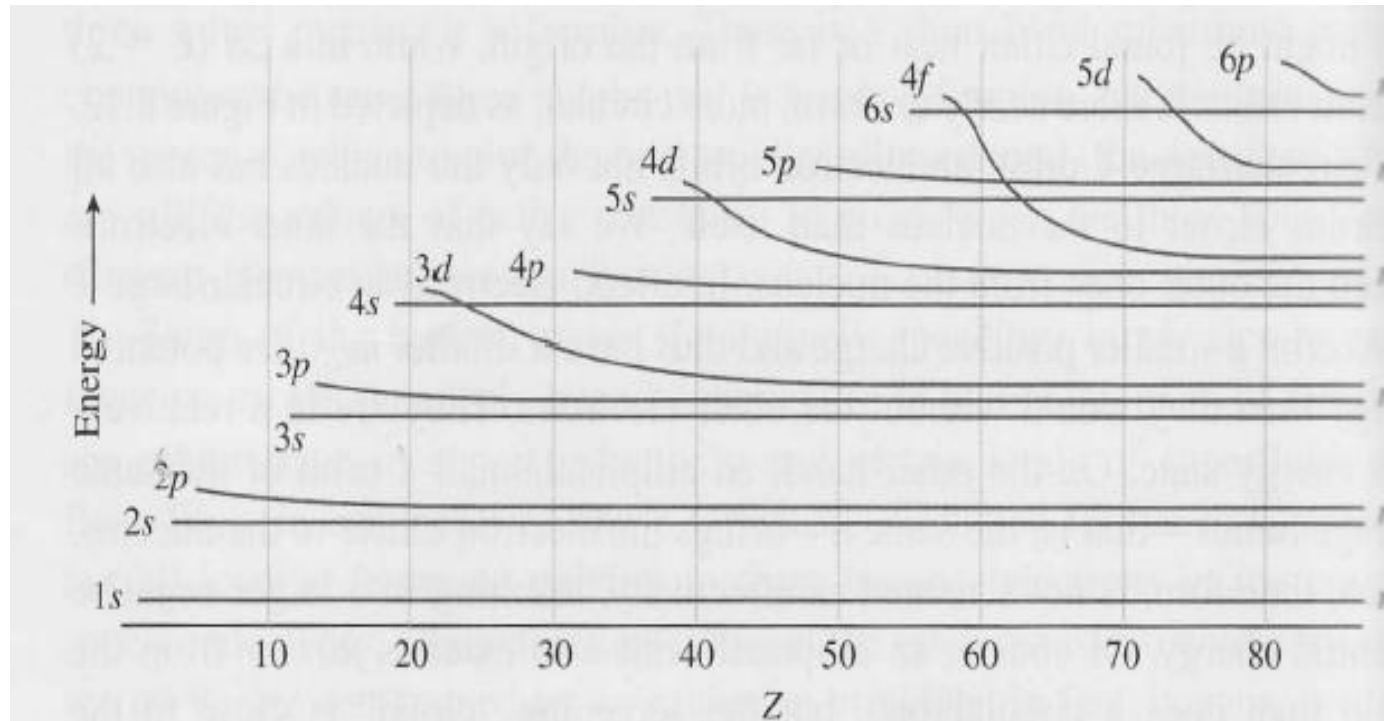
Subshell $n\ell$	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s	4f	5d	6p	7s	5f	6d
$n + \ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell + 1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10

	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	5s	5p	5d	5f	6s	6p	6d	7s
1 H	1																	
2 He	2																	
3 Li	2	1																
4 Be	2	2																
5 B	2	2	1															
6 C	2	2	2															
7 N	2	2	3															
8 O	2	2	4															
9 F	2	2	5															
10 Ne	2	2	6															
11 Na	2	2	6	1														
12 Mg	2	2	6	2														
13 Al	2	2	6	2	1													
14 Si	2	2	6	2	2													
15 P	2	2	6	2	3													
16 S	2	2	6	2	4													
17 Cl	2	2	6	2	5													
18 Ar	2	2	6	2	6													
19 K	2	2	6	2	6						1							
20 Ca	2	2	6	2	6						2							
21 Sc	2	2	6	2	6	1	2											
22 Ti	2	2	6	2	6	2	2											
23 V	2	2	6	2	6	3	2											
24 Cr	2	2	6	2	6	5	1											
25 Mn	2	2	6	2	6	5	2											
26 Fe	2	2	6	2	6	6	2											
27 Co	2	2	6	2	6	7	2											
28 Ni	2	2	6	2	6	8	2											
29 Cu	2	2	6	2	6	10	1											
30 Zn	2	2	6	2	6	10	2											
31 Ga	2	2	6	2	6	10	2	1										
32 Ge	2	2	6	2	6	10	2	2										
33 As	2	2	6	2	6	10	2	3										
34 Se	2	2	6	2	6	10	2	4										
35 Br	2	2	6	2	6	10	2	5										
36 Kr	2	2	6	2	6	10	2	6										
37 Rb	2	2	6	2	6	10	2	6			1							
38 Sr	2	2	6	2	6	10	2	6			2							
39 Y	2	2	6	2	6	10	2	6	1		2							
40 Zr	2	2	6	2	6	10	2	6	2		2							
41 Nb	2	2	6	2	6	10	2	6	4		1							
42 Mo	2	2	6	2	6	10	2	6	5		1							
43 Tc	2	2	6	2	6	10	2	6	5		2							
44 Ru	2	2	6	2	6	10	2	6	7		1							
45 Rh	2	2	6	2	6	10	2	6	8		1							
46 Pd	2	2	6	2	6	10	2	6	10		1							
47 Ag	2	2	6	2	6	10	2	6	10		1							
48 Cd	2	2	6	2	6	10	2	6	10		2							
49 In	2	2	6	2	6	10	2	6	10		2	1						
50 Sn	2	2	6	2	6	10	2	6	10		2	2						
51 Sb	2	2	6	2	6	10	2	6	10		2	3						
52 Te	2	2	6	2	6	10	2	6	10		2	4						

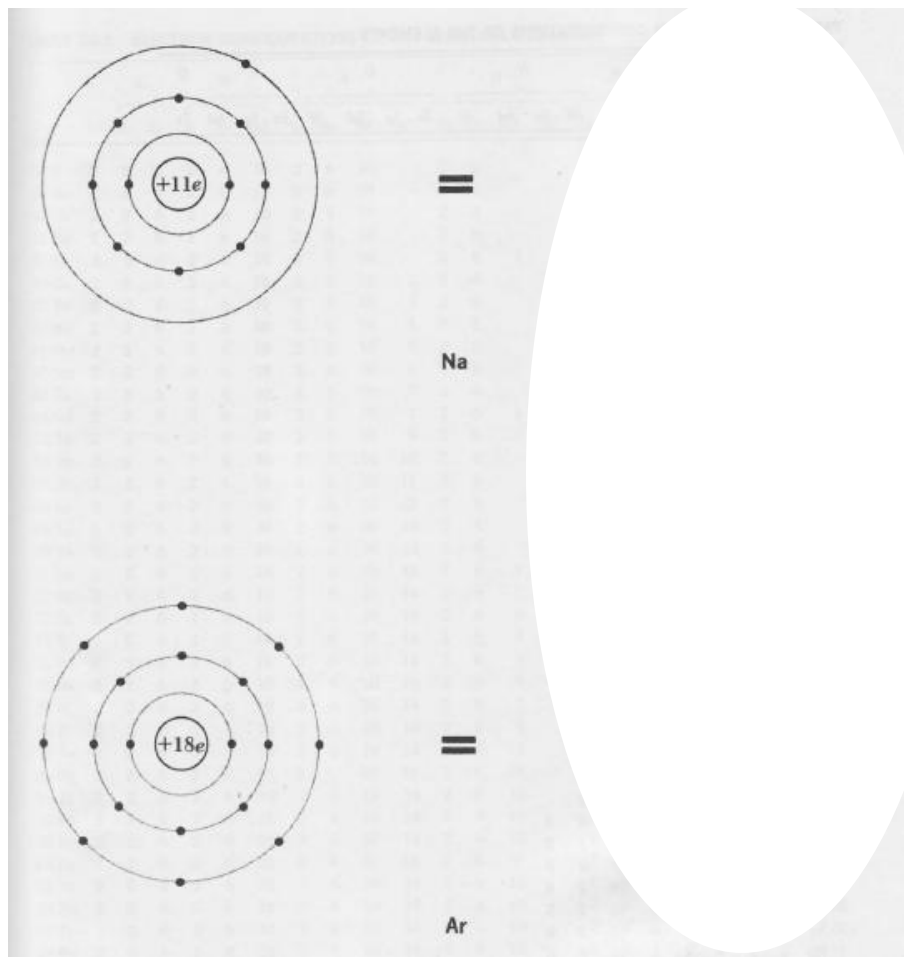
	1s	2s	2p	3s	3p	3d	4s	4p	4d	4f	5s	5p	5d	5f	6s	6p	6d	7s
53 I	2	2	6	2	6	10	2	6	10		2	5						
54 Xe	2	2	6	2	6	10	2	6	10		2	6						
55 Cs	2	2	6	2	6	10	2	6	10		2	6					1	
56 Ba	2	2	6	2	6	10	2	6	10		2	6					2	
57 La	2	2	6	2	6	10	2	6	10		2	6	1				2	
58 Ce	2	2	6	2	6	10	2	6	10		2	2	6				2	
59 Pr	2	2	6	2	6	10	2	6	10		3	2	6				2	
60 Nd	2	2	6	2	6	10	2	6	10		4	2	6				2	
61 Pm	2	2	6	2	6	10	2	6	10		5	2	6				2	
62 Sm	2	2	6	2	6	10	2	6	10		6	2	6				2	
63 Eu	2	2	6	2	6	10	2	6	10		7	2	6				2	
64 Gd	2	2	6	2	6	10	2	6	10		7	2	6	1			2	
65 Tb	2	2	6	2	6	10	2	6	10		9	2	6				2	
66 Dy	2	2	6	2	6	10	2	6	10		10	2	6				2	
67 Ho	2	2	6	2	6	10	2	6	10		11	2	6				2	
68 Er	2	2	6	2	6	10	2	6	10		12	2	6				2	
69 Tm	2	2	6	2	6	10	2	6	10		13	2	6				2	
70 Yb	2	2	6	2	6	10	2	6	10		14	2	6				2	
71 Lu	2	2	6	2	6	10	2	6	10		14	2	6	1			2	
72 Hf	2	2	6	2	6	10	2	6	10		14	2	6	2			2	
73 Ta	2	2	6	2	6	10	2	6	10		14	2	6	3			2	
74 W	2	2	6	2	6	10	2	6	10		14	2	6	4			2	
75 Re	2	2	6	2	6	10	2	6	10		14	2	6	5			2	
76 Os	2	2	6	2	6	10	2	6	10		14	2	6	6			2	
77 Ir	2	2	6	2	6	10	2	6	10		14	2	6	7			2	
78 Pt	2	2	6	2	6	10	2	6	10		14	2	6	9			1	
79 Au	2	2	6	2	6	10	2	6	10		14	2	6	10			1	
80 Hg	2	2	6	2	6	10	2	6	10		14	2	6	10			2	
81 Tl	2	2	6	2	6	10	2	6	10		14	2	6	10			2	1
82 Pb	2	2	6	2	6	10	2	6	10		14	2	6	10			2	2
83 Bi	2	2	6	2	6	10	2	6	10		14	2	6	10			2	3
84 Po	2	2	6	2	6	10	2	6	10		14	2	6	10			2	4
85 At	2	2	6	2	6	10	2	6	10		14	2	6	10			2	5
86 Rn	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
87 Fr	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
88 Ra	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
89 Ac	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
90 Th	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
91 Pa	2	2	6	2	6	10	2	6	10		14	2	6	10			2	6
92 U	2	2	6	2	6	10	2	6	10		14	2	6	10			3	2
93 Np	2	2	6	2	6	10	2	6	10		14	2	6	10			4	2
94 Pu	2	2	6	2	6	10	2	6	10		14	2	6	10			5	2
95 Am	2	2	6	2	6	10	2	6	10		14	2	6	10			6	2
96 Cm	2	2	6	2	6	10	2	6	10		14	2	6	10			7	2
97 Bk	2	2	6	2	6	10	2	6	10		14	2	6	10			8	2
98 Cf	2	2	6	2	6	10	2	6	10		14	2	6	10			10	2
99 E	2	2	6	2	6	10	2	6	10		14	2	6	10			11	2
100 Fm	2	2	6	2	6	10	2	6	10		14	2	6	10			12	2
101 Md	2	2	6	2	6	10	2	6	10		14	2	6	10			13	2
102 No	2	2	6	2	6	10	2	6	10		14	2	6	10			14	2
103 Lw	2	2	6	2	6	10	2	6	10		14	2	6	10			14	2

**TABLE 8.2** Subshell ordering and capacity

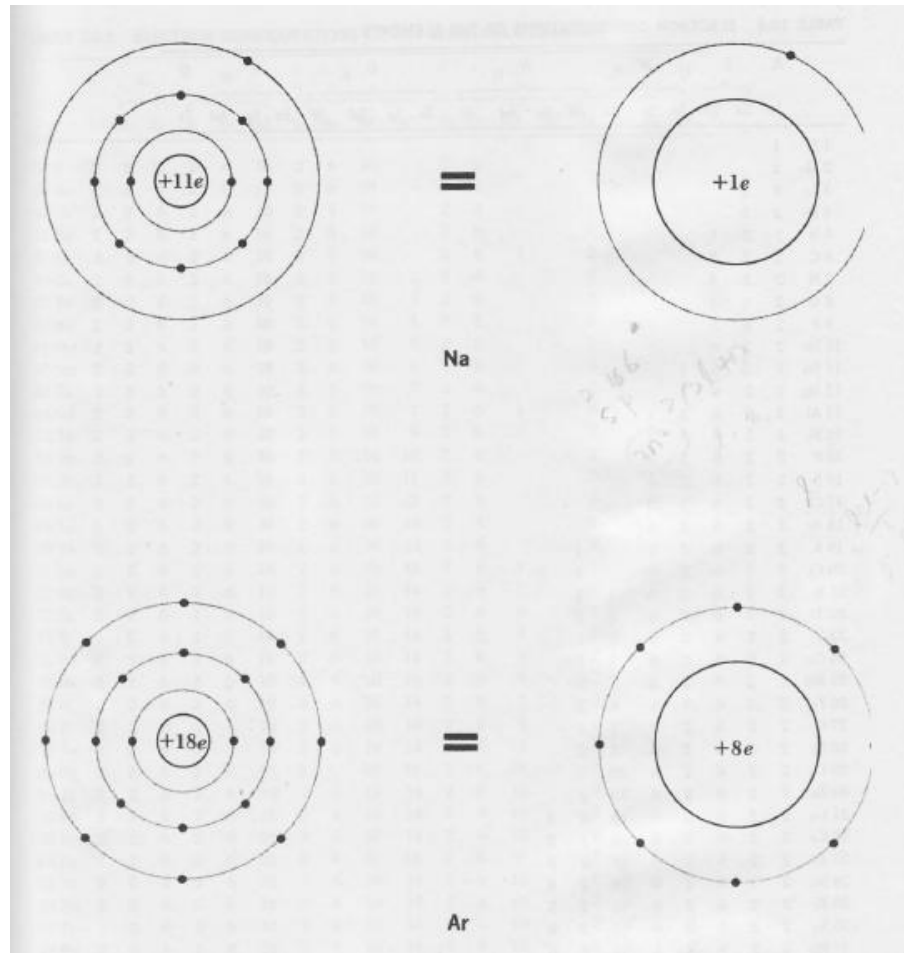
Subshell $n\ell$	1s	2s	2p	3s	3p	4s	3d	4p	5s	4d	5p	6s	4f	5d	6p	7s	5f	6d
$n + \ell$	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
Number of electrons $2(2\ell + 1)$	2	2	6	2	6	2	10	6	2	10	6	2	14	10	6	2	14	10



# Effective charge of the nucleus



# Effective charge of the nucleus



# Noble Gases

Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Period 1	1																	2	
1	H																		He
2	3	4											5	6	7	8	9	10	
	Li	Be											B	C	N	O	F	Ne	
3	11	12											13	14	15	16	17	18	
	Na	Mg											Al	Si	P	S	Cl	Ar	
4	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	
	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
5	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	
	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
6	55	56	57*	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	
	Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
7	87	88	89**	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	
	Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Uuq	Uup	Uuh	Uus	Uuo	

○ Non Metals	● Noble Gases
● Alkali Metals	● Metalloids
● Alkaline Metals	● Halogens
● Transition Metals	● Other Metals
● Rare Earth Elements	

*Lanthanides	58	59	60	61	62	63	64	65	66	67	68	69	70	71
	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu
**Actinides	90	91	92	93	94	95	96	97	98	99	100	101	102	103
	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

# Noble Gases

He (2) =  $1s^2$

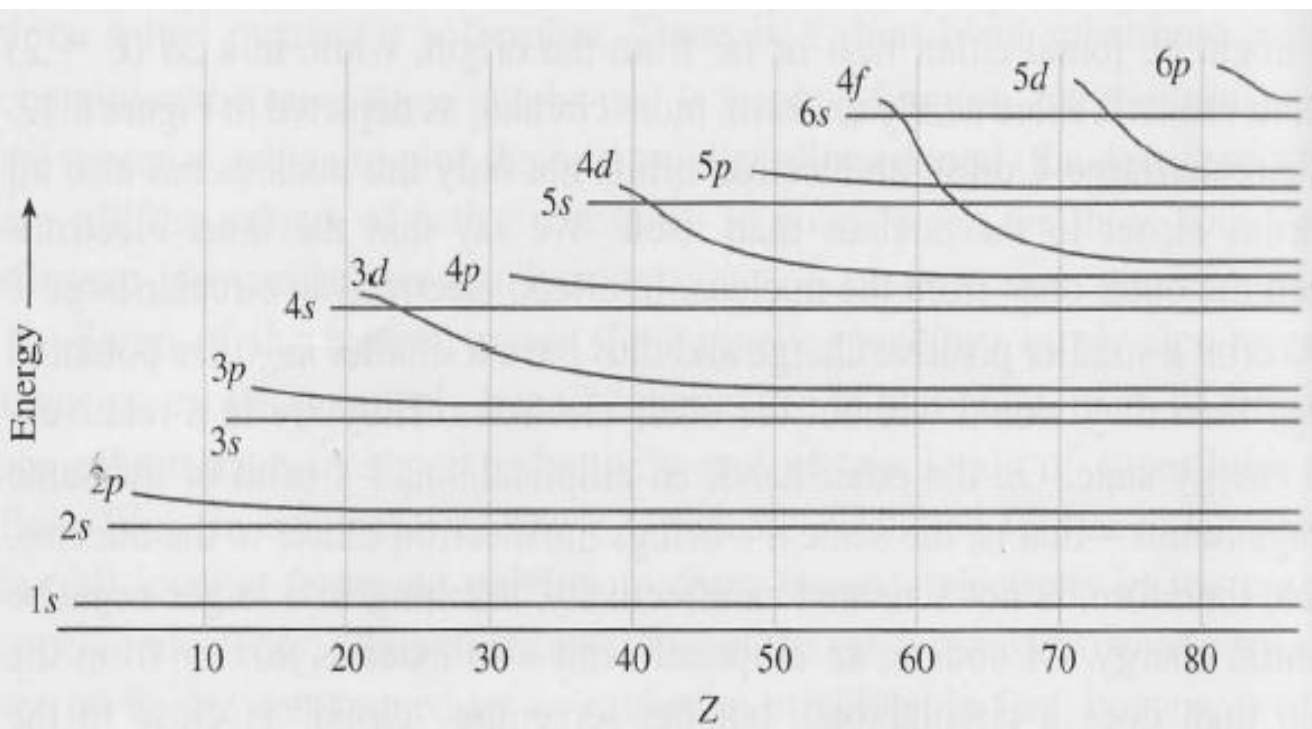
Ne (10) =  $1s^2 2s^2 2p^6$

Ar (18) =  $1s^2 2s^2 2p^6 3s^2 3p^6$

Kr (36) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$

Xe (54) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$

Rn (86) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$



# Noble Gases

He (2) =  $1s^2$

Ne (10) =  $1s^2 2s^2 2p^6$

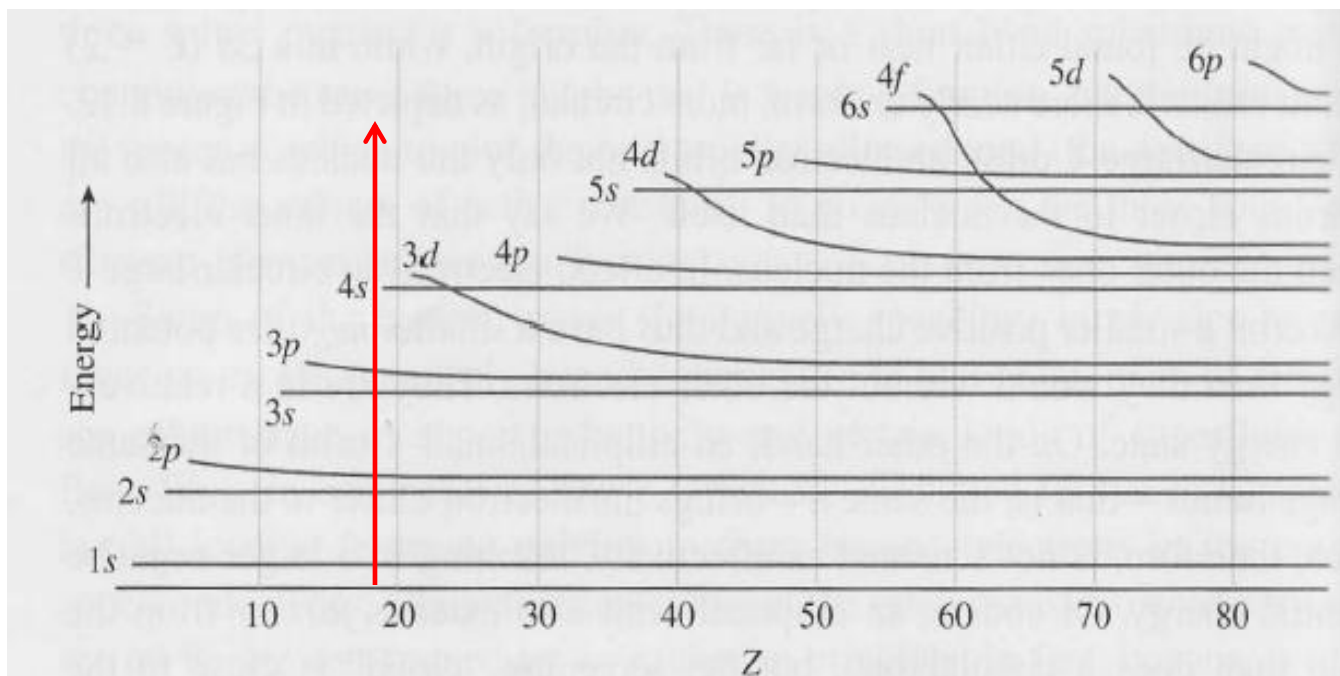
Ar (18) =  $1s^2 2s^2 2p^6 3s^2 3p^6$

Kr (36) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d_{10} 4s^2 4p^6$

Xe (54) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d_{10} 4s^2 4p^6 4d_{10} 5s^2 5p^6$

Rn (86) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d_{10} 4s^2 4p^6 4d_{10} 4f_{14} 5s^2 5p^6 5d_{10} 6s^2 6p^6$

3d ———  
3p ———  
3s ———  
2p ———  
2s ———  
1s ———



# Noble Gases

He (2) =  $1s^2$

Ne (10) =  $1s^2 2s^2 2p^6$

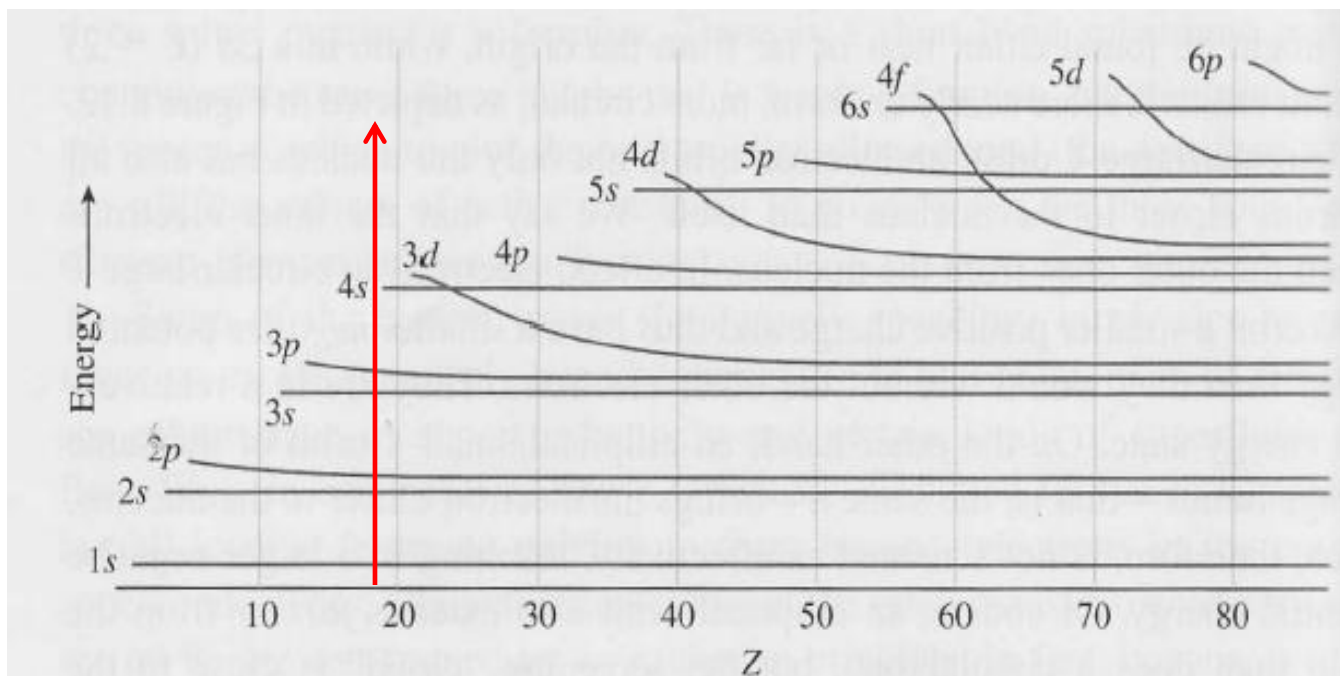
Ar (18) =  $1s^2 2s^2 2p^6 3s^2 3p^6$

Kr (36) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$

Xe (54) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$

Rn (86) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$

3d ———  
3p ———  
3s ———  
2p ———  
2s ———  
1s ———



# Noble Gases



He (2) =  $1s^2$

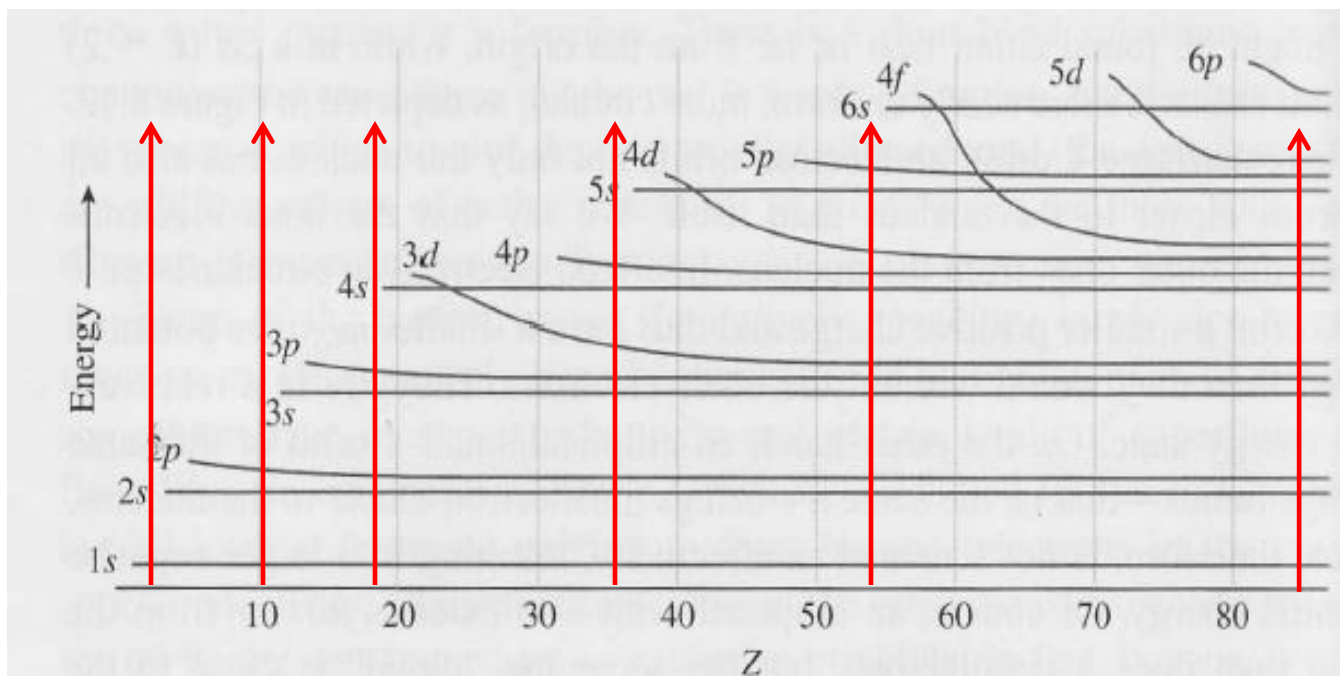
Ne (10) =  $1s^2 2s^2 2p^6$

Ar (18) =  $1s^2 2s^2 2p^6 3s^2 3p^6$

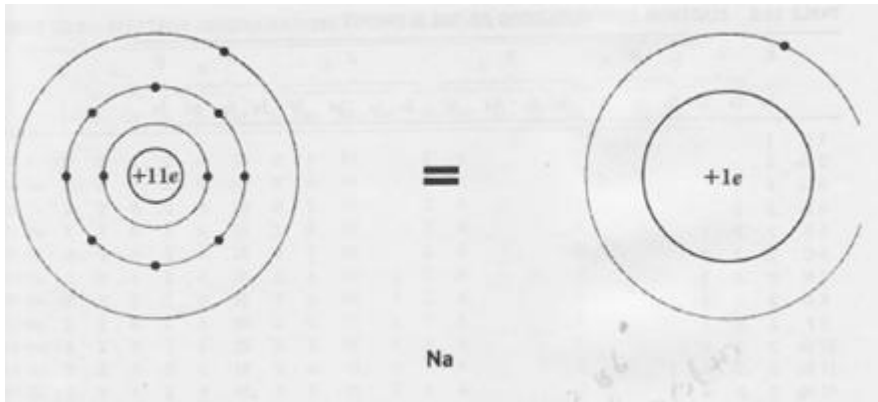
Kr (36) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6$

Xe (54) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 5s^2 5p^6$

Rn (86) =  $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 6s^2 6p^6$



# Alkali Metals



Group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Period 1	1																	2
1	H																	He
2	Li	Be											5	6	7	8	9	10
3	Na	Mg											13	14	15	16	17	18
4	K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
6	Cs	Ba	La*	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
7	Fr	Ra	Ac**	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Uut	Uuq	Uup	Uuh	Uus	Uuo

- Non Metals
- Alkali Metals
- Alkaline Metals
- Transition Metals
- Rare Earth Elements
- Noble Gases
- Metalloids
- Halogens
- Other Metals

\*Lanthanides

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

\*\*Actinides

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr

# Angular Probability Density

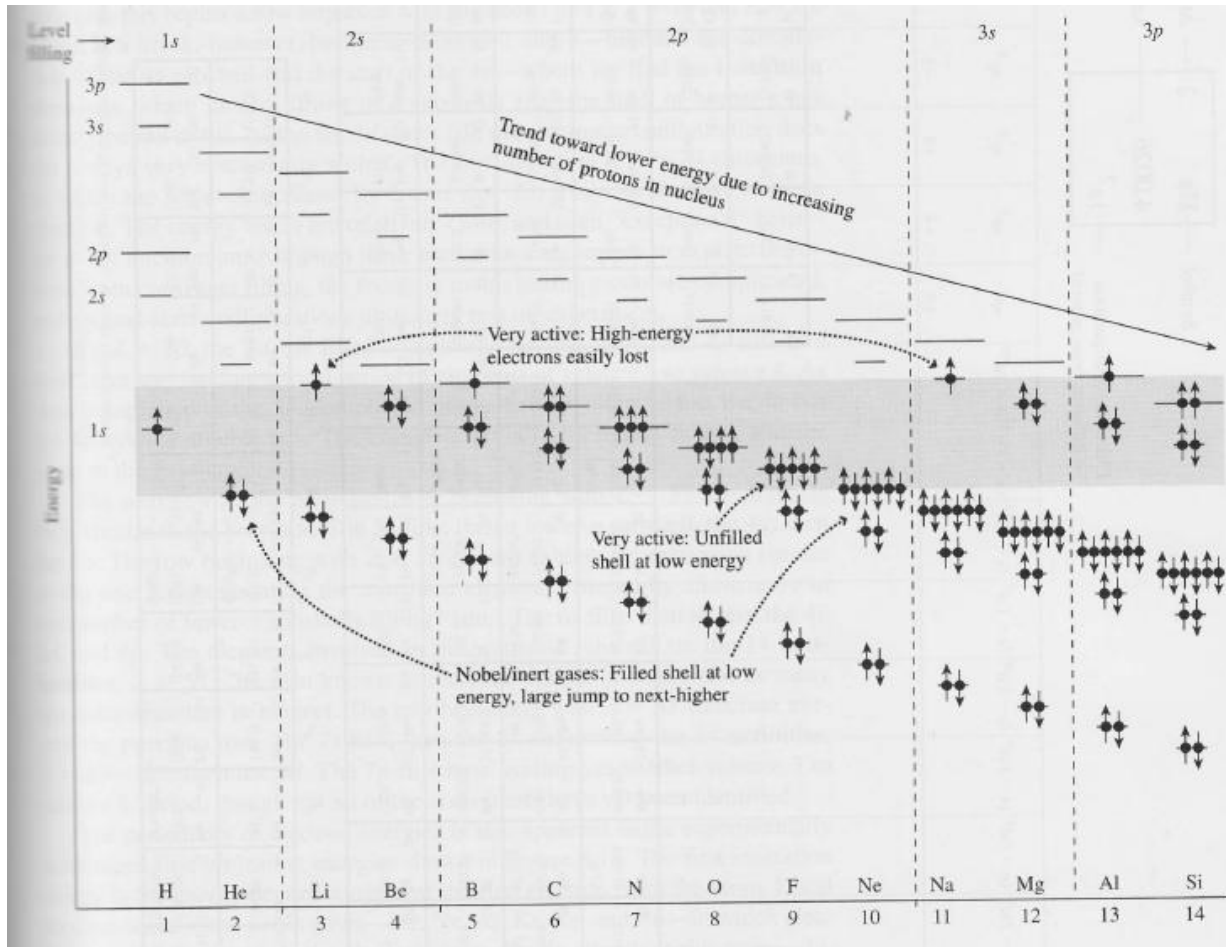
$$\Theta(\theta)^2 \Phi(\phi)^2 \equiv Y_l^{m_l} \cdot Y_l^{m_l}$$

$n$	$l$	$m_l$	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r, \theta, \phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi} a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2} a_0} r e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi} a_0^{3/2} a_0} r e^{-r/2a_0} \cos \theta$
2	1	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{1}{2\sqrt{6} a_0^{3/2} a_0} r e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi} a_0^{3/2} a_0} r e^{-r/2a_0} \sin \theta e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3} a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi} a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \cos \theta$
3	1	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin \theta$	$\frac{4}{81\sqrt{6} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \sin \theta e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3 \cos^2 \theta - 1)$	$\frac{4}{81\sqrt{30} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0} (3 \cos^2 \theta - 1)$
3	2	$\pm 1$	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin \theta \cos \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0} \sin \theta \cos \theta e^{\pm i\phi}$
3	2	$\pm 2$	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2 \theta$	$\frac{4}{81\sqrt{30} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi} a_0^{3/2} a_0^2} r^2 e^{-r/3a_0} \sin^2 \theta e^{\pm 2i\phi}$

Unsöld's theorem

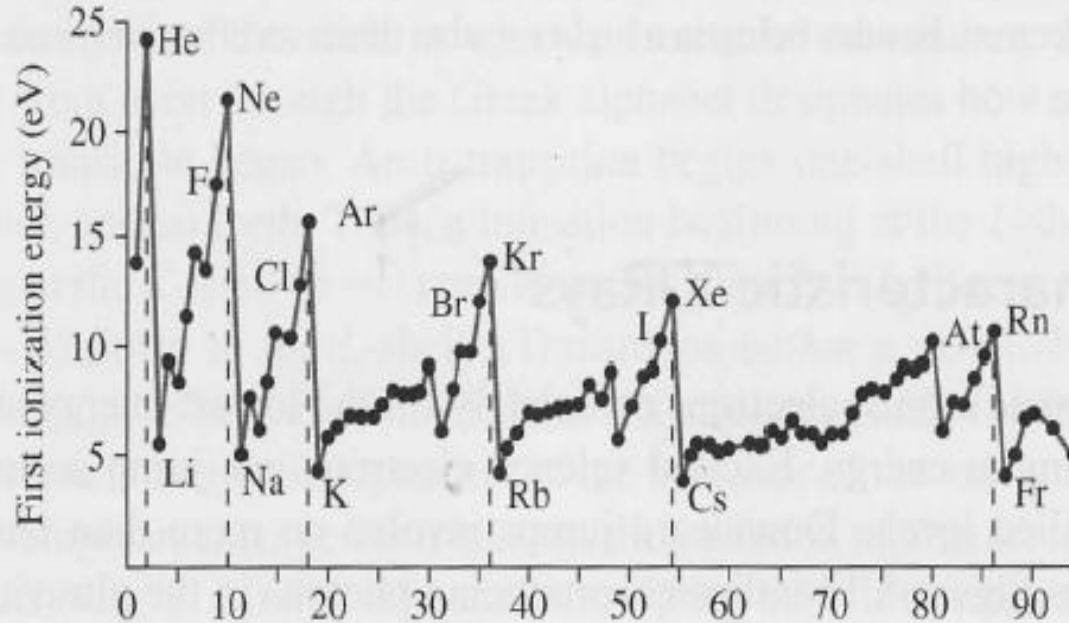
$$\sum_{m_l=-l}^l |\Theta_{l,m_l}|^2 |\Phi_{m_l}|^2 = \text{Constant}$$

# Orbital Energy levels

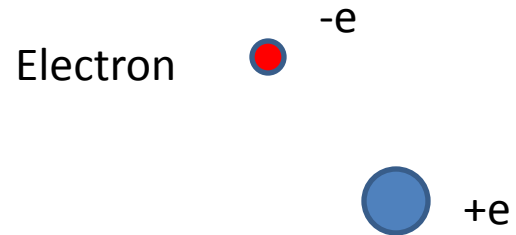


# Ionization Energy

Figure 8.16 First ionization energies of the elements.

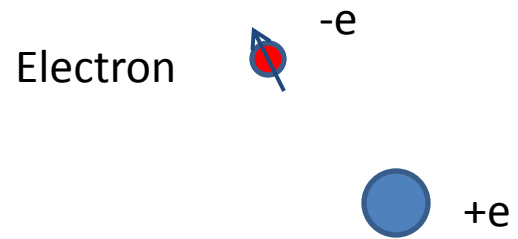


# Hydrogen atom



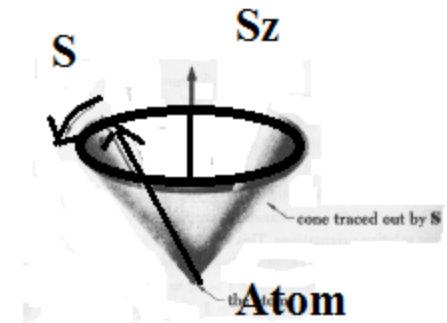
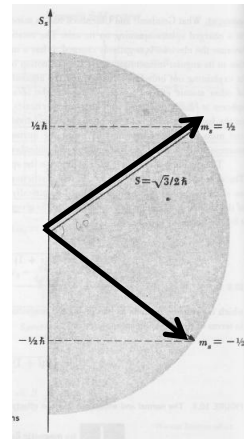
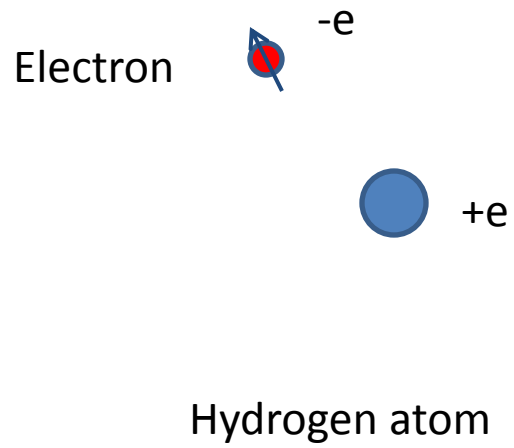
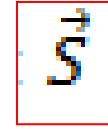
Hydrogen atom

# Spin

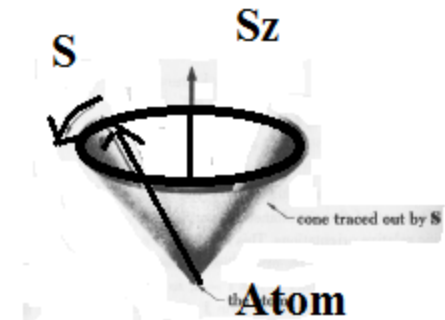
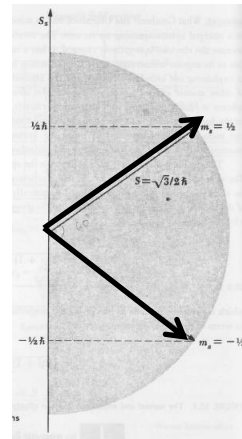
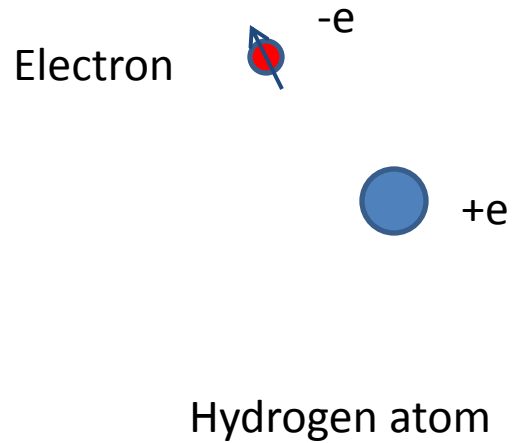
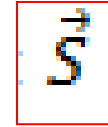


Hydrogen atom

# Spin



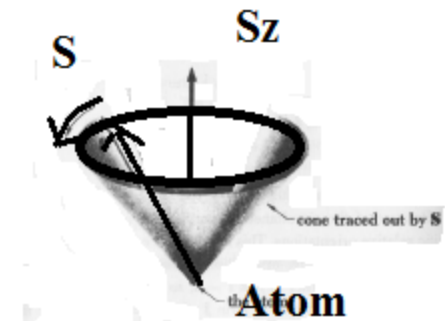
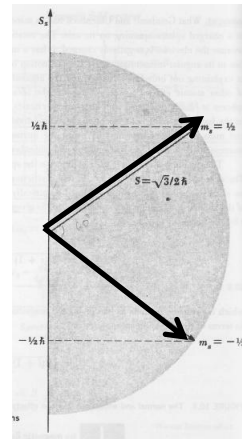
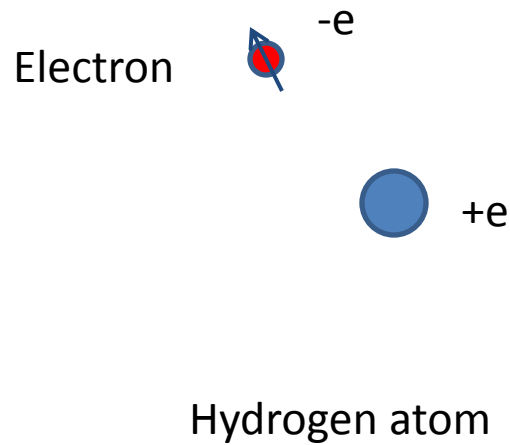
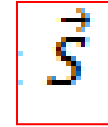
# Spin



$$|\vec{S}| = \sqrt{s(s+1)}\hbar$$
$$S_z = m_s \hbar$$

where  $s$  is a number intrinsic to a given particle  
where  $m_s = -s, -s+1, \dots, s-1, s$

# Spin



$$|\vec{S}| = \sqrt{s(s+1)}\hbar$$

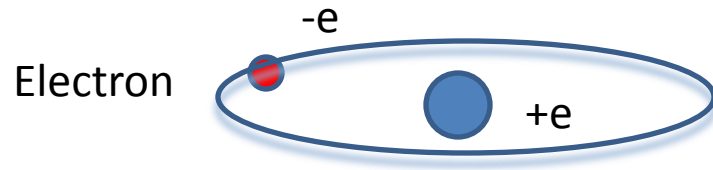
$$S_z = m_s \hbar$$

$$\vec{\mu}_s = -\left(\frac{e}{m}\right)\vec{S}$$

where  $s$  is a number intrinsic to a given particle  
 where  $m_s = -s, -s + 1, \dots, s - 1, s$

Magnetic moment under an external magnetic field

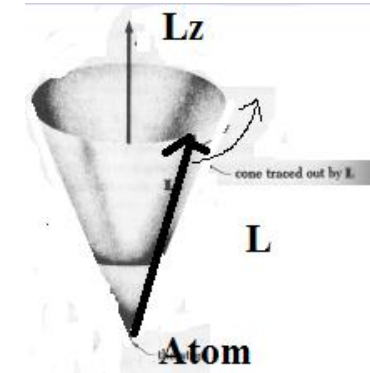
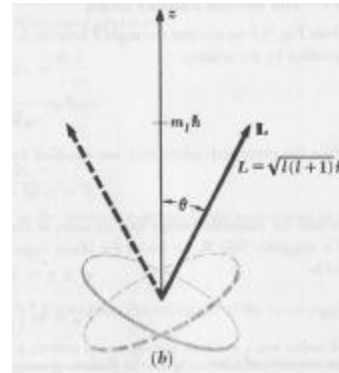
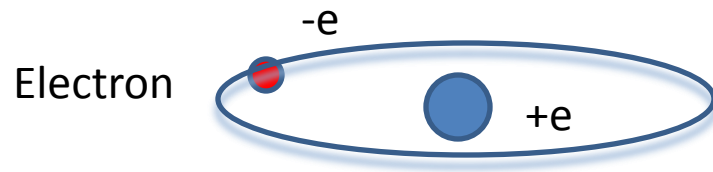
# Orbital angular momentum

 $\vec{L}$ 

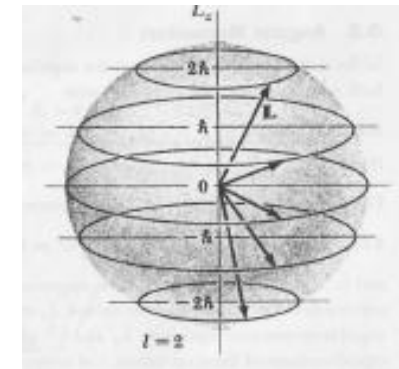
Hydrogen atom

# Orbital angular momentum

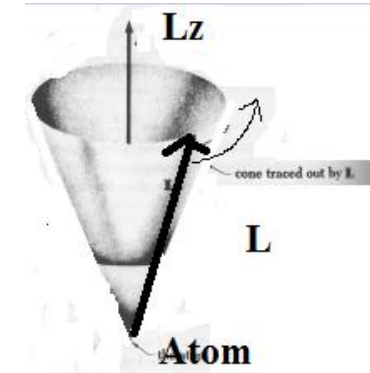
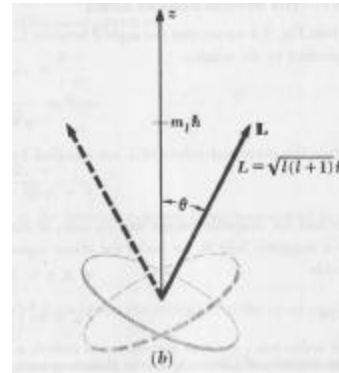
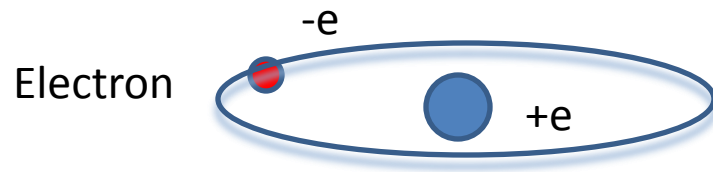
$$\vec{L}$$



Hydrogen atom



# Orbital angular momentum

 $\vec{L}$ 


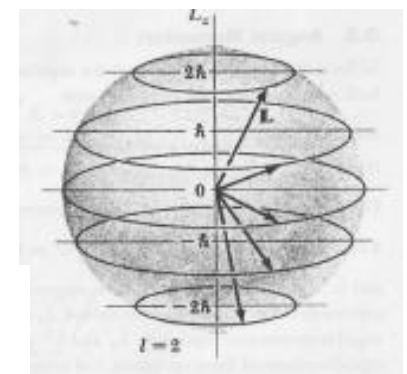
Hydrogen atom

$$|\vec{L}| = \sqrt{l(l+1)}\hbar$$

$$L_z = m_l \hbar$$

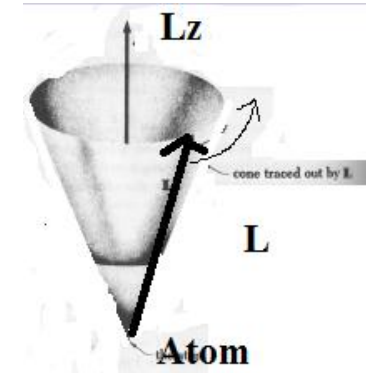
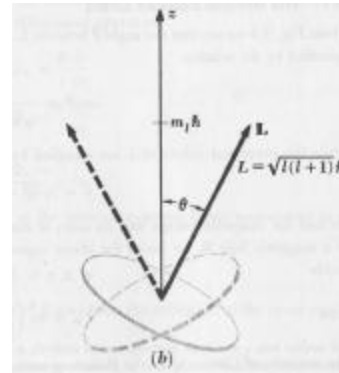
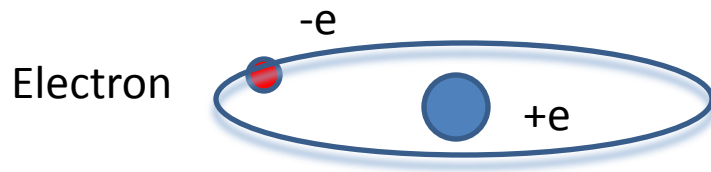
where  $l = 0, 1, 2, \dots, n - 1$

where  $m_l = -l, -l + 1, \dots, l - 1, l$



# Orbital angular momentum

$$\vec{L}$$



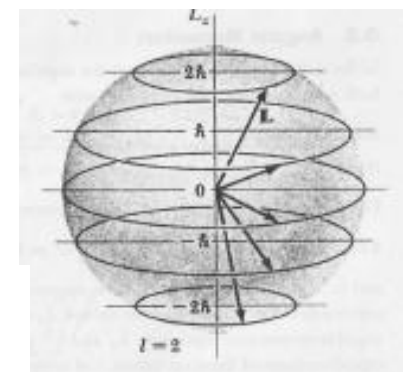
Hydrogen atom

$$|\vec{L}| = \sqrt{l(l+1)} \hbar$$

$$L_z = m_l \hbar$$

where  $l = 0, 1, 2, \dots, n - 1$

where  $m_l = -l, -l + 1, \dots, l - 1, l$



$$\vec{\mu}_L = - \left( \frac{e}{2m} \right) \vec{L}$$

Magnetic moment under an external magnetic field

# How strong is strong?

- What is the amount of energy comparable to LS coupling?
- Consider: a 2p electron
  - $n=2$ ,  $l=1$ , and  $r \approx n^2 a_0 \approx 4a_0$

$$U = -\vec{\mu}_S \cdot \vec{B}_{\text{due to } \vec{L}}$$
$$\vec{B} = \frac{\mu_0 e}{4\pi m r^3} \vec{L}$$

$$B = \frac{\mu_0 e}{4\pi m (4a_0)^3} \sqrt{1 \cdot 2} \hbar = 0.28 \text{ T}$$

$$U = \mu_S B = 2 \times 10^{-5} \text{ eV}$$

# How strong is strong?

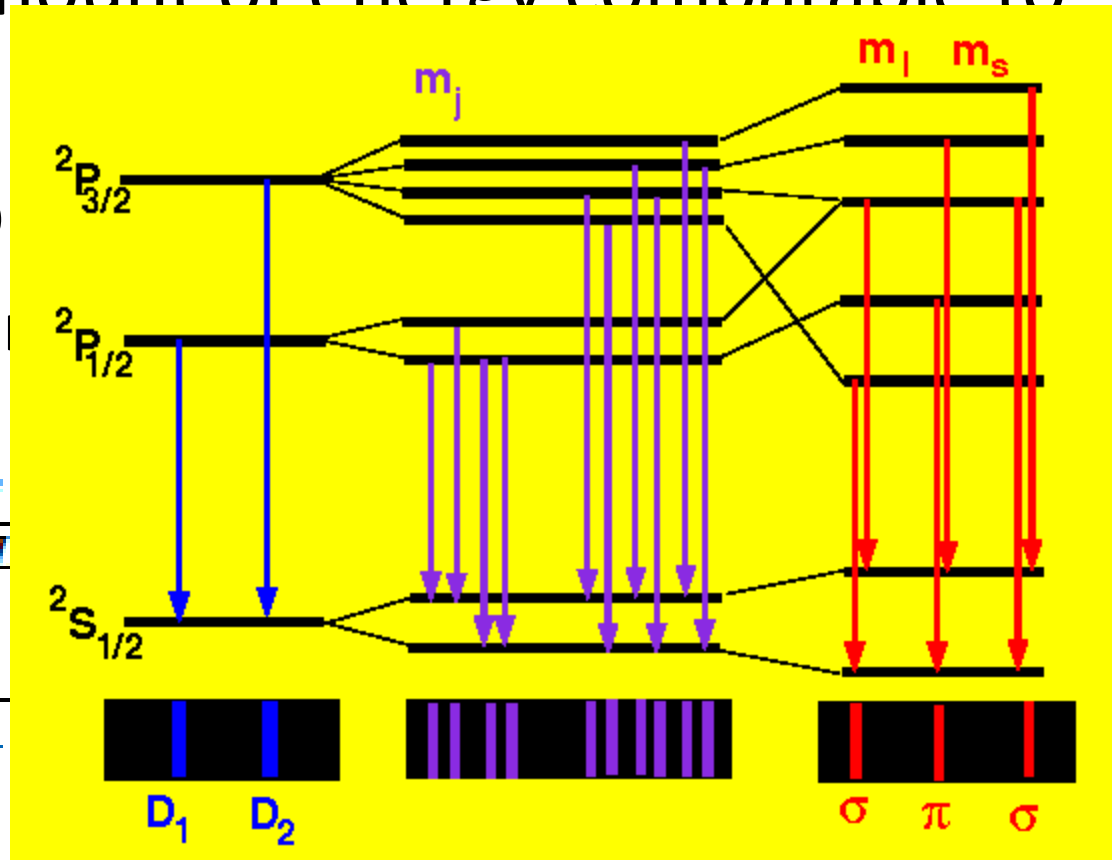
- What is the amount of energy comparable to LS coupling?
- Consider: a 2p
  - $n=2, l=1$ , and

$$U = -\vec{\mu}_S \cdot \vec{B}_{due}$$

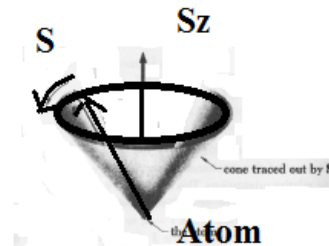
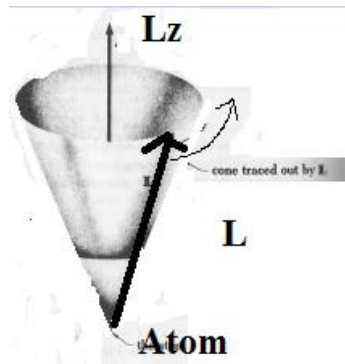
$$\vec{B} = \frac{\mu_0 e}{4\pi m r^3} \vec{L}$$

$$B = \frac{\mu_0 e}{4\pi m (4a_0)^3} \sqrt{1}$$

$$U = \mu_S B = 2 \times 10^{-5} \text{ eV}$$



# Strong $\vec{B}$ : Paschen-Back Effect



$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$

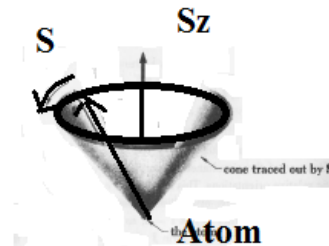
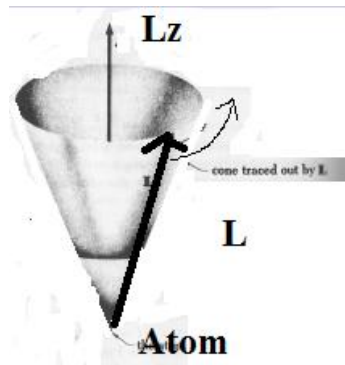
$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

$$U_L = -\vec{\mu}_L \cdot \vec{B} = -\mu_{L_z} B_z = -\left(-\frac{e}{2m} L_z\right) B_z = \frac{e}{2m} m_l \hbar B_z$$

$$U_S = -\vec{\mu}_S \cdot \vec{B} = -\mu_{S_z} B_z = -\left(-\frac{e}{m} S_z\right) B_z = \frac{e}{m} m_s \hbar B_z$$

$$E_{\text{strong magnetic field}} = E_n + \frac{e}{2m} (m_l + 2m_s) \hbar B_z$$

# Strong $\vec{B}$ : Paschen-Back Effect



$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$

$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

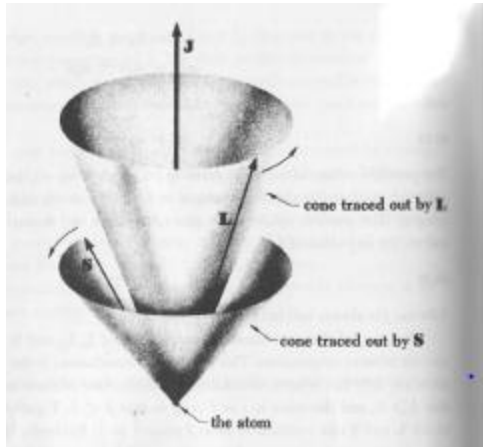
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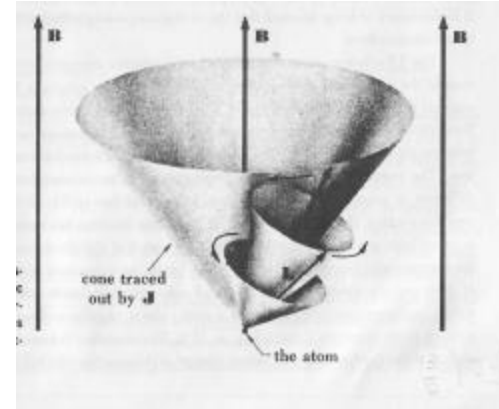
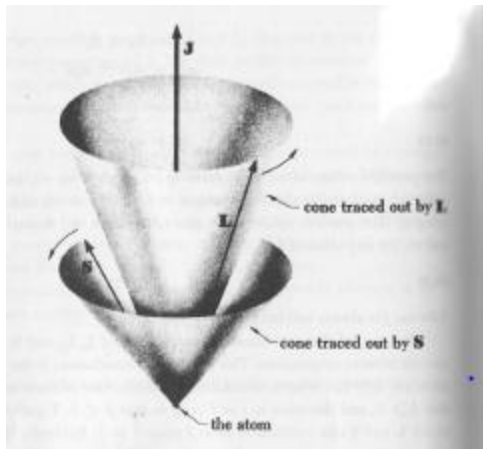
For a 2p electron, how many states are available and how much energy is deviated from the  $E_n$  value for each state?

# Weak $\vec{B}$ : Zeeman Effect



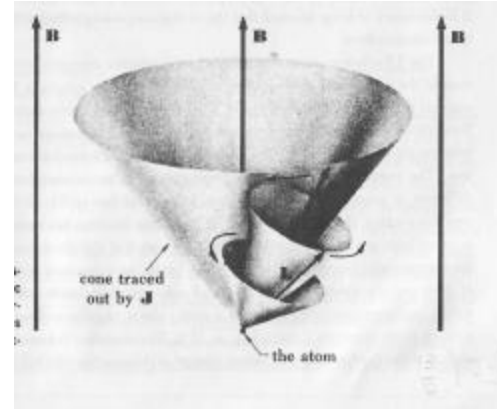
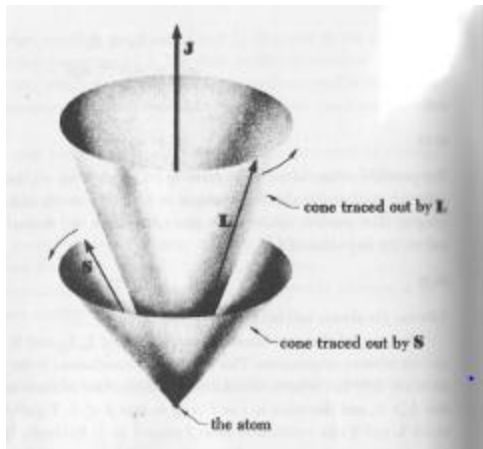
$$\vec{J} = \vec{L} + \vec{S}$$

# Weak $\vec{B}$ : Zeeman Effect



$$\vec{J} = \vec{L} + \vec{S}$$

# Weak $\vec{B}$ : Zeeman Effect



$$\vec{J} = \vec{L} + \vec{S}$$

$$|\vec{J}| = \sqrt{j(j+1)}\hbar$$

$$J_z = m_j \hbar$$

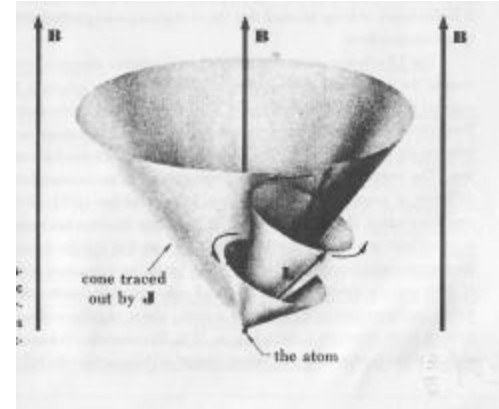
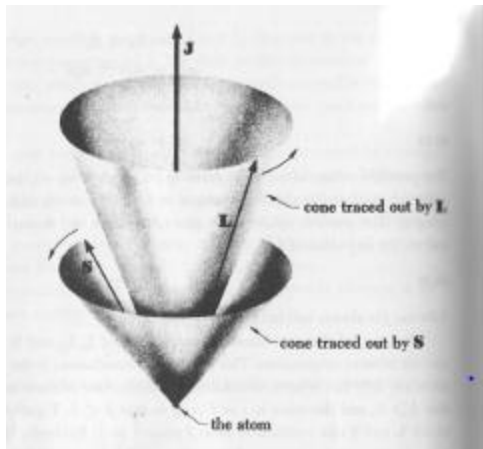
$$J_z = L_z \pm S_z$$

$$m_j \hbar = m_l \hbar + m_s \hbar$$

where  $j = |l - s|, |l - s| + 1, \dots, |l + s| - 1, |l + s|$

where  $m_j = -j, -j + 1, \dots, j - 1, j$

# Weak $\vec{B}$ : Zeeman Effect



$$\vec{J} = \vec{L} + \vec{S}$$

$$|\vec{J}| = \sqrt{j(j+1)}\hbar$$

$$J_z = m_j \hbar$$

$$J_z = L_z \pm S_z$$

$$m_j \hbar = m_l \hbar + m_s \hbar$$

where  $j = |l - s|, |l - s| + 1, \dots, |l + s| - 1, |l + s|$   
 where  $m_j = -j, -j + 1, \dots, j - 1, j$

For a 2p electron, how many states are available?

$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S = -\left(\frac{e}{2m}\right)(\vec{L} + 2\vec{S})$$

$$\begin{aligned}\vec{\mu}_L &= -\left(\frac{e}{2m}\right)\vec{L} \\ \vec{\mu}_S &= -\left(\frac{e}{m}\right)\vec{S}\end{aligned}$$

$$|\vec{\mu}_J| |\vec{J}| \cos(\pi + \delta) = \vec{\mu}_J \cdot \vec{J} \quad \cos \delta = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|}$$

$$|\vec{\mu}_J|_{\text{avg}} = |\vec{\mu}_J| \cos \delta = |\vec{\mu}_J| \left( -\frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|} \right) = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\vec{J}|}$$

avg

$$\vec{\mu}_L = - \left( \frac{e}{2m} \right) \vec{L}$$

$$\vec{\mu}_S = - \left( \frac{e}{m} \right) \vec{S}$$

$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S = - \left( \frac{e}{2m} \right) (\vec{L} + 2\vec{S})$$

$$|\vec{\mu}_J| |\vec{J}| \cos(\pi + \delta) = \vec{\mu}_J \cdot \vec{J} \quad \cos \delta = - \frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|}$$

$$|\vec{\mu}_J|_{\text{avg}} = |\vec{\mu}_J| \cos \delta = |\vec{\mu}_J| \left( - \frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|} \right) = - \frac{\vec{\mu}_J \cdot \vec{J}}{|\vec{J}|} = \frac{e}{2m} \frac{(\vec{L} + 2\vec{S}) \cdot (\vec{L} + \vec{S})}{|\vec{J}|}$$

$$= \frac{e}{2m} \frac{|\vec{L}|^2 + 2|\vec{S}|^2 + 3\vec{L} \cdot \vec{S}}{|\vec{J}|} \quad \text{since } \vec{L} \cdot \vec{S} = \frac{1}{2} (|\vec{J}|^2 - |\vec{L}|^2 - |\vec{S}|^2)$$

avg

$$\vec{\mu}_L = -\left(\frac{e}{2m}\right)\vec{L}$$

$$\vec{\mu}_S = -\left(\frac{e}{m}\right)\vec{S}$$

$$\vec{\mu}_J = \vec{\mu}_L + \vec{\mu}_S = -\left(\frac{e}{2m}\right)(\vec{L} + 2\vec{S})$$

$$|\vec{\mu}_J| |\vec{J}| \cos(\pi + \delta) = \vec{\mu}_J \cdot \vec{J} \quad \cos \delta = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|}$$

$$|\vec{\mu}_J|_{\text{avg}} = |\vec{\mu}_J| \cos \delta = |\vec{\mu}_J| \left( -\frac{\vec{\mu}_J \cdot \vec{J}}{|\mu_J| |\vec{J}|} \right) = -\frac{\vec{\mu}_J \cdot \vec{J}}{|\vec{J}|} = \frac{e}{2m} \frac{(\vec{L} + 2\vec{S}) \cdot (\vec{L} + \vec{S})}{|\vec{J}|}$$

$$= \frac{e}{2m} \frac{|\vec{L}|^2 + 2|\vec{S}|^2 + 3\vec{L} \cdot \vec{S}}{|\vec{J}|} \quad \text{since } \vec{L} \cdot \vec{S} = \frac{1}{2}(|\vec{J}|^2 - |\vec{L}|^2 - |\vec{S}|^2)$$

$$|\vec{\mu}_J|_{\text{avg}} = \frac{e}{2m} \frac{3|\vec{J}|^2 - |\vec{L}|^2 + |\vec{S}|^2}{2|\vec{J}|} = \frac{e}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2\sqrt{j(j+1)}} \hbar$$

$\vec{B} = B_z \hat{z}$ , then potential energy  since  $\cos\theta = \frac{J_z}{|\vec{J}|} = \frac{J_z}{\sqrt{j(j+1)\hbar}}$

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$$U = -\vec{\mu}_J \cdot \vec{B} = |\vec{\mu}_J| B_z \cos\theta = |\vec{\mu}_J| B_z \frac{J_z}{\sqrt{j(j+1)}\hbar} = \frac{e J_z B_z}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

$\vec{B} = B_z \hat{z}$ , then potential energy   since  $\cos\theta = \frac{J_z}{|\vec{J}|} = \frac{J_z}{\sqrt{j(j+1)}\hbar}$

$$|\vec{\mu}_J| = \frac{e}{2m} \frac{3|\vec{J}|^2 - |\vec{L}|^2 + |\vec{S}|^2}{2|\vec{J}|} = \frac{e}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2\sqrt{j(j+1)}} \hbar$$

$$\begin{aligned} U = -\vec{\mu}_J \cdot \vec{B} &= |\vec{\mu}_J| B_z \cos\theta = |\vec{\mu}_J| B_z \frac{J_z}{\sqrt{j(j+1)}\hbar} = \frac{e J_z B_z}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)} \\ &= g_{Lande} \frac{e J_z B_z}{2m} = g_{Lande} \frac{e B_z}{2m} (m_j \hbar) \end{aligned}$$

$$\text{Where } g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

$\vec{B} = B_z \hat{z}$ , then potential energy   since  $\cos\theta = \frac{J_z}{|\vec{J}|} = \frac{J_z}{\sqrt{j(j+1)}\hbar}$

$$|\vec{\mu}_J| = \frac{e}{2m} \frac{3|\vec{J}|^2 - |\vec{L}|^2 + |\vec{S}|^2}{2|\vec{J}|} = \frac{e}{2m} \frac{3j(j+1) - l(l+1) + s(s+1)}{2\sqrt{j(j+1)}} \hbar$$

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$$\text{Where } g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

$$E_{\text{weak external magnetic field}} = E_n + g_{Lande} \frac{e B_z}{2m} (m_j \hbar)$$

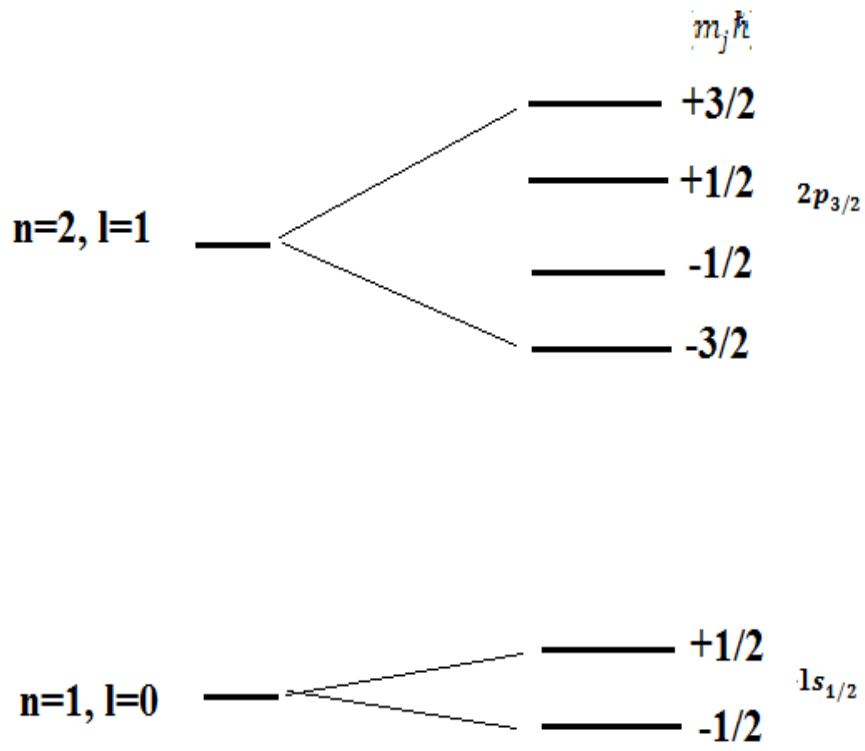
**:  $2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**

Obtain energy split for each.

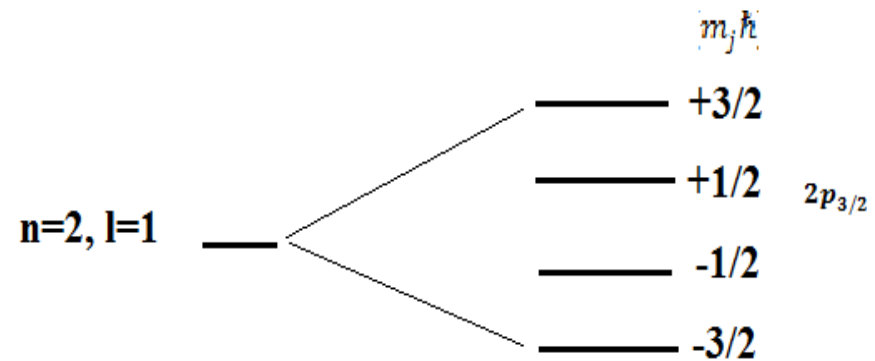
Draw an energy diagram without B and with B.

Indicate how the energy without B is split when B is present.

**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**

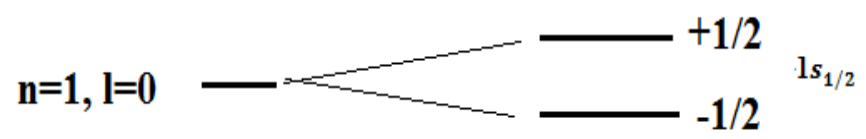


**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**

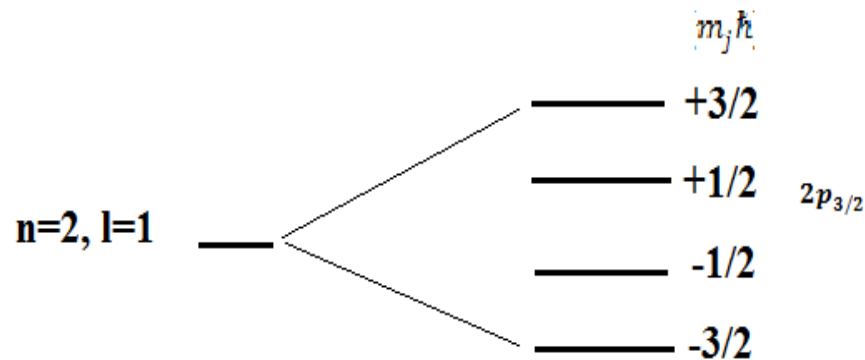


Calculate

$$g_{Lande} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$



**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**

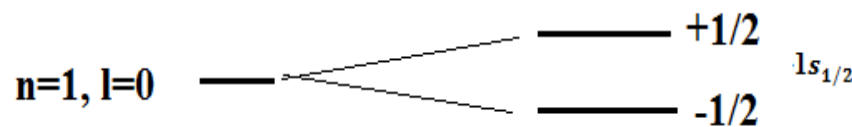


Calculate

$$g_{Lands} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

For  $1s_{1/2}$ ,  $l = 0, s = \frac{1}{2}, j = \frac{1}{2}$ ;  $g_{Lands} = 2$ .

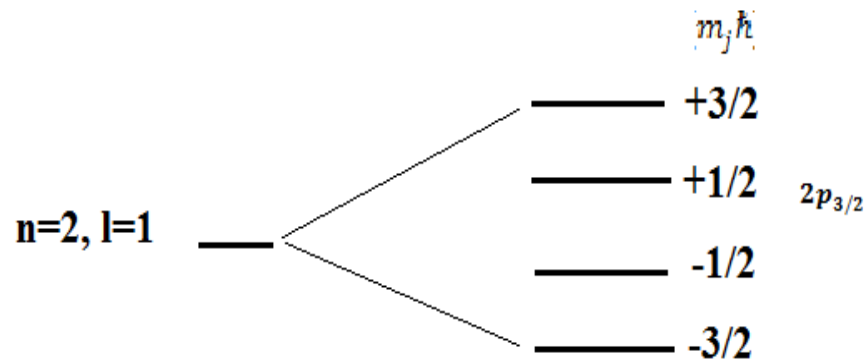
For  $2p_{3/2}$ ,  $l = 1, s = \frac{1}{2}, j = \frac{3}{2}$ ;  $g_{Lands} = \frac{4}{3}$



Calculate

$$\Delta E = g_{Lands} \frac{eB_z}{2m} (m_j \hbar)$$

**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**

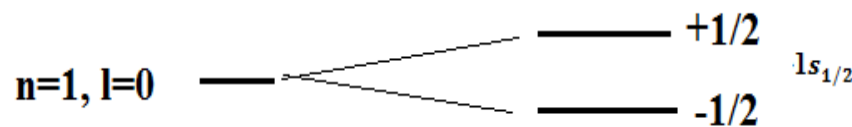


Calculate

$$g_{Lands} = \frac{3j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

For  $1s_{1/2}, l = 0, s = \frac{1}{2}, j = \frac{1}{2}; g_{Lands} = 2$

For  $2p_{3/2}, l = 1, s = \frac{1}{2}, j = \frac{3}{2}; g_{Lands} = \frac{4}{3}$

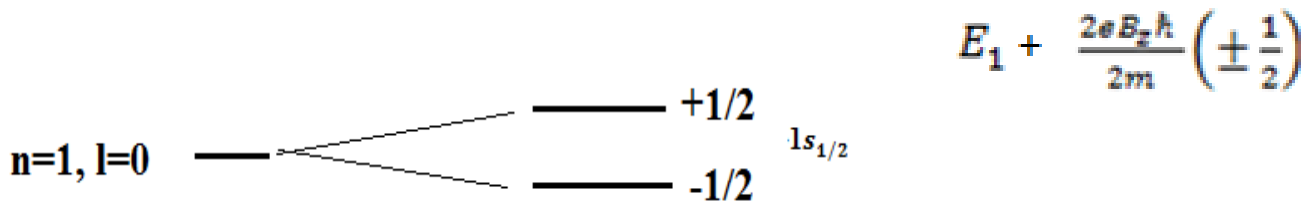
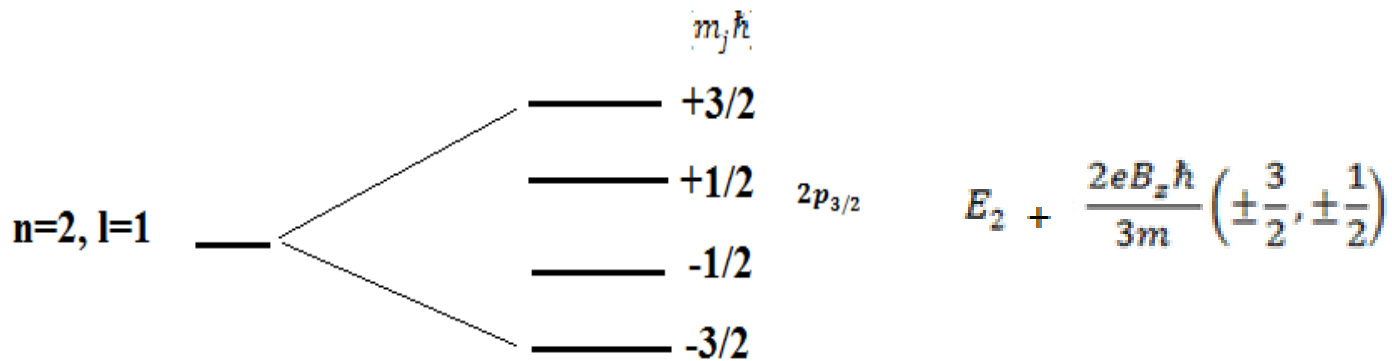


Calculate

$$\Delta E = g_{Lands} \frac{eB_z}{2m} (m_j \hbar) = \frac{2eB_z \hbar}{2m} (m_j) = \frac{2eB_z \hbar}{2m} \left( \pm \frac{1}{2} \right)$$

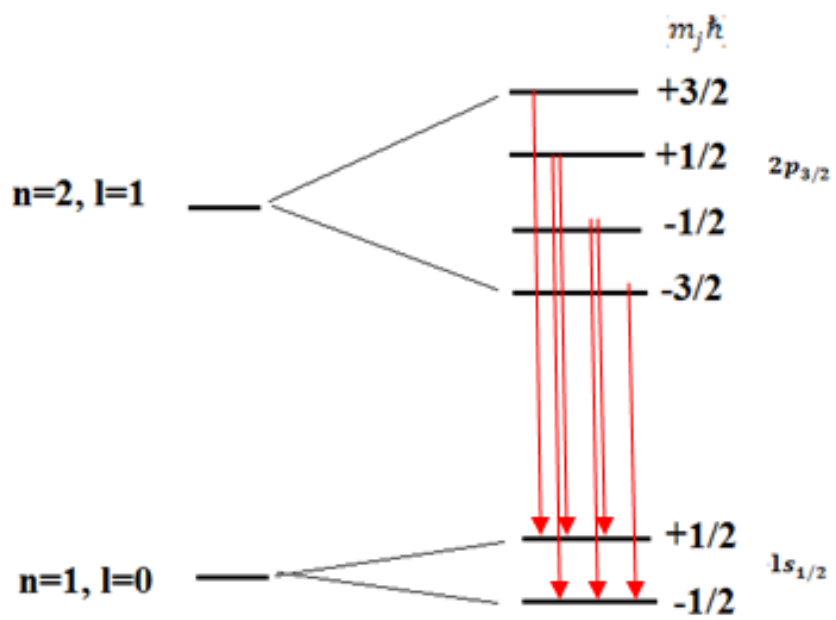
$$\Delta E = g_{Lands} \frac{eB_z}{2m} (m_j \hbar) = \left( \frac{4}{3} \right) \frac{eB_z \hbar}{2m} (m_j) = \frac{2eB_z \hbar}{3m} \left( \pm \frac{3}{2}, \pm \frac{1}{2} \right)$$

**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**



Selection Rule  $\Delta m_j = 0, \pm 1$

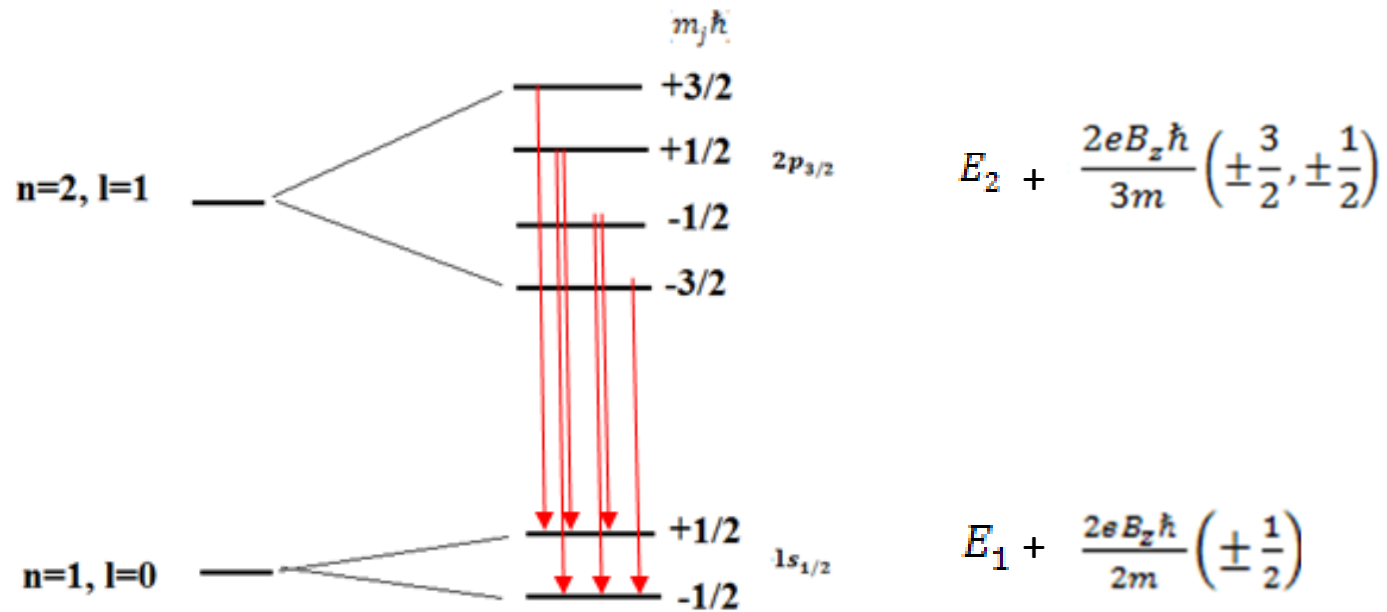
**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**



Selection Rule

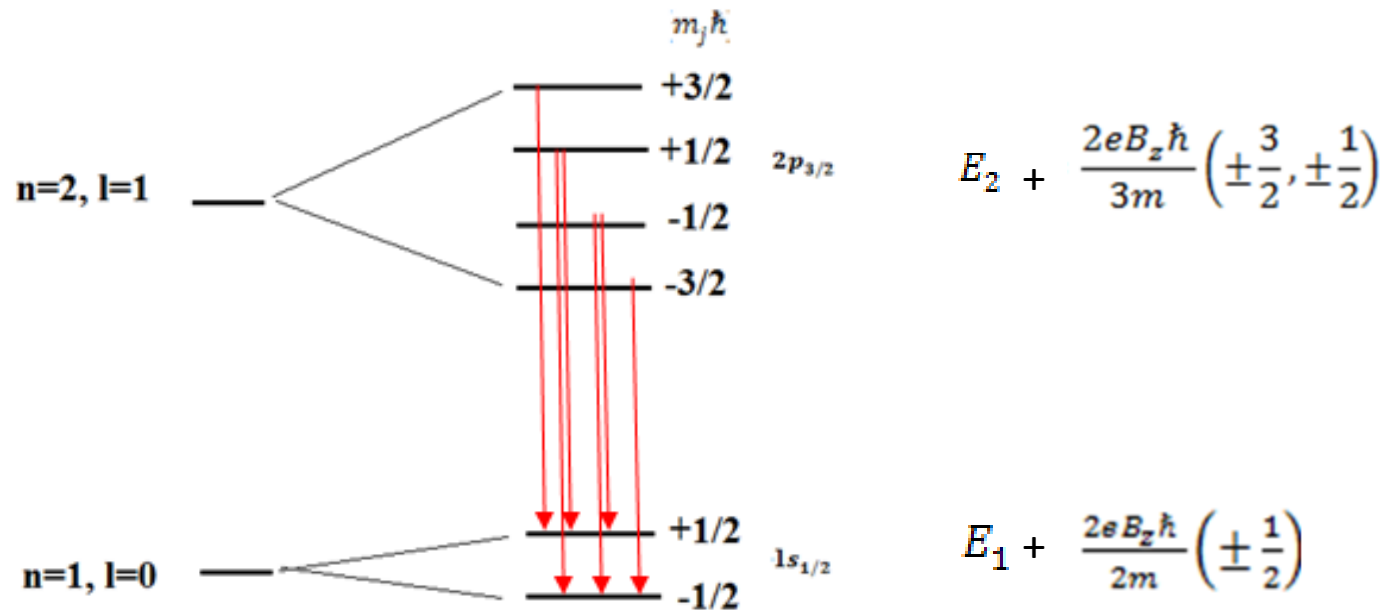
$$\Delta m_j = 0, \pm 1$$

**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**



Calculate photon energy:

**$2p_{3/2} \rightarrow 1s_{1/2}$  in a weak external magnetic field of .05T**



Calculate photon energy:

$$\Delta E = \Delta E_{\text{without magnetic field}} + \frac{2eB_z\hbar}{2m} \left( \pm\frac{5}{3}, \pm\frac{3}{3}, \pm\frac{1}{3} \right)$$