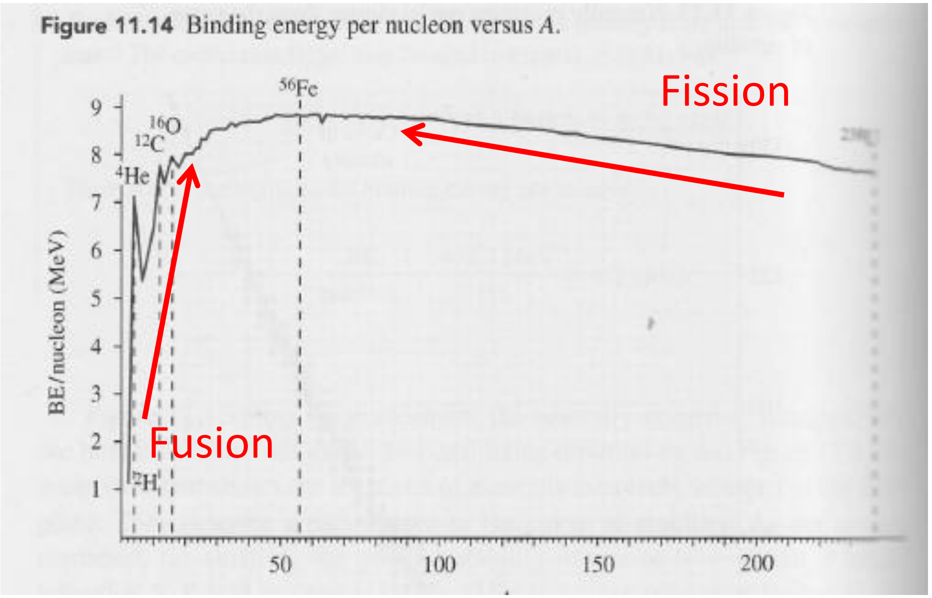
Lecture 19 Topics

- Nuclear reactions
  - Fission
    - Spontaneous fission
    - Induced fission
      - Happened big time as weapon
      - Happens in nuclear power plants
  - Fusion
    - Happens in stars
    - Has yet to happen for everyday use
- Fundamental forces
  - New definition involving mediating bosons
- Antiparticles
  - Klein-Gordon equation

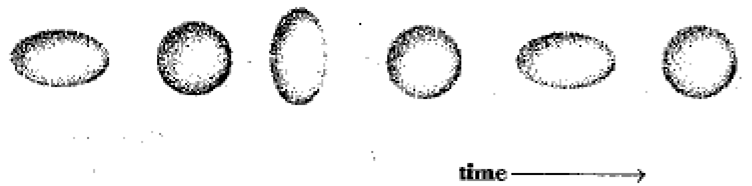
Binding energy/nucleon vs. A



Liquid drop model: Fission

When a large nucleus is excited…

FIGURE 24.11 The oscillations of a liquid drop.



Surface tension due to strong force vs. Repulsive Coulomb force

Will balance as gamma rays are emitted and
the nucleus finds a stable energy state.

Liquid drop model: Fission

FIGURE 24.11 The oscillations of a liquid drop.

When a large nucleus is TOO excited…

FIGURE 24.12 Nuclear fission according to the liquid-drop model.

Surface tension due to strong force < Repulsive Coulomb force

How to excite a large nucleus? Shoot highly energetic neutrons!!!

Liquid drop model: Fission

FIGURE 24.11 The oscillations of a liquid drop.

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FIGURE 24.12
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Nuclide	Half-life	Fission prob. per decay	Neutrons per fission	Neutrons per gram-second
^{235}U	7.04×10^{8} years	2.0×10^{-9}	1.86	3.0×10^{-4}
^{238}U	4.47×10^{9} years	5.4×10^{-7}	2.07	0.0136
^{239}Pu	2.41×10^{4} years	4.4×10^{-12}	2.16	0.022
^{240}Pu	6569 years	5.0×10^{-8}	2.21	920
^{250}Cm	6900 years	0.61	3.31	1.6×10^{10}
^{252}Cf	2.638 years	3.09×10^{-2}	3.73	2.3×10^{12}

Surface tension due to strong force < Repulsive Coulomb force

How to excite a large nucleus? Shoot highly energetic neutrons!!!

Nuclear fission

A heavy nucleus breaks into smaller nuclei

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n$$
$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$$
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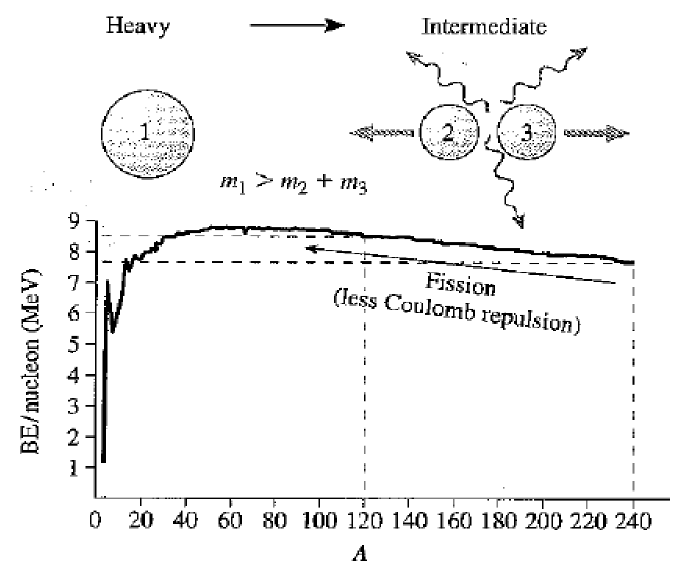
Figure 11.27 Decreasing BE/nucleon via fission.



What is the driving force?

Why neutrons are released?

How much energy can be released if we assume the bonding energy difference between A=240 nuclei and A=120 nuclei is about 0.9 MeV?

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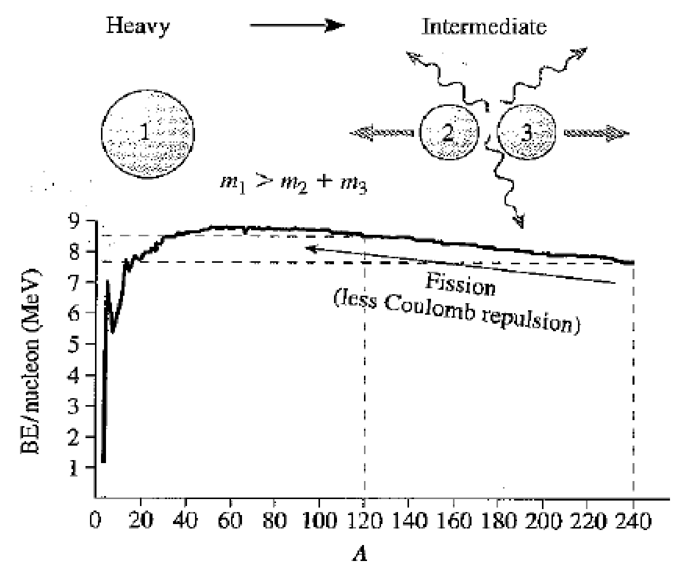

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At the order of 200 MeV….consider that
-typical chemical reactions are at the order of 1-10 eV.
-spontaneous decays are at the order of a few MeV

Fission: Chain reaction

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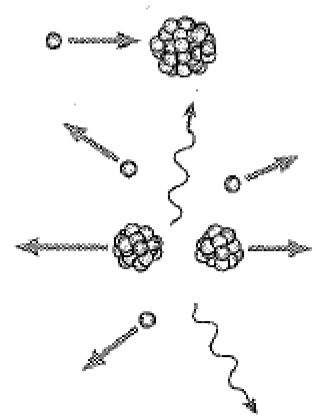
Figure 11.28 Neutron-induced fission, freeing more neutrons.



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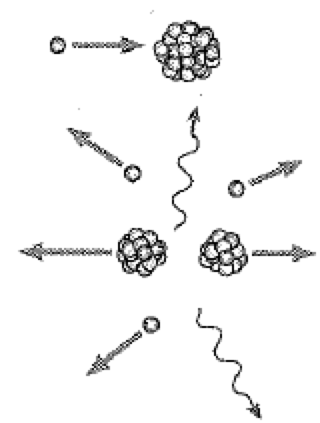

Since each nuclear fission reaction generates highly energetic neutrons, each of which also can induce another fission reaction with another large nucleus.

$$E_j = E_0 n^j$$

where E_0 is energy released for the fission reaction
n neutrons are generated in each fission
By the jth time, released energy can be up to E_j

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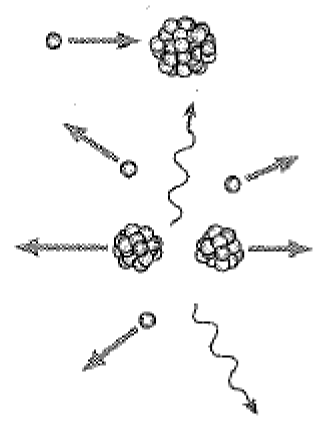

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Since there are multiple fission pathways, consider k as a net number of neutrons generated, then

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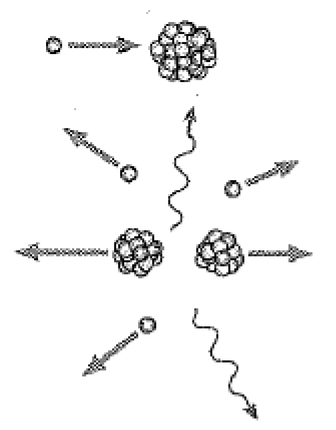

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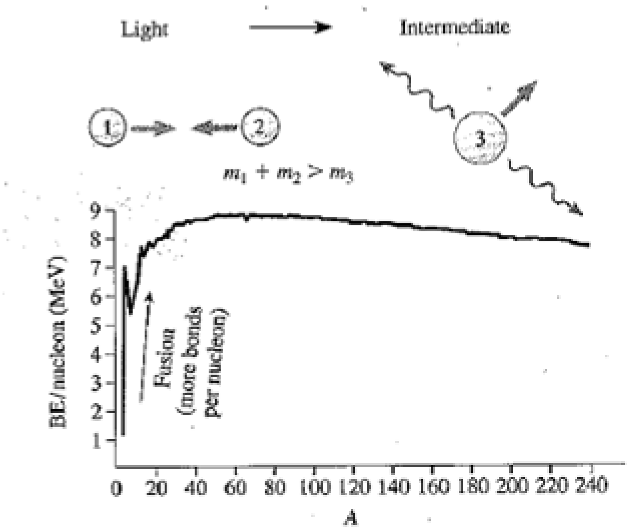
Since there are multiple fission pathways, consider k as a net number of neutrons generated, then

k=1
k<1
k>1

Nuclear fusion

Small nuclei fuse to become a larger nucleus

Figure 11.30 Decreasing BE/nucleon via fusion.



Example:

$${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

$Q =$

What is the driving force?

Is chain reaction possible?

Nuclear fusion

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Example:

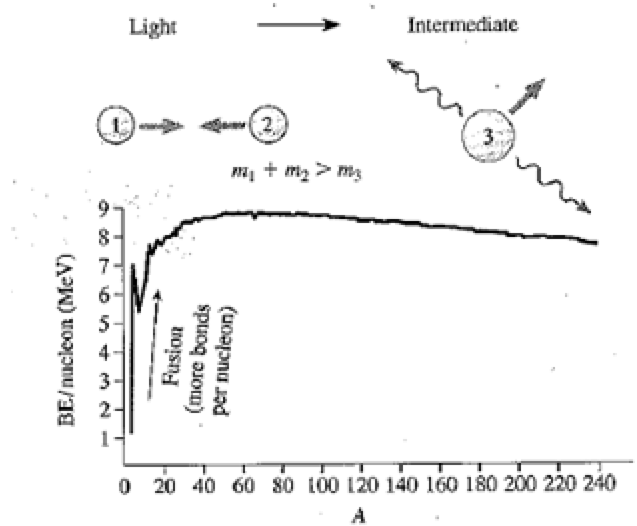
$$ {}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma $$

$$ Q = (2.0141 + 1.0078 - 3.0160)uc^2 $$
$$ = 5.48\ MeV $$

What is the driving force?

Is chain reaction possible?

Figure 11.30 Decreasing BE/nucleon via fusion.



Proton-Proton cycle

Step 1: $^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + {}_{+1}^{0}\beta + \nu$ (Q=0.42 MeV)

Step 2: $^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He$ (Q=5.48 MeV)

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Step 3: $^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$ (Q=12.9 MeV)

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Net Results:

Proton-Proton cycle

Step 1: ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{+1}\beta + \nu$ (Q=0.42 MeV)

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Net Results: Ins and Outs

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Net Results:

4 protons are fused to create one He nucleus
Two positrons and two neutrinos are generated
Total energy generated = 24.7 MeV

Proton-Proton cycle

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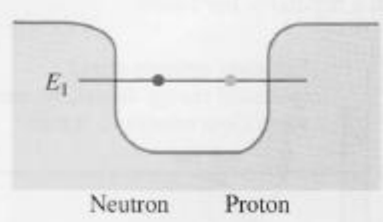
Step 1 is a relatively slow process, why?

Figure 11.5 The deuteron's neutron and proton bound in a well resulting from their attractive potential energy.



Carbon cycle

He nuclei in abundance can be fused to create Carbon. Once Carbon is available,

Step 1: $^{1}_{1}H + ^{12}_{6}C \rightarrow ^{13}_{7}N$

Step 2: $^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{+1}\beta + \nu$

Step 3: $^{1}_{1}H + ^{13}_{6}C \rightarrow ^{14}_{7}N$

Step 4: $^{1}_{1}H + ^{14}_{7}N \rightarrow ^{15}_{8}O$

Step 5: $^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{+1}\beta + \nu$

Step 6: $^{1}_{1}H + ^{15}_{7}N \rightarrow ^{12}_{6}C + ^{4}_{2}He$

Net effect:

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Spontaneous fusion not possible!

Increase in Z means that
-protons will need to be fused or added to existing nucleus.
-Coulomb repulsion should be overcome before both nuclei are bound by strong force.

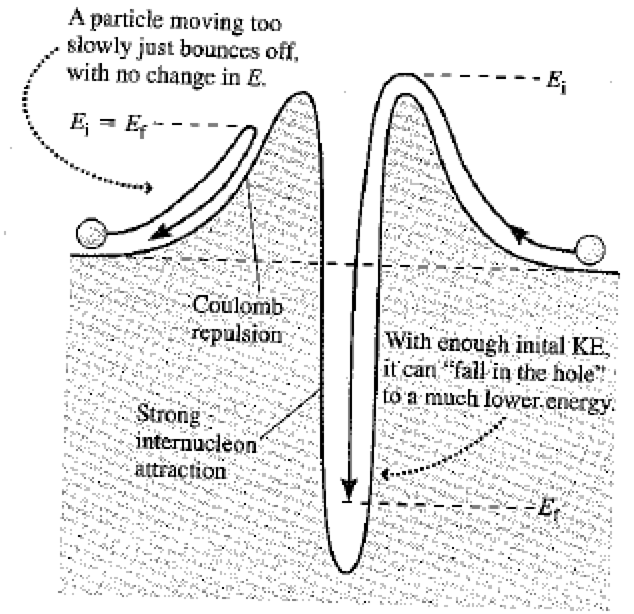
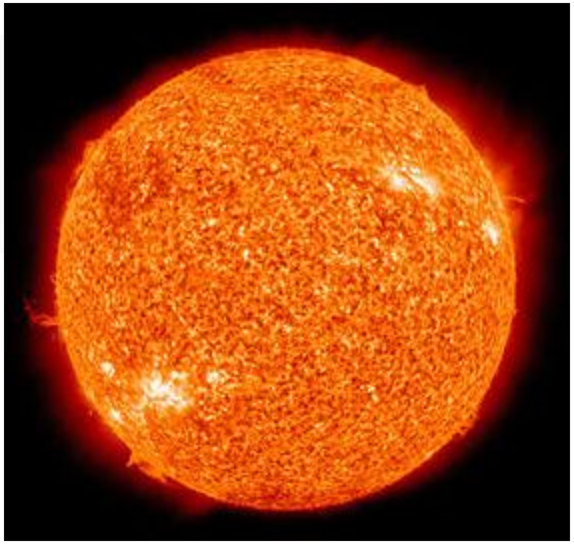


Figure 11.32 Nuclear fusion: over the Coulomb hurdle, then into the strong force well.

Fusion does not occur in ordinary conditions

Temperature	Center (modeled): ~1.57×10^7 K [1]
	Photosphere (effective): 5,778 K [1]
	Corona: ~5×10^6 K



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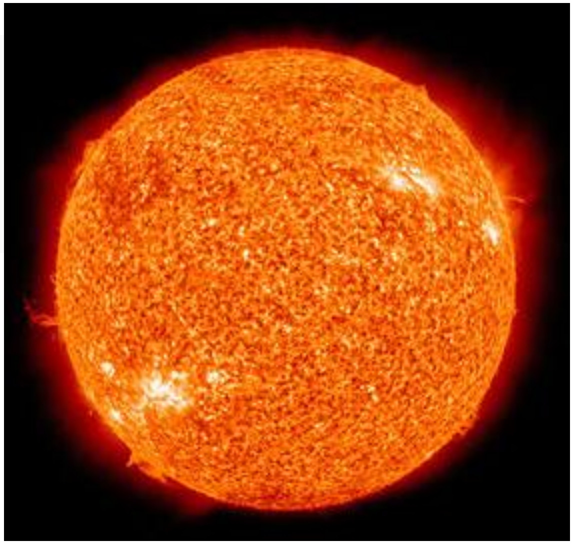

For nuclear fusion to occur in a Hydrogen Bomb, an atomic bomb is initially detonated.

Fusion/Fission: Clear energy solutions?

	Fission	**Fusion**
Fuel	Uranium and Thorium: Not rare and should be mined $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n$ $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2\,^{1}_{0}n$ $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3\,^{1}_{0}n$	Deuterium: Abundant and non-toxic $^{2}_{1}D + ^{3}_{1}T \rightarrow ^{4}_{2}He + ^{1}_{0}n \quad Q=17.6\ MeV$
Waste	Highly toxic Radioactive with long half-life Disposal is a problem	He isotopes (harmless) and Tritium (radioactive with short half life and not chemically hazardous)
Chain reaction	Chain reactions are possible, thus the fission process should be controlled	Chain reactions are not possible.

Elementary particles

What are the fundamental building blocks of the Universe (living and nonliving)?

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-Molecules
-Atoms → Elements
-Nucleus + electrons
-Nucleons (protons and neutrons) in the nucleus
-Quarks in nucleons

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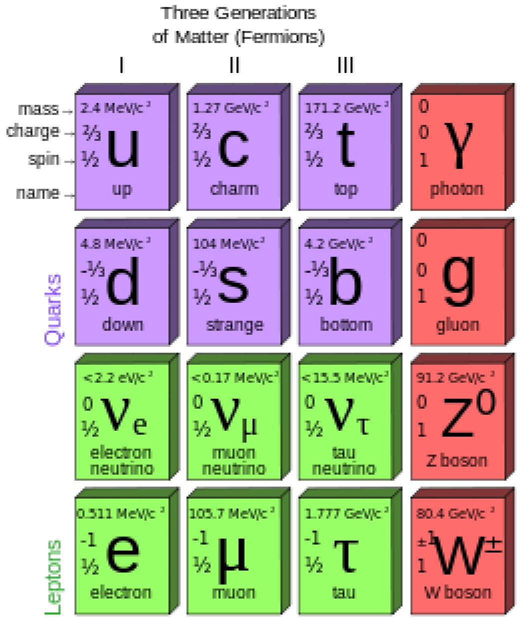

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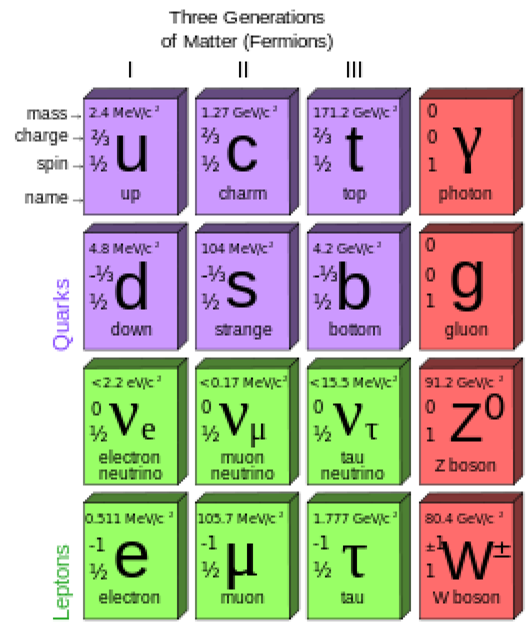


Three classes of particles:

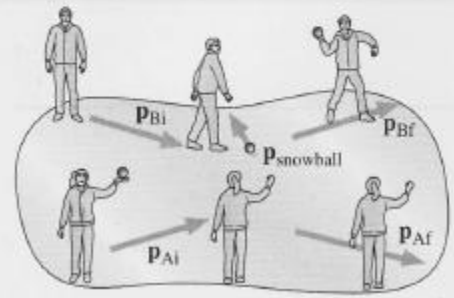
- Six quarks → strong force
- Six leptons → electroweak force
- Four mediating particles called field quanta
  - → Graviton: gravitational force
  - → Photon: electromagnetic force
  - → W^+, W^-, Z^0: weak force
  - → Gluon: strong force

New definition for force

Force is exerted by exchanging a mediating particle.

Anna is throwing a snowball on a frictionless frozen pond.

Figure 12.1 A force between students conveyed by exchange of a snowball.



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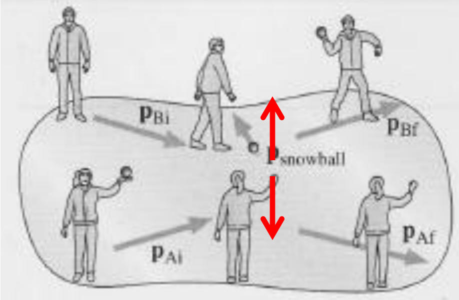
$$\vec{p}_{Anna\,(after\ snowball\ throwing)} = \vec{p}_{Anna} - \vec{p}_{snowball}$$

$$\vec{p}_{Bob\,(after\ snowball\ receiving)} = \vec{p}_{Bob} + \vec{p}_{snowball}$$



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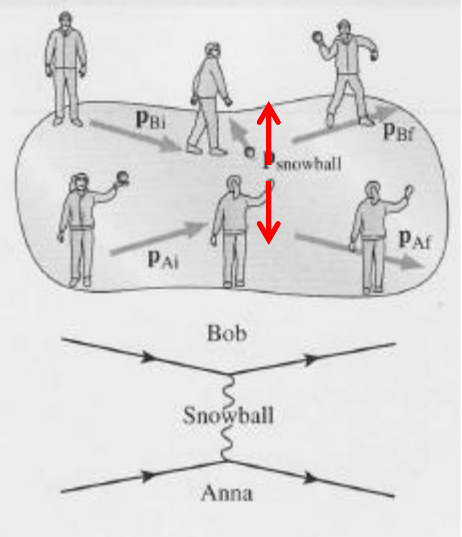

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Limitations of snowball analogy:
-Attractive fundamental force cannot be shown
-Mediating particles exist during the exchange, not before or after
-Mediating particles do not act like snowball (classical physical entities)

Force range

From uncertainty principle

$$\Delta t \Delta E \approx \hbar \quad \longrightarrow \quad \Delta t \approx \frac{\hbar}{\Delta E}$$

Energy of the mediating particle

$$\Delta E = mc^2$$

$$\Delta t \approx \frac{\hbar}{\Delta E} \approx$$

Force range

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Force range

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

- When the mediating particle has mass
- When it doesn't

Force range

TABLE 12.1 Fundamental forces and particles

Force	Gravitation	Electroweak			Strong	Residual
Property	Mass/energy	Charge/weak charge			Color charge	
Strength	$\sim 10^{-39}$	$\sim 10^{-2}$	$\sim 10^{-6}$		1	
Range	$1/r^2$	$1/r^2$	10^{-3} fm		short	1 fm
Mediating Bosons	Graviton?	Photon, γ	W^+, W^-	Z^0	Gluon	$\pi^\pm, \pi^0$
Spin	2?	1	1	1	1	0
Mass	0?	$< 6 \times 10^{-22}$	80.4×10^3	91.2×10^3	< 10	140, 135
Charge	—	0	+1, −1	0	0	$\pm 1, 0$
Color charge	—	—	—	—	r, g, or b + $\bar{r}$, $\bar{g}$, or $\bar{b}$	Neutral

$$\Delta x \approx c\Delta t \approx \frac{\hbar}{c}\frac{1}{m}$$

- When the mediating particle has mass
- When it doesn’t

How fundamental forces are connected?

- Particles
  - Fundamental particles experiencing force are fermions
  - Fermions should have a property associated with the force if they are engaged in that force
    - Mass/energy for gravitational force
    - Charge/weak charge for electroweak force
    - Color charge for strong force
- Mediating particles
  - Spin: particles that mediate force are bosons
  - Mass determines the range of the force

Schrodinger Equation

Hamiltonian operator (H)

$$H\Psi(x,t) = E\Psi(x,t)$$

Since H = Total Energy = Kinetic energy (T) + Potential energy (U)

$$\left(\frac{p^2}{2m} + U\right)\Psi(x,t) = E\Psi(x,t)$$

$$p = \frac{\hbar}{i}\frac{\partial}{\partial x} \quad \text{and} \quad \mathrm{E} = i\hbar\,\frac{\partial}{\partial t}$$

Time-Dependent Schrodinger Equation

$$\frac{-\hbar^2}{2m}\,\frac{\partial^2\Psi\,(x,t)}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\,\frac{\partial\,\Psi(x,t)}{\partial t}$$

Klein-Gordon Equation

$$E = mc^2$$

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$$E^2 = m^2 c^4 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}} c^4$$

Klein-Gordon Equation

$$E = mc^2$$

$$E^2 = m^2c^4 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}}c^4 :$$

$$= {m_0}^2c^4\left(\frac{1}{1 - \frac{v^2}{c^2}}\right) = {m_0}^2c^4\left(\frac{1}{1 - \frac{v^2}{c^2}} - 1 + 1\right)$$

Klein-Gordon Equation

$$E = mc^2$$

$$E^2 = m^2c^4 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}} c^4$$

$$= {m_0}^2c^4 \left(\frac{1}{1 - \frac{v^2}{c^2}} \right) = {m_0}^2c^4 \left(\frac{1}{1 - \frac{v^2}{c^2}} - 1 + 1 \right)$$

$$= {m_0}^2c^4 + {m_0}^2c^4 \left(\frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}} \right) = {m_0}^2c^4 + c^2v^2 \frac{{m_0}^2}{1 - \frac{v^2}{c^2}}$$

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$$= {m_0}^2c^4 + {m_0}^2c^4\left(\frac{\frac{v^2}{c^2}}{1 - \frac{v^2}{c^2}}\right) = {m_0}^2c^4 + c^2v^2\frac{{m_0}^2}{1 - \frac{v^2}{c^2}}$$

$$= {m_0}^2c^4 + c^2v^2m^2 = {m_0}^2c^4 + p^2c^2$$

$$E^2 = {m_0}^2c^4 + p^2c^2$$

Klein-Gordon Equation

$$E^2 = {m_0}^2 c^4 + p^2 c^2$$

$$E^2 \psi = {m_0}^2 c^4 \psi + p^2 c^2 \psi$$

Using operator notations

$p = -i\hbar\nabla$ and $E = i\hbar \frac{\partial}{\partial t}$

Klein-Gordon Equation

$$E^2 = m_0{}^2 c^4 + p^2 c^2$$

$$E^2 \psi = m_0{}^2 c^4 \psi + p^2 c^2 \psi$$

Using operator notations
$p = -i\hbar\nabla$ and $E = i\hbar \frac{\partial}{\partial t}$

$$-c^2 \hbar^2 \nabla^2 \psi + m_0{}^2 c^4 \psi = -\hbar^2 \frac{\partial^2}{\partial t^2} \psi$$

Klein-Gordon equation works for spinless particles
Use Dirac equation for particles with spin

Schrodinger vs. Klein-Gordon Eq

	Schrodinger Eq (free particle)	Klein-Gordon Eq (free particle)
Energy relation	$E = T$	$E^2 = m_0{}^2c^4 + p^2c^2$
Equation	$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t}$	$-c^2\hbar^2\nabla^2\psi + m_0{}^2c^4\psi = -\hbar^2\frac{\partial^2}{\partial t^2}\psi$
Energy	$E = \frac{p^2}{2m}$, E is positive	Positive and negative
	$\psi^*\psi$ Probability density Does not change with time	$i\Psi^*\frac{\partial}{\partial t}\Psi - i\Psi\frac{\partial}{\partial t}\Psi^*$ This does not change with time and it represents charge density and both positive and negative values are possible: positive for particle and negative for antiparticle

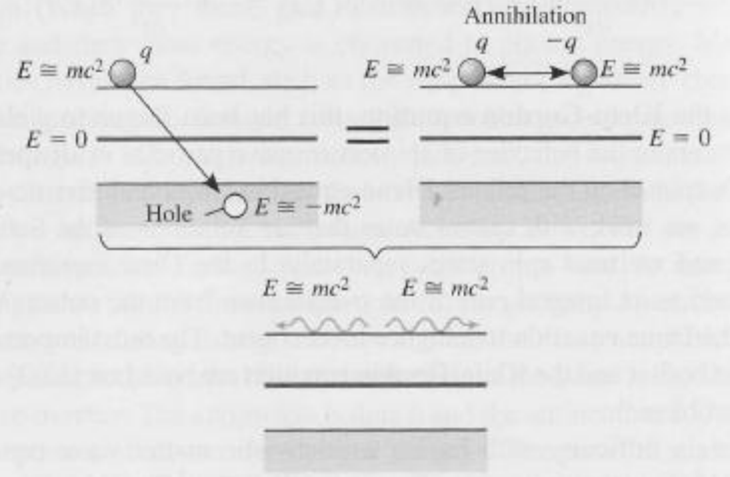
Particle and antiparticle

The antiparticle state is similar to a hole in a sea of allowed but filled negative energy states of the particle.

-If filled, the antiparticle state cannot be observed.

-When a hole is available, a particle can be annihilated, emitting photon energy twice of the particle's energy.

Figure 12.2 Whether a positive-energy particle drops into an empty negative-energy state, or two positive-energy particles annihilate, the result is the same.



Antiparticles

- Antiparticle: particle with the same properties except charge. $p \; and \; \bar{p}, n \; and \; \bar{n}$

  $e^- \; and \; e^+, \mu^+ \; and \; \mu^-$

- Experimentally detected.
- Particle-antiparticle can be produced and disappear simultaneously with the energy involved.

Quarks for Strong force

- Particles that experience strong force are Quarks
- Six types
  - Spin ½
  - Have different mass
  - Have different charge
  - Have all three color charge
  - Have antiparticles
  - Not separable
- Gluons (massless, spin-1) are mediating particles

Quarks
Participants in gravitation, electroweak, and strong

	Spin	Mass	Charge	Color charge
Up, u	$\frac{1}{2}$	~5	$+\frac{2}{3}$	r, g, b
Down, d	$\frac{1}{2}$	~10	$-\frac{1}{3}$	r, g, b
Strange, s	$\frac{1}{2}$	~100	$-\frac{1}{3}$	r, g, b
Charm, c	$\frac{1}{2}$	$\sim 1.3 \times 10^3$	$+\frac{2}{3}$	r, g, b
Bottom, b	$\frac{1}{2}$	$\sim 4.5 \times 10^3$	$-\frac{1}{3}$	r, g, b
Top, t	$\frac{1}{2}$	$\sim 180 \times 10^3$	$+\frac{2}{3}$	r, g, b

Color charge

- Three types: red (r), green (g) and blue (b)
- Each color has anti-color charge: anti-red, anti-blue, and anti-green
- Gluons carry a color-anticolor pair (blue-antigreen) and thus interact with one another.

	Electromagnetic interactions	**Strong interactions**
Property	Charge	Color charge
Fermions	Charged particles	Quarks
Mediating bosons	Photons	Gluons
	Do not carry charge	Carry color charge
	Do not interact with themselves	Interact with themselves

Hadrons

- Hadrons consist of quarks:
  - Two quarks: mesons
  - Three quarks: baryons
- Hadrons are color charge neutral
  - Two quarks system = quark (color) + quark (anticolor) e.g. red + antired
  - Three quarks system: a quark carries one each of red, blue, and green colors
  - Zero net color leads to attraction

Hadrons

TABLE 12.2 Commonly produced hadrons

Baryons	Mass (MeV/c^2)	Spin	Strange-ness	I, I_3	Lifetime, τ (or width $\hbar/\tau$)
p (uud)	938	$\frac{1}{2}$	0	$\frac{1}{2}, +\frac{1}{2}$	$>10^{32}$ yr
n (udd)	940	$\frac{1}{2}$	0	$\frac{1}{2}, -\frac{1}{2}$	889 s
Σ^+ (uus)	1189	$\frac{1}{2}$	−1	1, +1	8.0×10^{-11} s
Σ^0 (uds)	1193	$\frac{1}{2}$	−1	1, 0	7.4×10^{-20} s
Λ^0 (uds)	1116	$\frac{1}{2}$	−1	0, 0	2.6×10^{-10} s
Σ^- (dds)	1197	$\frac{1}{2}$	−1	1, − 1	1.5×10^{-10} s
Ξ^0 (uss)	1315	$\frac{1}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	2.9×10^{-10} s
Ξ^- (dss)	1321	$\frac{1}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	1.6×10^{-10} s
Δ^{++} (uuu)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, +\frac{3}{2}$	120 MeV
Δ^+ (uud)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, +\frac{1}{2}$	120 MeV
Δ^0 (udd)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, -\frac{1}{2}$	120 MeV
Δ^- (ddd)	1232	$\frac{3}{2}$	0	$\frac{3}{2}, -\frac{3}{2}$	120 MeV
Σ^{*+} (uus)	1383	$\frac{3}{2}$	−1	1, +1	~40 MeV
Σ^{*0} (uds)	1384	$\frac{3}{2}$	−1	1, 0	~40 MeV
Σ^{*-} (dds)	1387	$\frac{3}{2}$	−1	1, −1	~40 MeV
Ξ^{*0} (uss)	1532	$\frac{3}{2}$	−2	$\frac{1}{2}, +\frac{1}{2}$	~10 MeV
Ξ^{*-} (dss)	1535	$\frac{3}{2}$	−2	$\frac{1}{2}, -\frac{1}{2}$	~10 MeV
Ω^- (sss)	1672	$\frac{3}{2}$	−3	0, 0	8.2×10^{-11} s

Mesons	Mass (MeV/c^2)	Spin	Strange-ness	I, I_3	Lifetime, τ (or width $\hbar/\tau$)
$\pi^+(u\bar{d})$	140	0	0	1, +1	2.6×10^{-8} s
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$\pi^-(d\bar{u})$	140	0	0	1, −1	2.6×10^{-8} s
$K^+(u\bar{s})$	494	0	+1	$\frac{1}{2}, +\frac{1}{2}$	1.2×10^{-8} s
$K^0_S(d\bar{s}, s\bar{d})$	498	0	mix	$\frac{1}{2}$, mix	8.9×10^{-11} s
$K^0_L(d\bar{s}, s\bar{d})$	498	0	mix	$\frac{1}{2}$, mix	5.2×10^{-8} s
$K^-(s\bar{u})$	494	0	−1	$\frac{1}{2}, -\frac{1}{2}$	1.2×10^{-8} s
$\rho^+(u\bar{d})$	769	1	0	1, +1	151 MeV
$\rho^0(u\bar{u} + d\bar{d})$	769	1	0	1, 0	151 MeV
$\rho^-(d\bar{u})$	769	1	0	1, −1	151 MeV
$K^{*+}(u\bar{s})$	892	1	+1	$\frac{1}{2}, +\frac{1}{2}$	50 MeV
$K^{*0}(d\bar{s})$	896	1	+1	$\frac{1}{2}, -\frac{1}{2}$	51 MeV
$\overline{K^{*0}}(s\bar{d})$	896	1	−1	$\frac{1}{2}, +\frac{1}{2}$	51 MeV
$K^{*-}(s\bar{u})$	892	1	−1	$\frac{1}{2}, -\frac{1}{2}$	50 MeV
Heavy mesons—containing quarks beyond the strange					
J/ψ ($c\bar{c}$)	3100	1	0	0, 0	87 keV
Υ ($b\bar{b}$)	9460	1	0	0, 0	~50 keV

50 MeV corresponds to a lifetime of 10^{-23} second

Hadrons

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Participants in gravitation, electroweak, and strong

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Construct charge
Color charge
Compare mass

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