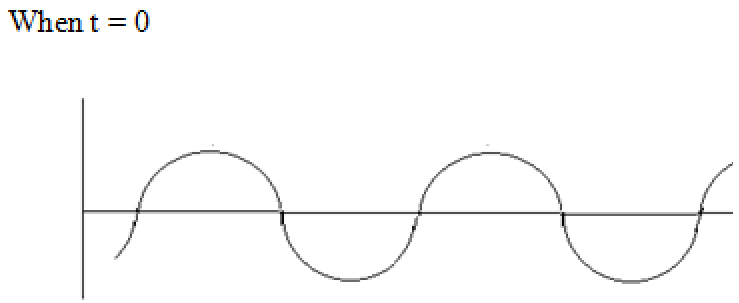
Phys 102: Modern Physics

- Course survey
- Course introduction
- Office hour
- Powers of ten
- Reviews
  - Wave properties
  - Schrodinger equation
  - Bound and unbound states

Office hours choices

- Mon noon to 1 pm
- Mon 1 to 2 pm
- Tues 2 to 3 pm
- Thurs 2 to 3 pm

Wave properties



1.1 When t=0

Amplitude =

Wavelength (λ) =

1.2 When position is fixed

Period (T) =

Frequency (ν) =

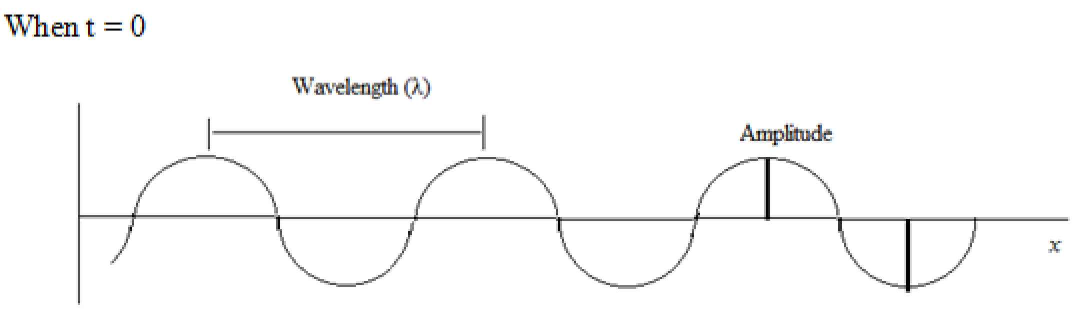
Relationship between T and ν:

1.3 Define

Angular frequency(ω) =

Angular wave number(k) =

Wave properties



1.1 When t=0

Amplitude = Distance from the middle to the crest
Wavelength (λ) = Distance from crest to crest

1.2 When position is fixed

Period (T) = Time between passage of two successive crests
Frequency (ν) = Number of crest passing per unit time
Relationship between T and ν: $\nu = 1/T$

1.3 Define

Angular frequency(ω) = $2\pi\nu$
Angular wave number(k) = $2\pi/\lambda$

Schrodinger Equation

Hamiltonian operator (H)

$$H\Psi(x,t) = E\Psi(x,t)$$

Schrodinger Equation

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Since H = Total Energy = Kinetic energy (T) + Potential energy (U)

Schrodinger Equation

Hamiltonian operator (H)

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Since H = Total Energy = Kinetic energy (T) + Potential energy (U)

$$\left(\frac{p^2}{2m} + U\right)\Psi(x,t) = E\Psi(x,t)$$

Schrodinger Equation

Hamiltonian operator (H)

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$$p = \frac{\hbar}{i}\frac{\partial}{\partial x} \quad \text{and} \quad \mathrm{E} = i\hbar\,\frac{\partial}{\partial t}$$

Schrodinger Equation

Hamiltonian operator (H)

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Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\,\frac{\partial^2\Psi\,(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\,\frac{\partial\,\Psi(x,t)}{\partial t}$$

Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

$U(x)$ = Potential energy (position dependent)

$\Psi(x,t)$ = Wave function of a particle

$\Psi(x,t)$ involves real and imaginary parts (Re $\Psi(x,t)$ + i Im $\Psi(x,t)$) where $i^2 = -1$.

Schrodinger Equation

Time-Dependent Schrodinger Equation

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- $U(x)$ = Potential energy (position dependent)
- $\Psi(x,t)$ = Wave function of a particle
- $\Psi(x,t)$ involves real and imaginary parts (Re $\Psi(x,t)$ $+ i$ Im $\Psi(x,t)$) where $i^2 = -1$.
- $\int_a^b \Psi^*(x,t)\Psi(x,t)dx = \int_a^b |\Psi(x,t)|^2\, dx$ = P (Probability of finding the particle between a and b at time t).
- $\Psi^*(x,t)\Psi(x,t) = |\Psi(x,t)|^2$ represents probability *density*.
- $\int_{-\infty}^{\infty} |\Psi(x,t)|^2\, dx = 1$ (normalization)
- $\Psi(x,t)$ should be normalized for $|\Psi(x,t)|^2$ to represent probability density.
- $\Psi(x,t)$ and its partial derivative ($= \frac{\partial\Psi(x,t)}{\partial x}$) should be continuous everywhere.

Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Separation of Variables

$$\Psi(x,t) = \psi(x)\,\phi(t)$$

Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

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Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Separation of Variables $\Psi(x,t)=\psi(x)\ \phi(t)$

$$\frac{-\hbar}{2m}\frac{\partial^2\psi(x)\ \phi(t)}{\partial x^2}+U(x)\psi(x)\ \phi(t)=i\hbar\frac{\partial\ \psi(x)\ \phi(t)}{\partial t}$$

$$\frac{-\hbar^2\ \phi(t)}{2m}\frac{\partial^2\ \psi(x)}{\partial x^2}+U(x)\psi(x)\ \phi(t)=\psi(x)i\hbar\frac{\partial\ \phi(t)}{\partial t}$$

Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Separation of Variables $\Psi(x,t) = \psi(x)\,\phi(t)$

$$\frac{-\hbar}{2m}\frac{\partial^2\psi(x)\,\phi(t)}{\partial x^2} + U(x)\psi(x)\,\phi(t) = i\hbar\frac{\partial\,\psi(x)\,\phi(t)}{\partial t}$$

$$\frac{-\hbar^2\,\phi(t)}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x)\,\phi(t) = \psi(x)i\hbar\frac{\partial\,\phi(t)}{\partial t}$$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + U(x) = i\hbar\frac{1}{\phi(t)}\frac{\partial\,\phi(t)}{\partial t}$$

Schrodinger Equation

: **Time-Dependent Schrodinger Equation**

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Separation of Variables $\Psi(x,t)=\psi(x)\,\phi(t)$

$$\frac{-\hbar}{2m}\frac{\partial^2\psi(x)\,\phi(t)}{\partial x^2}+U(x)\psi(x)\,\phi(t)=i\hbar\frac{\partial\,\psi(x)\,\phi(t)}{\partial t}$$

$$\frac{-\hbar^2\phi(t)}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)\,\phi(t)=\psi(x)i\hbar\frac{\partial\,\phi(t)}{\partial t}$$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)=i\hbar\frac{1}{\phi(t)}\frac{\partial\,\phi(t)}{\partial t}$$

= Constant (C) = Constant (C)

Temporal part

$$i\hbar \ \frac{1}{\phi(t)}\frac{\partial\, \phi(t)}{\partial t} = \mathrm{C}$$

Temporal part

$$i\hbar \; \frac{1}{\phi(t)} \frac{\partial \, \phi(t)}{\partial t} = \mathrm{C}$$

$$\phi(t) =$$

Temporal part

$$i\hbar \; \frac{1}{\phi(t)} \frac{\partial \phi(t)}{\partial t} = \mathrm{C}$$

$$\phi(t) = \; Ae^{(C/i\hbar)t} = Ae^{-i(C/\hbar)t}$$

$$\Psi(x,t) =$$

$$\Psi^*(x,t)\Psi\;(x,t) =$$

Temporal part

$$i\hbar \;\frac{1}{\phi(t)}\frac{\partial\,\phi(t)}{\partial t} = \mathrm{C}$$

$$\phi(t) = \; Ae^{(C/i\hbar)t} = Ae^{-i(C/\hbar)t}$$

$$\Psi(x,t) = \psi(x)\;\phi(t) = \psi(x)\;e^{-i(C/\hbar)t}$$

$$\Psi^*(x,t)\Psi\;(x,t) = \psi(x)^*\;e^{+i(C/\hbar)t}\;\psi(x)\;e^{-i(C/\hbar)t} = \psi(x)^*\;\psi(x)$$

Schrodinger Equation

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

Separation of Variables $\Psi(x,t) = \psi(x)\,\phi(t)$

Spatial part of $\Psi(x,t)$: $\frac{-\hbar^2}{2m}\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + U(x) = C$

Temporal part of $\Psi(x,t)$: $i\hbar\frac{1}{\phi(t)}\frac{\partial\phi(t)}{\partial t} = C$

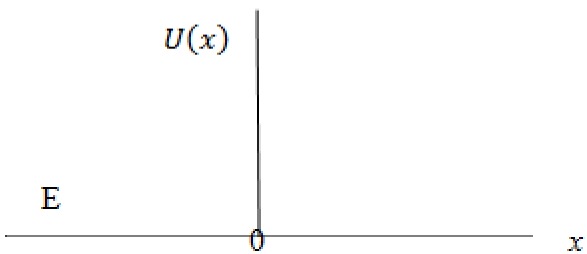
time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

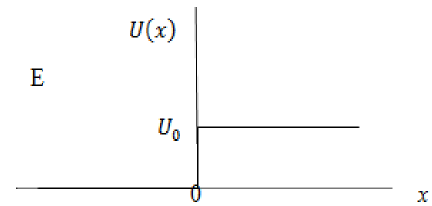
Bound vs. unbound states

time-independent Schrodinger Equation:

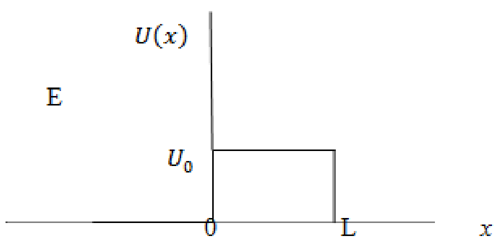
$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$



$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$



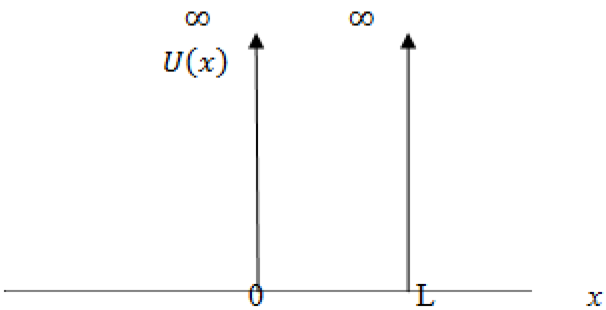
$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \leq x \leq L \\ 0 & x > L \end{cases}$$



Bound vs. unbound states

Infinite Well

$$U(x) = \begin{cases} \infty & x \le 0 \\ 0 & 0 < x < L \\ \infty & x \ge L \end{cases}$$



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Finite Well

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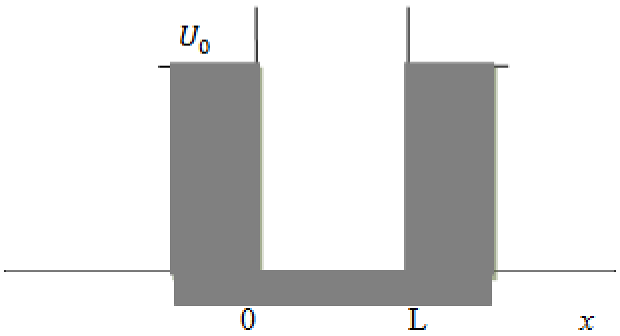

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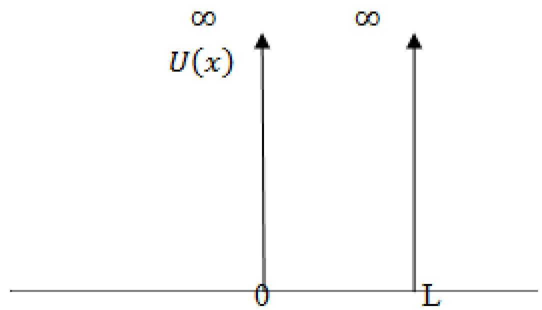

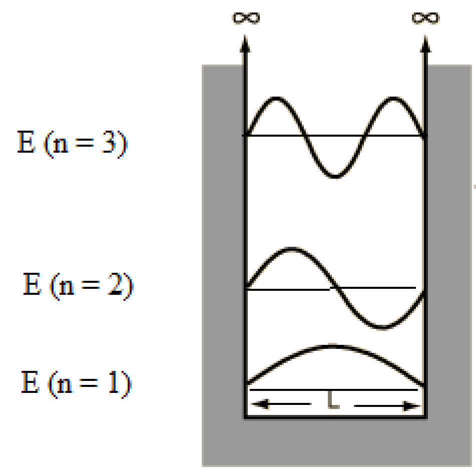

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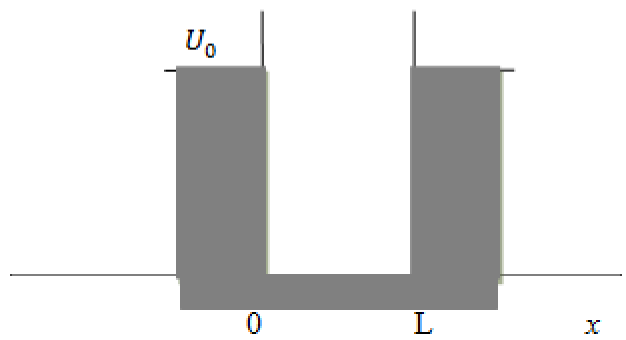
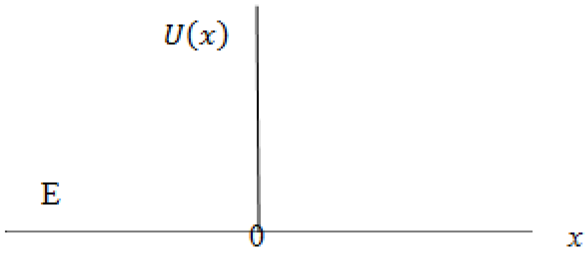


Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$



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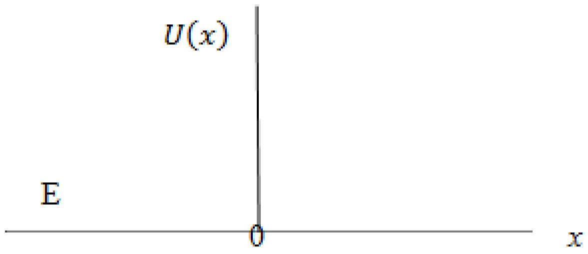

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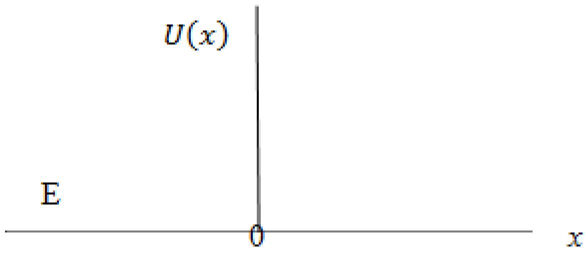


$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\,\psi(x)$$

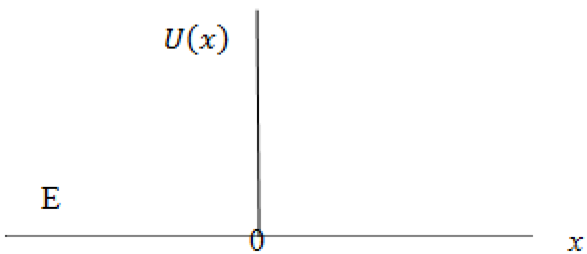
$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2mE}{\hbar^2}\psi(x)$$

Solutions:

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$



$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\,\psi(x)$$

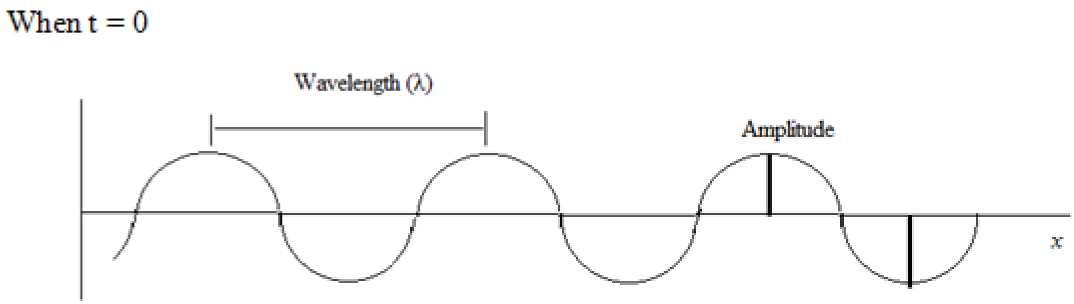
$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2mE}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

Solutions:

$e^{+ikx} = \cos kx + i\sin kx$ *for a particle* moving in the positive direction (→) on the x axis.

$e^{-ikx} = \cos kx - i\sin kx$ *for a particle* moving in the opposite direction (←) on the x axis.

For Free Particle

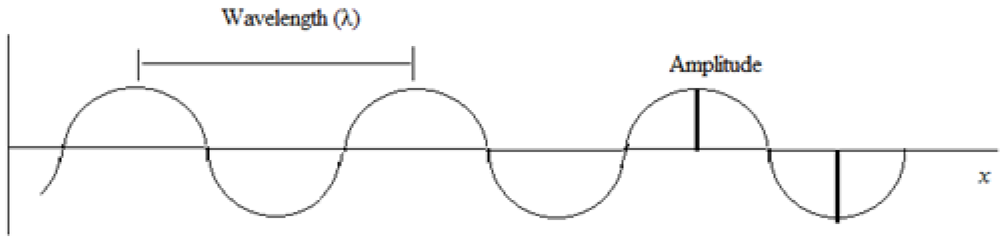


1.4 Momentum (p) =

1.5 Energy quanta (E) =

For Free Particle

When t = 0



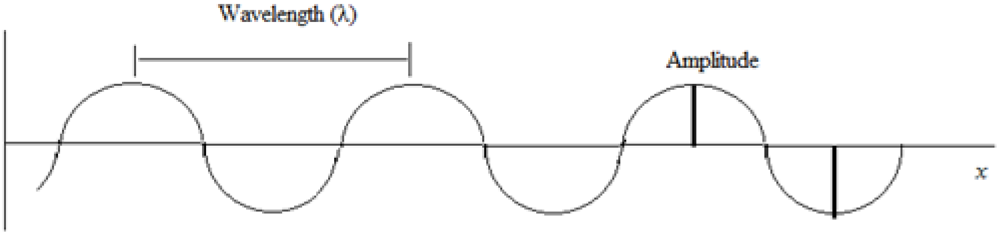
$$\Psi(x,t) = \psi(x)\ \phi(t) = \psi(x)\ e^{-i(C/\hbar)t}$$

1.4 Momentum (p) =

1.5 Energy quanta (E) =

For Free Particle

When t = 0



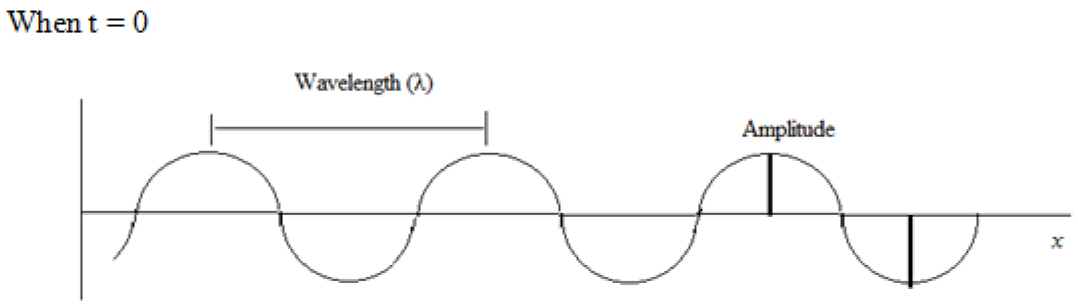
$$\Psi(x,t) = \psi(x)\,\phi(t) = \psi(x)\,e^{-i(C/\hbar)t}$$

$$\Psi = e^{ikx}e^{i\omega t} = e^{ikx-i\omega t}$$

1.4 Momentum (p) =

1.5 Energy quanta (E) =

For Free Particle



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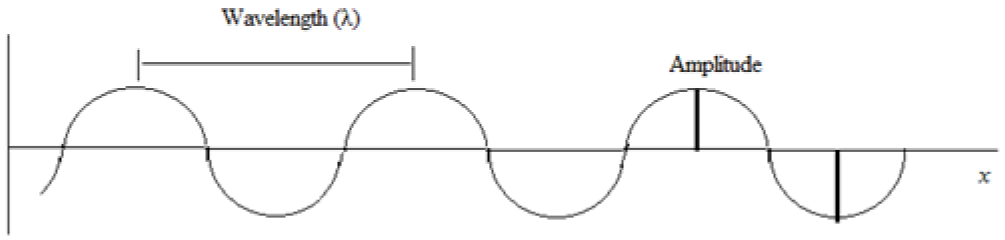

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$$\Psi = e^{ikx} e^{i\omega t} = e^{ikx-i\omega t}$$

1.4 Momentum (p) = $\hbar k = \hbar\,(\frac{2\pi}{\lambda}) = \frac{h}{\lambda}$ (de Broglie relation)

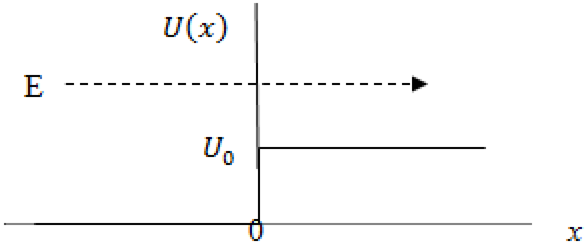
1.5 Energy quanta (E) = $\hbar\omega$

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$



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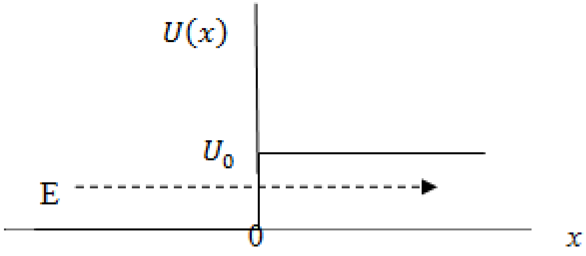

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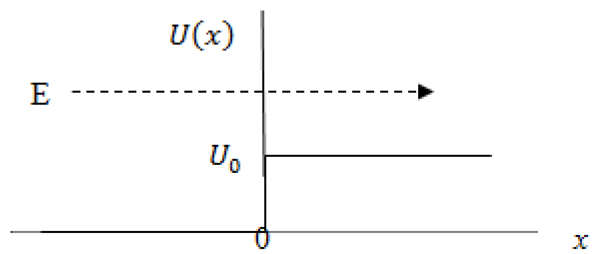

Where $x < 0$,

Where $x \geq 0$

Bound vs. unbound states

time-independent Schrodinger Equation:

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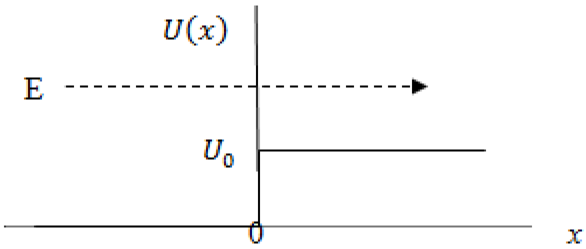

Where $x < 0$,

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

Bound vs. unbound states

time-independent Schrodinger Equation:

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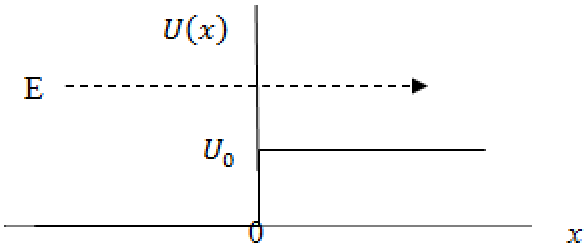

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$\psi_{x<0}$ = Incoming wave function + Reflected wave function

$$= \quad A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$$

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$\psi_{x\geq 0}$ = the transmitted wave function,

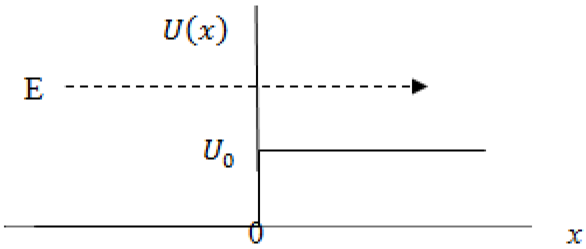
$$= Ce^{ik'x}$$

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$



Where $x < 0$,

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$\psi_{x<0}$ = Incoming wave function + Reflected wave function
= A e^{+ikx} + B e^{-ikx}

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$\psi_{x\geq 0}$ = the transmitted wave function,
$= Ce^{ik'x}$

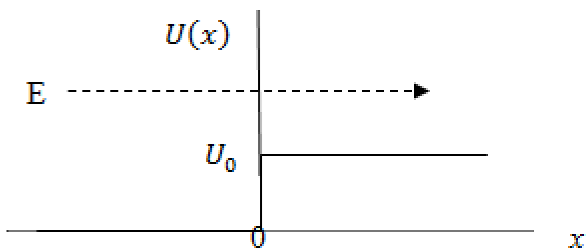
- $\psi_{x<0}(x=0) = \psi_{x\geq 0}(x=0)$ → A + B = C
- $\frac{d\psi_{x<0}}{dx}\Big|_{x=0}\ \frac{d\psi_{x\geq 0}}{dx}\Big|_{x=0}$ → k(A – B) = k'C

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$



Where $x < 0$,

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$\psi_{x<0}$ = Incoming wave function + Reflected wave function

$= \quad A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$\psi_{x\geq 0}$ = the transmitted wave function,

$= Ce^{ik'x}$

- $\psi_{x<0}(x=0) = \psi_{x\geq 0}(x=0)$ → $A + B = C$
- $\frac{d\psi_{x<0}}{dx}\big|_{x=0}\ \frac{d\psi_{x\geq 0}}{dx}\big|_{x=0}$ → $k(A-B) = k'C$

$$\text{Reflection probability} = \frac{\text{reflected particle flux}}{\text{incoming particle flux}} = \frac{|\psi_{reflected}|^2\,k}{|\psi_{incoming}|^2\,k} = \frac{B^*B}{A^*A}$$

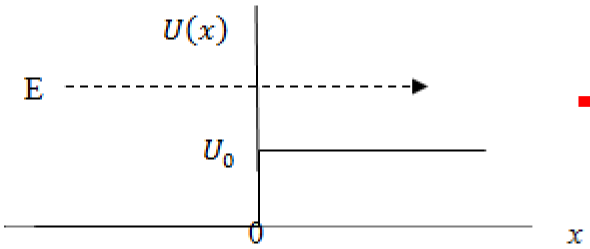
$$\text{Transmission probability} = \frac{\text{transmitted particle flux}}{\text{incoming particle flux}} = \frac{|\psi_{trans}|^2\,k'}{|\psi_{incoming}|^2\,k} = \frac{C^*C\,k'}{A^*A\,k}$$

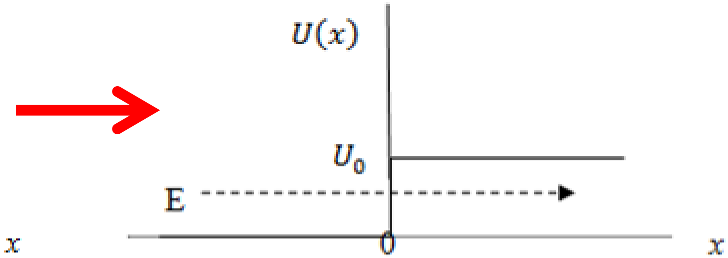
Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$





Where $x < 0$,

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$\psi_{x<0}$ = Incoming wave function + Reflected wave function

$= \quad A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$\psi_{x\geq 0}$ = the transmitted wave function,

$= Ce^{ik'x}$

- $\psi_{x<0}(x=0) = \psi_{x\geq 0}(x=0)$ → $A + B = C$
- $\frac{d\psi_{x<0}}{dx}\Big|_{x=0} \; \frac{d\psi_{x\geq 0}}{dx}\Big|_{x=0}$ → $k(A - B) = k'C$

$$\text{Reflection probability} = \frac{\text{reflected particle flux}}{\text{incoming particle flux}} = \frac{|\psi_{reflected}|^2\,k}{|\psi_{incoming}|^2\,k} = \frac{B^*B}{A^*A}$$

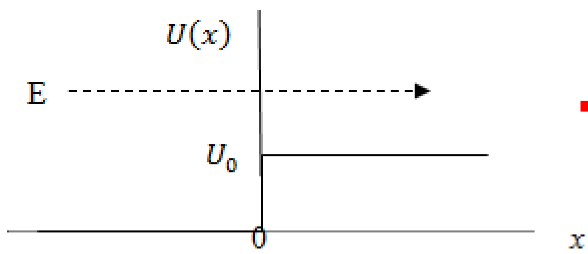
$$\text{Transmission probability} = \frac{\text{transmitted particle flux}}{\text{incoming particle flux}} = \frac{|\psi_{trans}|^2\,k'}{|\psi_{incoming}|^2\,k} = \frac{C^*C\,k'}{A^*A\,k}$$

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & x \geq 0 \end{cases}$$



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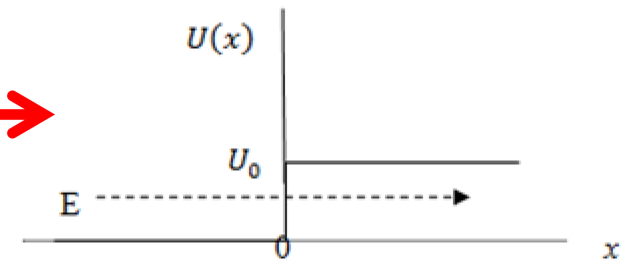


Where $x < 0$,

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m\,E}{\hbar^2}\psi(x) = -\,k^2\,\psi(x) \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$

$\psi_{x<0}$ = Incoming wave function + Reflected wave function

$= \quad A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$

Where $x \geq 0$

$$\frac{\partial^2\psi(x)}{\partial x^2} = \frac{-2m(E-U_0)}{\hbar^2}\psi(x) = -\,k'^2\,\psi(x) \quad \text{where } k' = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$$

$\psi_{x\geq 0}$ = the transmitted wave function,

$= Ce^{ik'x}$ → $Ce^{-\alpha x}$

$$\alpha = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}$$

- $\psi_{x<0}(x=0) = \psi_{x\geq 0}(x=0)$ → $A + B = C$
- $\frac{d\psi_{x<0}}{dx}\Big|_{x=0}$ $\frac{d\psi_{x\geq 0}}{dx}\Big|_{x=0}$ → $k(A-B) = -\alpha\,C$

$$\text{Reflection probability} = \frac{\text{reflected particle flux}}{\text{incoming particle flux}} = \frac{|\psi_{reflected}|^2\,k}{|\psi_{incoming}|^2\,k} = \frac{B^*B}{A^*A}$$

Transmission probability = 0

→ 0

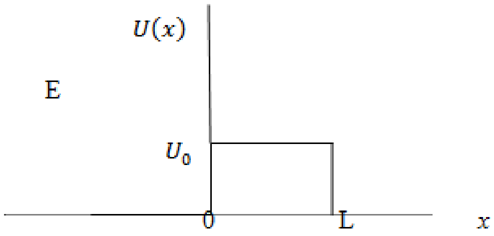
Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \le x \le L \\ 0 & x > L \end{cases}$$

E



Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$

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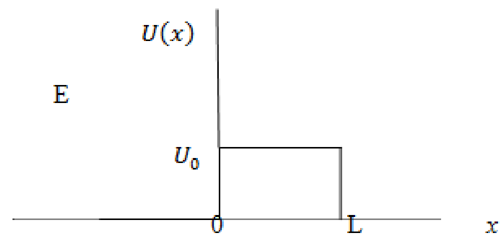
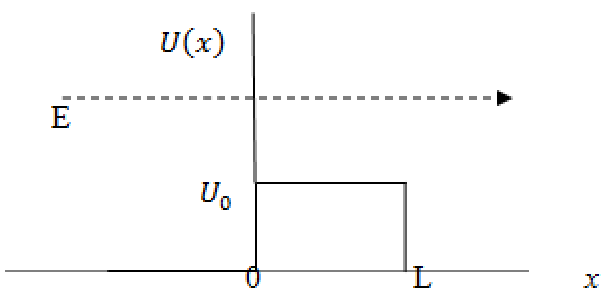
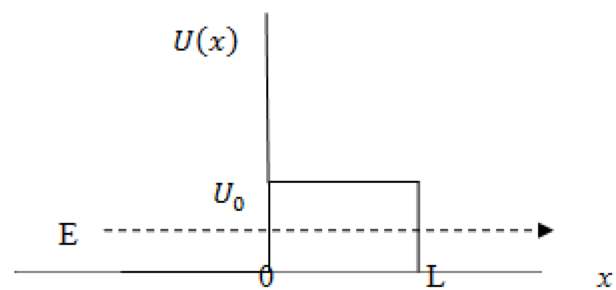


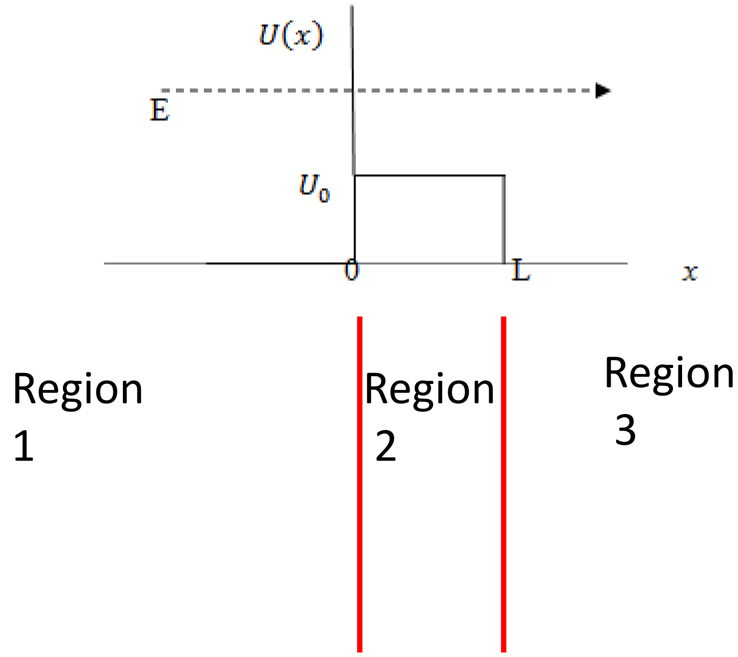




Bound vs. unbound states

time-independent Schrodinger Equation:

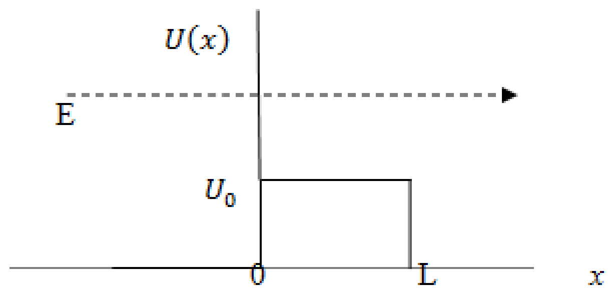
$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$



Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$



Region 1

$A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$

Region 2

$C\,e^{+ik'x} \quad + \quad D\,e^{-ik'x}$

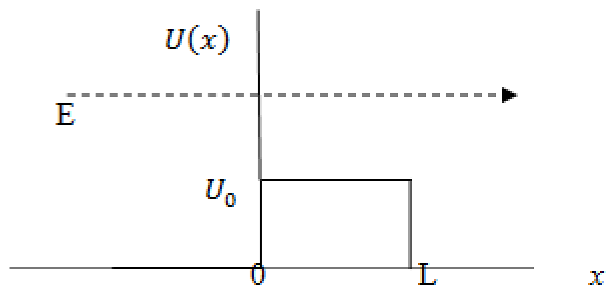
Region 3

Fe^{ikx}

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = E\,\psi(x)$$



Transmission probability $= \frac{F*F}{A^*A}$

Reflection probability $= \frac{B^*B}{A^*A}$

Region 1

Region 2

Region 3

$A\,e^{+ikx} \quad + \quad B\,e^{-ikx}$

$C\,e^{+ik'x} \quad + D\,e^{-ik'x}$

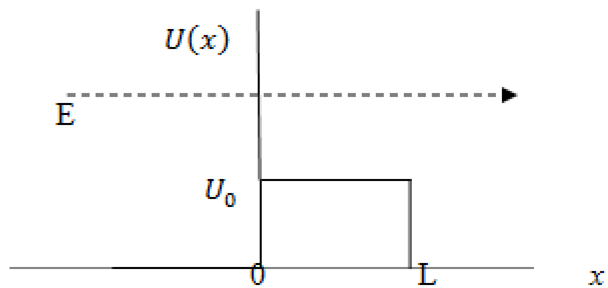
Fe^{ikx}

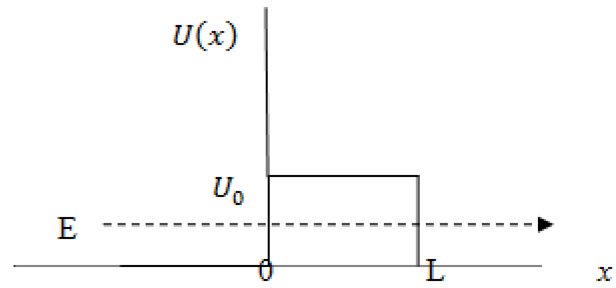
- $\psi_{x<0}(x=0) = \psi_{0\le x\le L}\,(x=0)$ → $A + B = C + D$
- $\frac{d\psi_{x<0}}{dx}\,|_{\,x=0} = \frac{d\psi_{0\le x\le L}}{dx}\,|_{\,x=0}$ → $k\,(A - B) = k'(C - D)$
- $\psi_{\psi_{0\le x\le L}}(x = L) = \psi_{0x>L}\,(x = L)$ → $Ce^{+ik'L} + De^{-ik'L} = Fe^{+ikL}$
- $\frac{d\psi_{0\le x\le L}}{dx}\,|_{\,x=L} = \frac{d\psi_{x>L}}{dx}\,|_{\,x=L}$ → $ik'\,(Ce^{+ik'L} - De^{-ik'L} = ik\,Fe^{+ikL}$

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$





Region 1 | Region 2 | Region 3

$\mathrm{A}\,e^{+ikx} \quad + \quad \mathrm{B}\,e^{-ikx}$

$\mathrm{C}\,e^{+ik'x} + \mathrm{D}\,e^{-ik'x}$

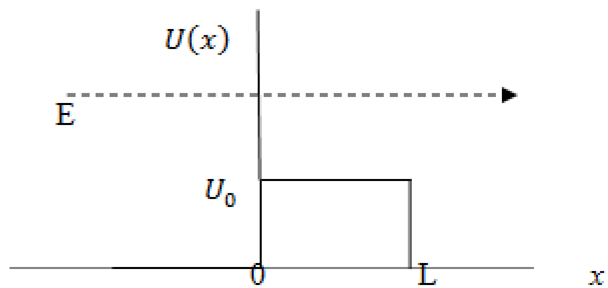
$\mathrm{F}e^{ikx}$

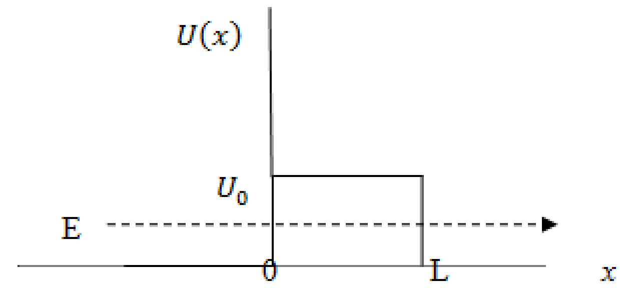
- $\psi_{x<0}(x=0) = \psi_{0\le x\le L}(x=0)$ → $\mathrm{A}+\mathrm{B} = \mathrm{C}+\mathrm{D}$
- $\frac{d\psi_{x<0}}{dx}\Big|_{x=0} = \frac{d\psi_{0\le x\le L}}{dx}\Big|_{x=0}$ → $k(\mathrm{A}-\mathrm{B}) = k'(\mathrm{C}-\mathrm{D})$
- $\psi_{\psi_{0\le x\le L}}(x=L) = \psi_{0x>L}(x=\mathrm{L})$ → $\mathrm{C}e^{+ik'L} + \mathrm{D}e^{-ik'L} = \mathrm{F}e^{+ikL}$
- $\frac{d\psi_{0\le x\le L}}{dx}\Big|_{x=\mathrm{L}} = \frac{d\psi_{x>L}}{dx}\Big|_{x=\mathrm{L}}$ → $ik'(\mathrm{C}e^{+ik'L} - \mathrm{D}e^{-ik'L} = ik\,\mathrm{F}e^{+ikL}$

Bound vs. unbound states

time-independent Schrodinger Equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\,\psi(x) = \mathrm{E}\,\psi(x)$$





Region 1: $\mathrm{A}\,e^{+ikx} + \mathrm{B}\,e^{-ikx}$

Region 2: $\mathrm{C}\,e^{+\alpha x} + \mathrm{D}\,e^{-\alpha x}$

Region 3: $\mathrm{F}e^{ikx}$