

Announcement

- Homework (15%) + Class participation (5%)
- Midterm 1 (20%) + Midterm 2 (20%)
- Final (40%) on 3/19, Wed, 8-11 am
 - Bring calculator, open-book and open-notes
 - 30% Nuclear
 - 25% Elementary
 - 45% Atomic, statistical, and solid state focusing on common underlying ideas:
 - Energy, Hamiltonian, conservation, system, symmetry, etc.

Lecture 18

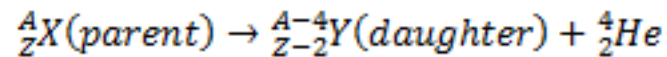
- Nuclear decay
 - Alpha decay
 - Beta decay
 - Gamma decay
- Nuclear reactions
 - Fission
 - Fusion

Radioactive decay

- An unstable nucleus (parent nucleus) can spontaneously emit small particles or energies to become a nucleus (daughter nucleus) in a more stable state.
 - Alpha decay ($\alpha = \text{He nucleus}$)
 - Beta decay: β^+ decay, β^- decay, electron capture
 - Gamma decay ($\gamma = \text{photons}$)

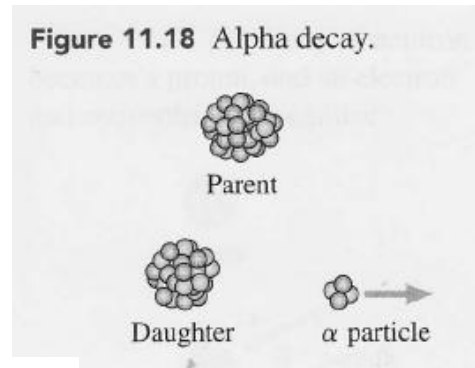
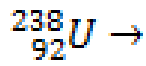
Alpha decay

- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons



- $Z_{\text{daughter}} =$
- $N_{\text{daughter}} =$
- $A_{\text{daughter}} =$

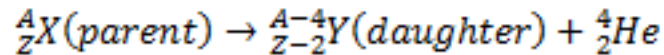
Released kinetic energy (Q)
 $Q =$



m: atomic mass

Alpha decay

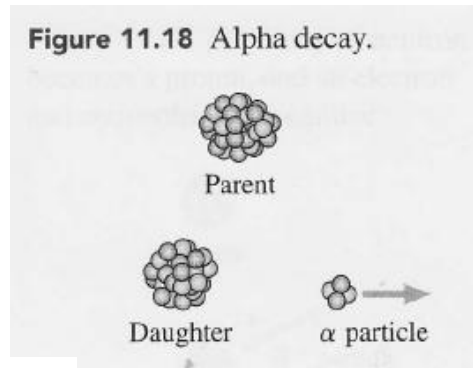
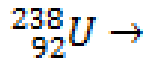
- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons



- $Z_{\text{daughter}} = Z_{\text{parent}} - 2$
- $N_{\text{daughter}} = N_{\text{parent}} - 2$
- $A_{\text{daughter}} = A_{\text{parent}} - 4$

Released kinetic energy (Q)

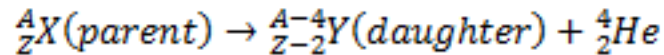
$$Q = (m_{\text{parent}} - m_{\text{daughter}} - m_{\text{He}}) c^2$$



m: atomic mass

Alpha decay

- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons

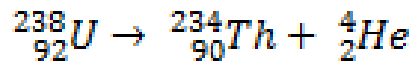


- $Z_{\text{daughter}} = Z_{\text{parent}} - 2$
- $N_{\text{daughter}} = N_{\text{parent}} - 2$
- $A_{\text{daughter}} = A_{\text{parent}} - 4$

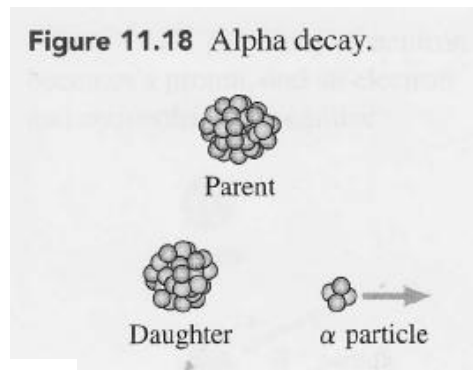
Released kinetic energy (Q)

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - m_{\text{He}}) c^2$$

m: atomic mass

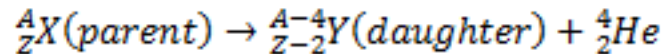


${}^{238}_{92} \text{U}$	234.050784 u
${}^{234}_{90} \text{Th}$	234.043593 u
${}^4_2 \text{He}$	4.002603 u
	$Qc^2 = 931.5 \text{ MeV}$



Alpha decay

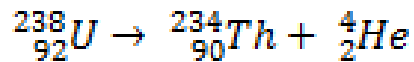
- Emits an alpha particle
- Alpha particle is He nucleus=2 protons +2 neutrons



- $Z_{\text{daughter}} = Z_{\text{parent}} - 2$
- $N_{\text{daughter}} = N_{\text{parent}} - 2$
- $A_{\text{daughter}} = A_{\text{parent}} - 4$

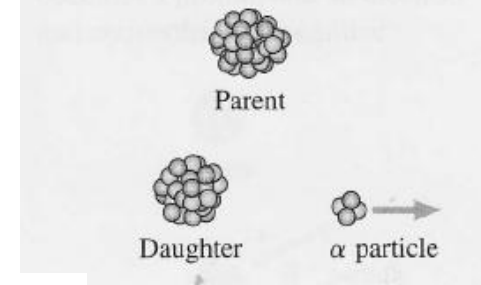
Released kinetic energy (Q)

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - m_{{}^4_2\text{He}}) c^2$$



$$\begin{aligned} Q &= (m_{\text{parent}} - m_{\text{daughter}} - m_{{}^4_2\text{He}}) c^2 \\ &= (238.050784 - 234.043593 - 4.002603) \text{u} c^2 \\ &= 0.004588 \times 931.5 \text{ MeV} = 4.27 \text{ MeV} \end{aligned}$$

Figure 11.18 Alpha decay.



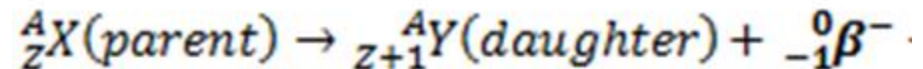
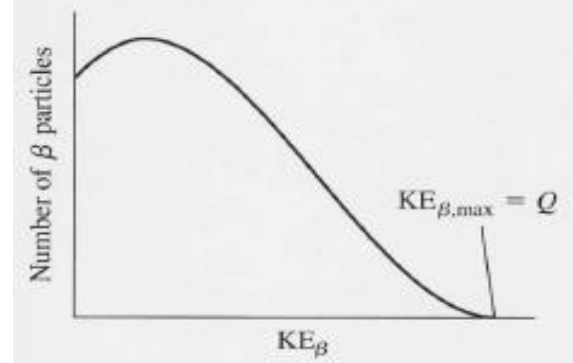
${}^{238}_{92}\text{U}$	234.050784 u
${}^{234}_{90}\text{Th}$	234.043593 u
${}^4_2\text{He}$	4.002603 u

$$\text{U}c^2 = 931.5 \text{ MeV}$$

Beta decay

- Emits β^+ , β^- particles (= positron, electron)
- Another particle is involved!
- To satisfy:
 - Energy conservation
 - Beta particles carry kinetic energies from 0 to maximum allowed
 - Charge conservation
 - β^+ , β^- carries +1 and -1
 - Angular momentum conservation
 - β^+ , β^- : spin $\frac{1}{2}$

Figure 11.19 The mysterious variation in β particle energies.

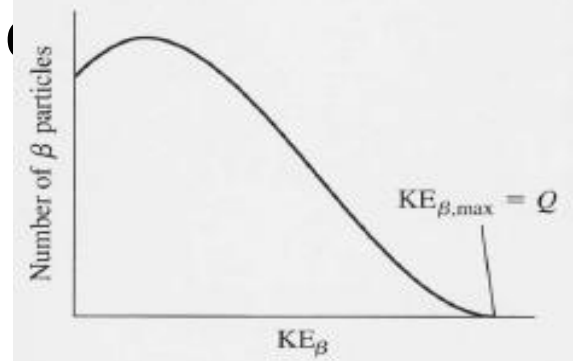


???

Beta decay

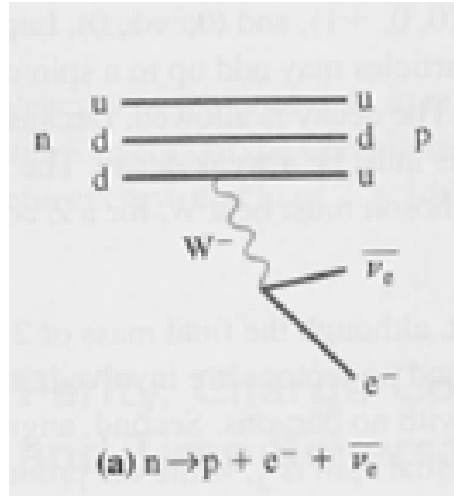
- Emits β^+ , β^- particles (= positron, electron)
- **Neutrino (ν)** is involved!
- To satisfy:
 - Energy conservation
 - Beta particles carry kinetic energies from 0 to maximum allowed **Neutrino: mass negligible**
 - Charge conservation
 - β^+ , β^- carries +1 and -1 **Neutrino: neutral**
 - Angular momentum conservation
 - β^+ , β^- : spin $\frac{1}{2}$ **Neutrino spin=1/2**

Figure 11.19 The mysterious variation in β particle energies.



Decay Possible?

Mass=940
 Charge = 0
 Spin=1/2
 Strangeness=0
 Baryon number=1
 Lepton number (e)=0



Strangeness not conserved in weak decay
 Some flexibility in angular momentum

Mass=938
 Spin=1/2
 Charge = +1
 Strangeness=0
 Baryon number=1
 Lepton number=0

Mass=0
 Spin=1/2
 Charge = -1
 Strangeness=0
 Baryon number=0
 Lepton number=-1

Mass=0.51
 Spin=1/2
 Charge = 0
 Strangeness=0
 Baryon number=0
 Lepton number=1

β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.



- $Z_{\text{daughter}} = Z_{\text{parent}} + 1$
- $N_{\text{daughter}} = N_{\text{parent}} - 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

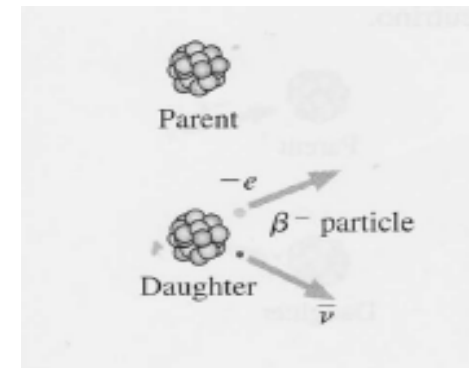
Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$

Example:



$$Q =$$



β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

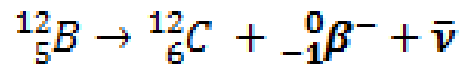


- $Z_{\text{daughter}} = Z_{\text{parent}} + 1$
- $N_{\text{daughter}} = N_{\text{parent}} - 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

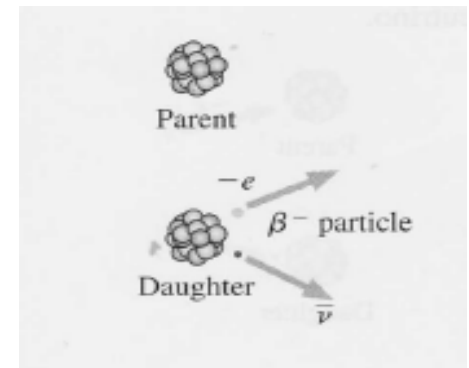
Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$

Example:



$Q =$



$${}^{12}_5 B = 12.014532 \text{ u}$$

$${}^{12}_6 C = 12 \text{ u}$$

$$1 \text{ u} c^2 = 931.5 \text{ MeV}$$

β^- decay

- Emits an electron and an anti-neutrino.
- Changes a neutron inside the nucleus into a proton.

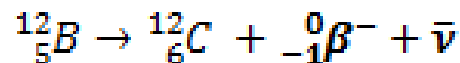


- $Z_{\text{daughter}} = Z_{\text{parent}} + 1$
- $N_{\text{daughter}} = N_{\text{parent}} - 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

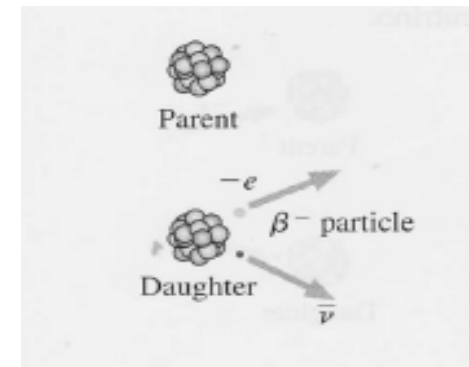
Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$

Example:



$$Q = (12.014352 - 12)uc^2 = 13.4 \text{ MeV}$$



$${}^{12}_5 B = 12.014532 \text{ u}$$

$${}^{12}_6 C = 12 \text{ u}$$

$$uc^2 = 931.5 \text{ MeV}$$

β^+ decay

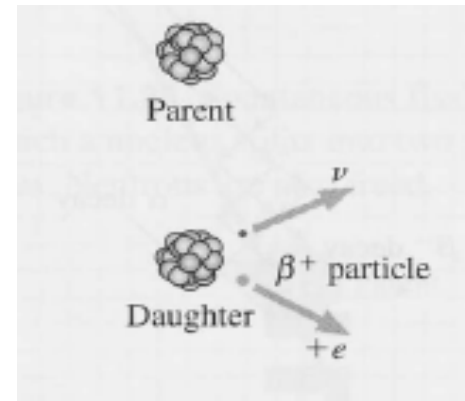
- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.



- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - 2m_{\text{electron}}) c^2$$



Example ${}^{12}_7 N \rightarrow$

$Q =$

β^+ decay

- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

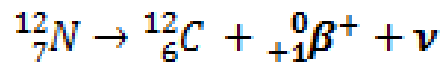


- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - 2m_{\text{electron}}) c^2$$

Example

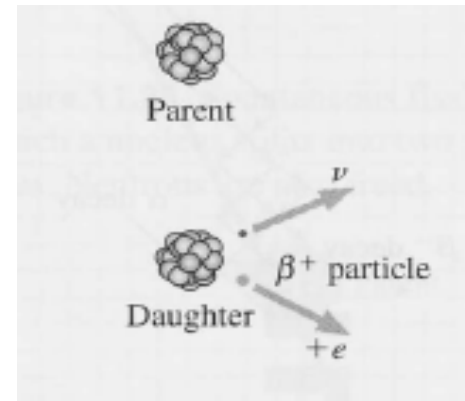


$Q =$

$${}^{12}_7 N = 12.018613$$

$${}^{12}_6 C = 12$$

$$uc^2 = 931.5 \text{ MeV}$$



β^+ decay

- Emits a positron and a neutrino
- Changes a proton inside the nucleus into a neutron.

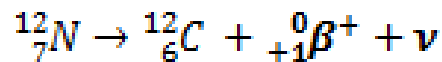


- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$

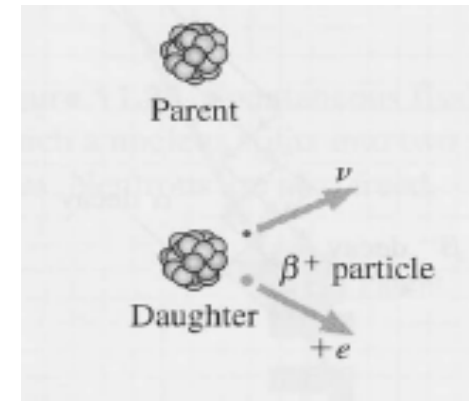
Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}} - 2m_{\text{electron}}) c^2$$

Example



$$Q = (12.018613 - 12 - 2 \times 0.0005486) uc^2 \\ = 16.3 \text{ MeV}$$



$${}^{12}_7 N = 12.018613$$

$${}^{12}_6 C = 12$$

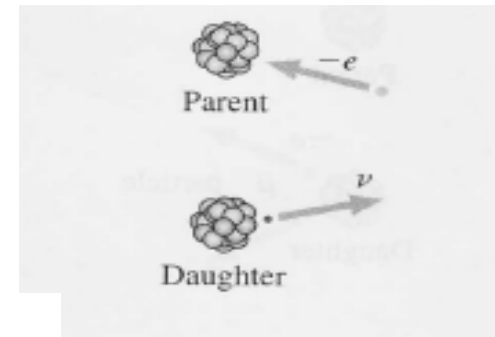
$$uc^2 = 931.5 \text{ MeV}$$

Electron capture

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than decay when an electron is already exists for a nucleus to capture.



- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$



Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$



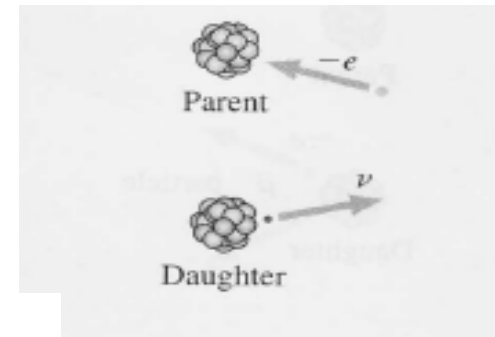
$Q =$

Electron capture

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- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$



Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$



$${}^{11}_6\text{C} = 11.01143 \text{ u}$$

$${}^{11}_5\text{B} = 11.009305 \text{ u}$$

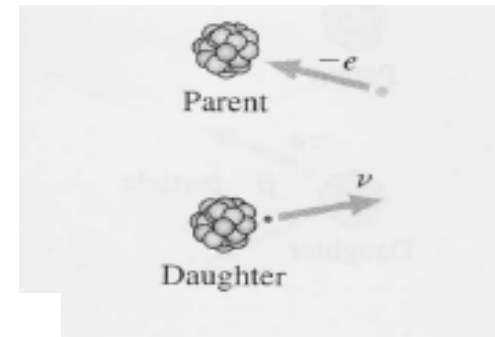
$$Q =$$

Electron capture

- A nucleus with too many protons can change a proton into a neutron by capturing an electron.
- Electron capture is easier than decay since an electron is already exists for a nucleus to capture.



- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$
- $N_{\text{daughter}} = N_{\text{parent}} + 1$
- $A_{\text{daughter}} = A_{\text{parent}}$



Released kinetic energy

$$Q = (m_{\text{parent}} - m_{\text{daughter}}) c^2$$



$$\begin{aligned} {}^{11}_6\text{C} &= 11.01143 \text{ u} \\ {}^{11}_5\text{B} &= 11.009305 \text{ u} \end{aligned}$$

$$Q = (11.01143 - 11.009305)uc^2 = 1.97 \text{ MeV}$$

Gamma decay

- A nucleus in an excited state emits photons (gamma particles, to go into a lower energy state.
- Gamma decay does not alter N or Z.
- Gamma energies are characteristic of a given isotope, and are thus used to identify the isotope.

Figure 11.24 Gamma decay.



Radioactive decay series

- An unstable nucleus can be involved in a series of decays until it finds a stable state.
- We can plot this process on a graph that represents N and Z numbers of each nucleus in the series.

Alpha decay is shown by an arrow

- $Z_{daughter} = Z_{parent} - 2$

- $N_{daughter} = N_{parent} - 2$

β^- decay

- $Z_{daughter} = Z_{parent} + 1$

- $N_{daughter} = N_{parent} - 1$

β^+ decay and electron capture

- $Z_{daughter} = Z_{parent} - 1$

- $N_{daughter} = N_{parent} + 1$

Radioactive decay series

- An unstable nucleus can keep decaying until it finds a stable nucleus.
- We can plot this process on a graph of N and Z numbers of each nucleus.

Alpha decay is shown by an arrow

- $Z_{\text{daughter}} = Z_{\text{parent}} - 2$

- $N_{\text{daughter}} = N_{\text{parent}} - 2$

β^- decay

- $Z_{\text{daughter}} = Z_{\text{parent}} + 1$

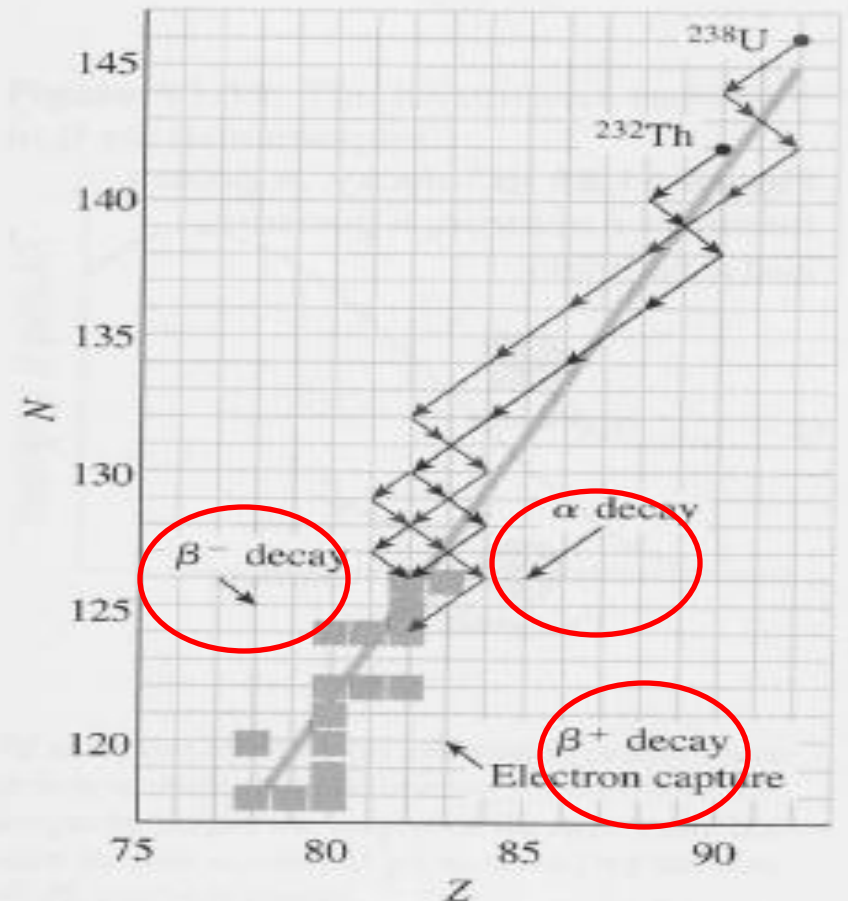
- $N_{\text{daughter}} = N_{\text{parent}} - 1$

β^+ decay and electron capture

- $Z_{\text{daughter}} = Z_{\text{parent}} - 1$

- $N_{\text{daughter}} = N_{\text{parent}} + 1$

Figure 11.23 The “directions” of α and β decays, and the decay series of uranium-238 and thorium-232.



Radioactive decay series

- An unstable nucleus can decay until it finds a stable nucleus
- We can plot this process on a graph of N and Z numbers of each nucleus

Alpha decay is shown by an arrow

- $Z_{daughter} = Z_{parent} - 2$

- $N_{daughter} = N_{parent} - 2$

β^- decay

- $Z_{daughter} = Z_{parent} + 1$

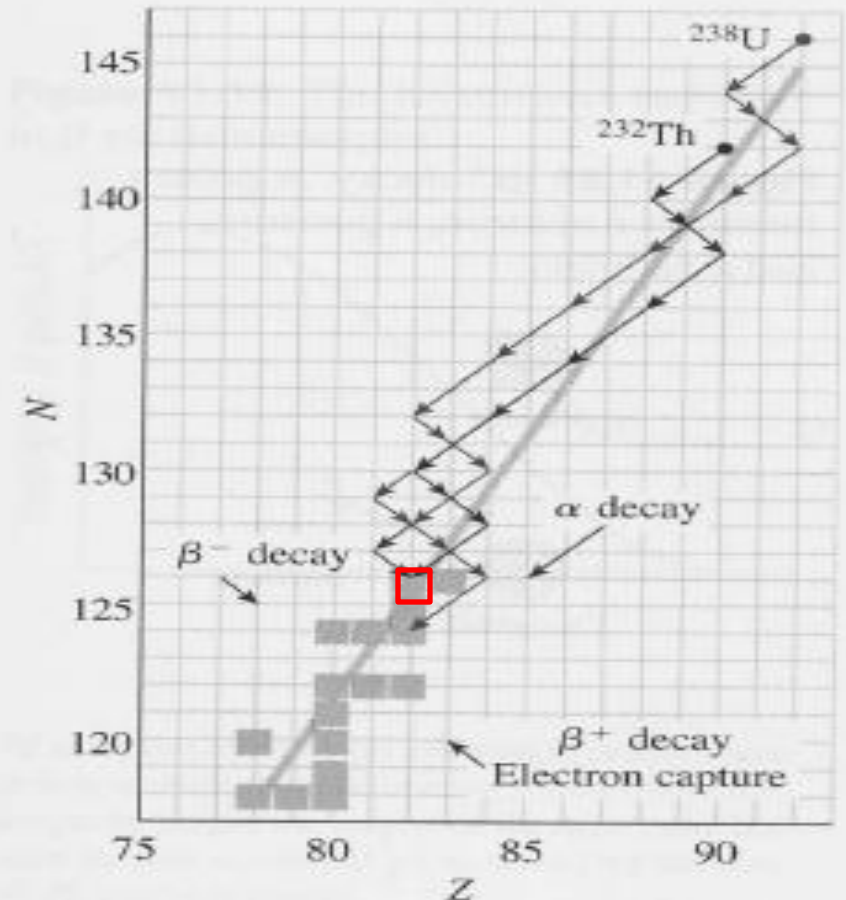
- $N_{daughter} = N_{parent} - 1$

β^+ decay and electron capture

- $Z_{daughter} = Z_{parent} - 1$

- $N_{daughter} = N_{parent} + 1$

Figure 11.23 The “directions” of α and β decays, and the decay series of uranium-238 and thorium-232.



Pb=82 protons +126 neutrons

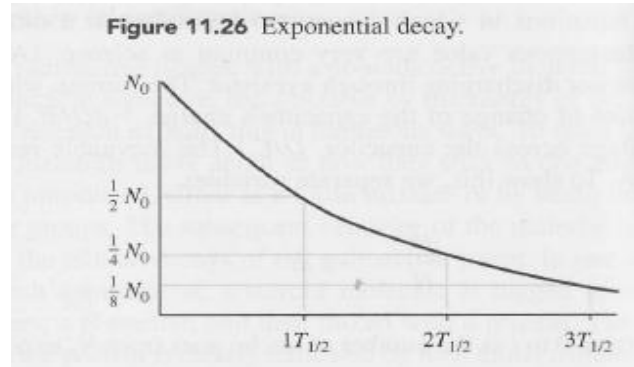
Radioactive decay law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

where N =Number of nuclei; λ =decay constant

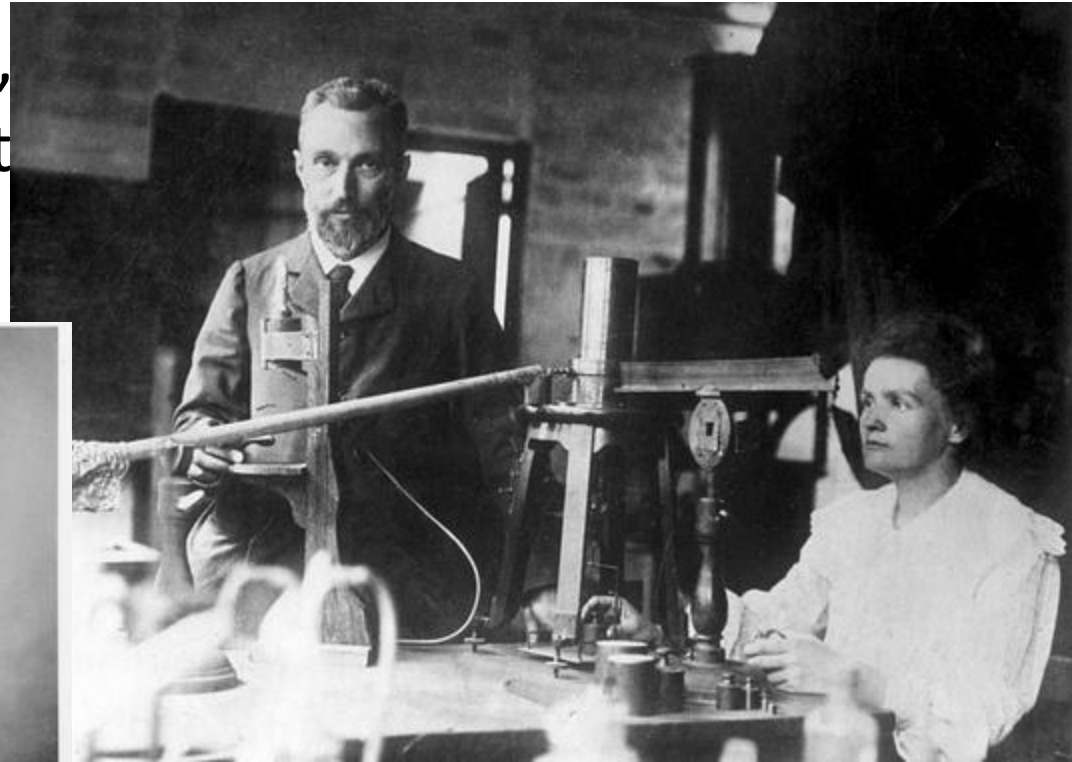


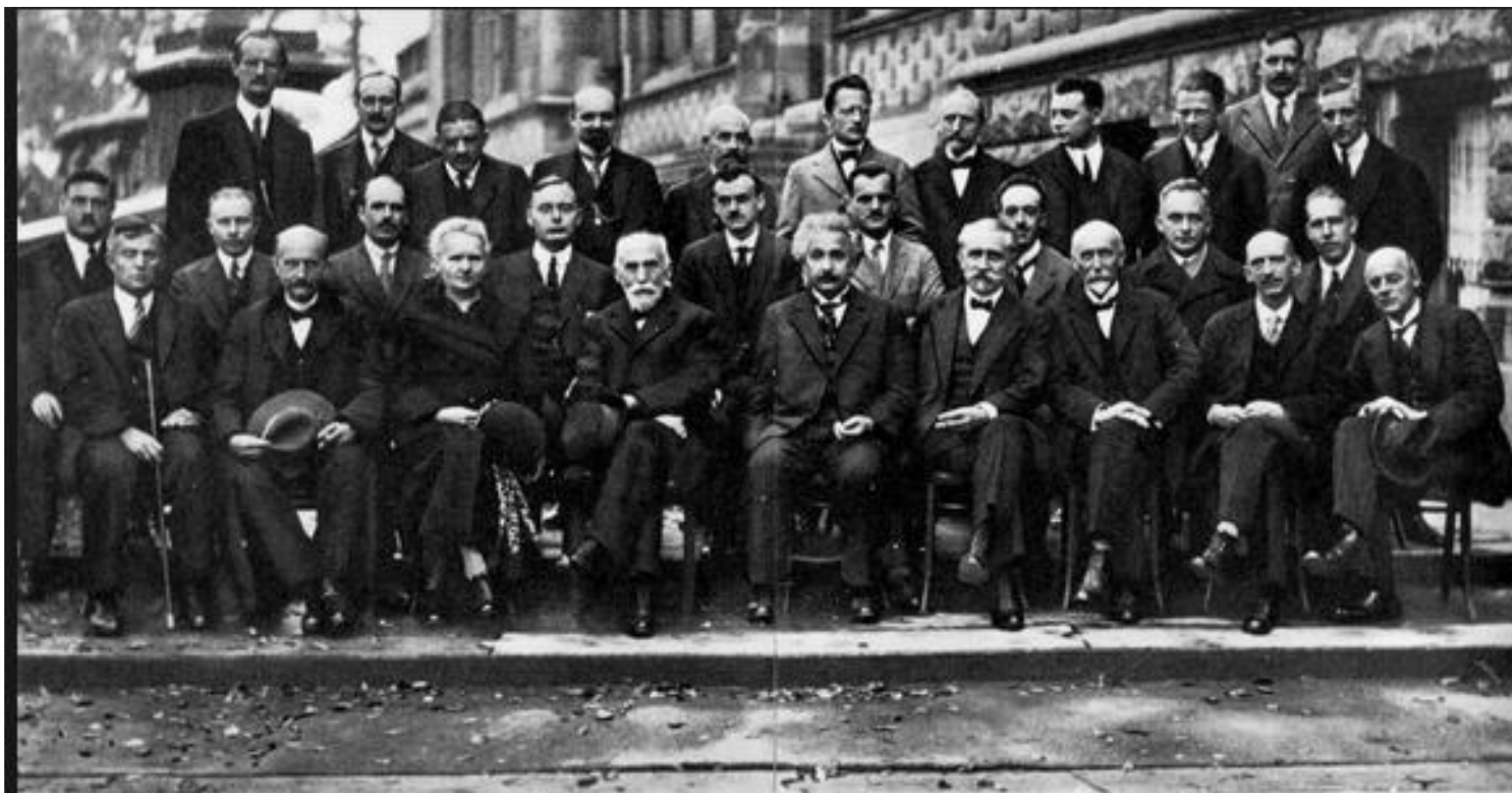
Radioactive decay law

- For all decays, proportional to

$$\frac{dN}{dt} \propto N$$

constant





Photographie Benjamin Couprie

28, Avenue Louise, Bruxelles

								R. H. FOWLER
A. PICCARD	E. HENRIOT		ED. HERZEN	TH. DE DONDER	E. SCHROEDINGER	W. PAULI	W. HEISENBERG	L. BRILLOUIN
		P. EHRENFEST				E. VERSCHAFFELT		
P. DEBYE	M. KNUDSEN	W. L. BRASS	H. A. KRAMERS	P. A. M. DIRAC	A. H. COMPTON	L. V. DE BROGLIE	M. BORN	N. BOHR
I. LANGMEIR	M. PLANCK	MADAME CURIE	H. A. LORENTZ	A. EINSTEIN	P. LANGEVIN	CH. E. GUYE	C. T. K. WILSON	
								O. W. RICHARDSON

Radioactive decay law

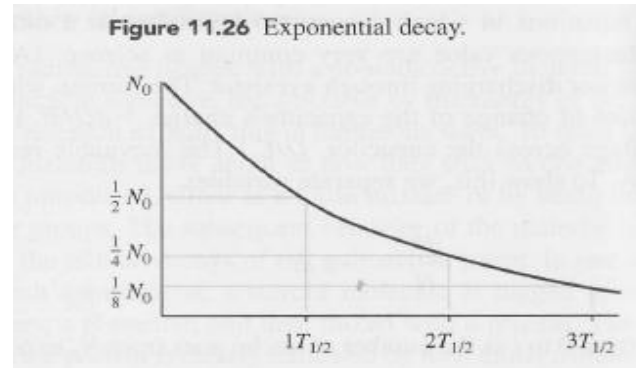
- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N \quad \frac{dN}{dt} = -\lambda N \quad \text{where } N = \text{Number of nuclei; } \lambda = \text{decay constant}$$

$$\frac{dN}{N} = -\lambda dt$$
$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$



Decay rate $R = \lambda N$ (decays per second)

Radioactive decay law

- For all decays, the rate of decay over time will be proportional to the sample size:

$$\frac{dN}{dt} \propto N \quad \frac{dN}{dt} = -\lambda N \quad \text{where } N = \text{Number of nuclei; } \lambda = \text{decay constant}$$

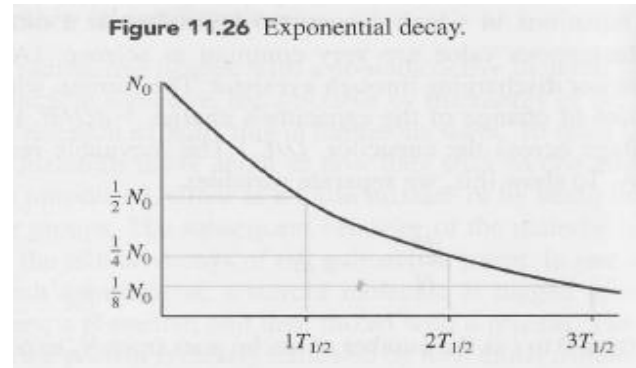
$$\frac{dN}{N} = -\lambda dt$$
$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{2} N_0 = N_0 e^{-\lambda T_{1/2}}$$

$$\lambda = \frac{\ln 2}{T_{1/2}}$$



Decay rate $R = \lambda N$ (decays per second)

Half-life ($T_{1/2}$)

Half-life

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad N = N_0 e^{-\lambda t}$$

Decay rate $R = \lambda N$ (decays per second)

TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}\text{Ca}$	β^+	50 ms
^3_1H	β^-	12.3 yr
$^{238}_{92}\text{U}$	α	4.5×10^9 yr

A vessel holds 2 μg of tritium.

(a) Initial decay rate?

Tritium's atomic mass = 3.02 u and 1 u = 1.66×10^{-27} kg

$$N = \frac{\text{sample mass}}{\text{atomic mass of Tritium}} \quad \lambda = \frac{\ln 2}{T_{1/2}} \quad T_{1/2} = 12.3 \text{ yr} = 12.3 \times 3.16 \times 10^7 \text{ sec}$$

Half-life

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad N = N_0 e^{-\lambda t}$$

Decay rate $R = \lambda N$ (decays per second)

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Tritium's atomic mass = 3.02 u and 1 u = 1.66×10^{-27} kg

$$N = \frac{\text{sample mass}}{\text{atomic mass of Tritium}} \quad \lambda = \frac{\ln 2}{T_{1/2}} \quad T_{1/2} = 12.3 \text{ yr} = 12.3 \times 3.16 \times 10^7 \text{ sec}$$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{\text{sample mass}}{\text{atomic mass of Tritium}} \quad R = 7.1 \times 10^8 \text{ decays/sec}$$

(a) Time elapse before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t}$$

$$\frac{1}{100} N_0 = N_0 e^{-\lambda t}$$

Half-life

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad N = N_0 e^{-\lambda t}$$

Decay rate $R = \lambda N$ (decays per second)

TABLE 11.3 Selected decays

Isotope	Decay Mode	Half-Life
$^{35}_{20}\text{Ca}$	β^+	50 ms
^3_1H	β^-	12.3 yr
$^{238}_{92}\text{U}$	α	4.5×10^9 yr

A vessel holds 2 μg of tritium.

(a) Initial decay rate?

Tritium's atomic mass = 3.02 u and 1 u = 1.66×10^{-27} kg

$$N = \frac{\text{sample mass}}{\text{atomic mass of Tritium}} \quad \lambda = \frac{\ln 2}{T_{1/2}} \quad T_{1/2} = 12.3 \text{ yr} = 12.3 \times 3.16 \times 10^7 \text{ sec}$$

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} \cdot \frac{\text{sample mass}}{\text{atomic mass of Tritium}} \quad R = 7.1 \times 10^8 \text{ decays/sec}$$

(a) Time elapse before the decay rate falls to 1% of its initial value?

$$N = N_0 e^{-\lambda t} \quad t = \frac{-\ln(\frac{1}{100})}{\lambda} = 2.6 \times 10^9 \text{ sec} = 81.7 \text{ years}$$

$$\frac{1}{100} N_0 = N_0 e^{-\lambda t}$$

Carbon 14 Dating

- Carbon-14's decay has a half-life of 5730 years.
- Carbon-14 dating only works for formerly living organisms.
- Carbon-14's amount is constantly maintained for living organisms since living organisms exchange Carbon with the environment. Ratio of naturally produced C14/C12= 1.3×10^{-12}
- When, a living organism dies, it stops the exchange process, thus C-14 in the dead organism decays exponentially.

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since $C^{14}/C^{12} = 1.3 \times 10^{-12}$ when the sample was alive
- Decay constant (λ)
- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec
- Elapsed time $N = N_0 e^{-\lambda t}$

$$1 \text{ year} = 3.16 \times 10^7 \text{ seconds}$$

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

- Since $C_{14}/C_{12} = 1.3 \times 10^{-12}$ when the sample was alive

$$N_0(C_{14}) = (1.3 \times 10^{-12}) \cdot \left(\frac{6 \text{ g}}{12 \text{ g}}\right) \cdot (6.02 \times 10^{23}) = 3.9 \times 10^{11}$$

- Decay constant (λ)
- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec
- Elapsed time $N = N_0 e^{-\lambda t}$

$$1 \text{ year} = 3.16 \times 10^7 \text{ seconds}$$

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- Decay constant (λ)

$$\lambda = \frac{\ln 2}{T_{1/2}} =$$

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

$$R = N \cdot \lambda =$$

- Elapsed time $N = N_0 e^{-\lambda t}$

$$1 \text{ year} = 3.16 \times 10^7 \text{ seconds}$$

Fossil age

Sample: 6 g of carbon and a decay rate (R) of 30 /min

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$$N_0(C_{14}) = (1.3 \times 10^{-12}) \cdot \left(\frac{6 \text{ g}}{12 \text{ g}}\right) \cdot (6.02 \times 10^{23}) = 3.9 \times 10^{11}$$

- Decay constant (λ)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5730 \text{ years} \cdot 3.16 \times 10^7 \text{ sec/year}} = 3.83 \times 10^{-12} \text{ /sec}$$

- Current decay rate (R)=30 /min=30/60 seconds= .5 /sec

$$R = N \cdot \lambda = N \cdot 3.83 \times 10^{-12} \text{ /sec} = 0.5 \text{ /sec}$$

$$N = 1.31 \times 10^{11}$$

- Elapsed time $N = N_0 e^{-\lambda t}$

$$1 \text{ year} = 3.16 \times 10^7 \text{ seconds}$$

$$t = -\frac{1}{\lambda} \ln \frac{N}{N_0} = -\frac{1}{3.83 \times 10^{-12} \text{ /sec}} \ln \frac{1.31 \times 10^{11}}{3.9 \times 10^{11}} = 2.86 \times 10^{11} \text{ sec} \sim 9000 \text{ years}$$

Nuclear Reactions

- Chemical reactions

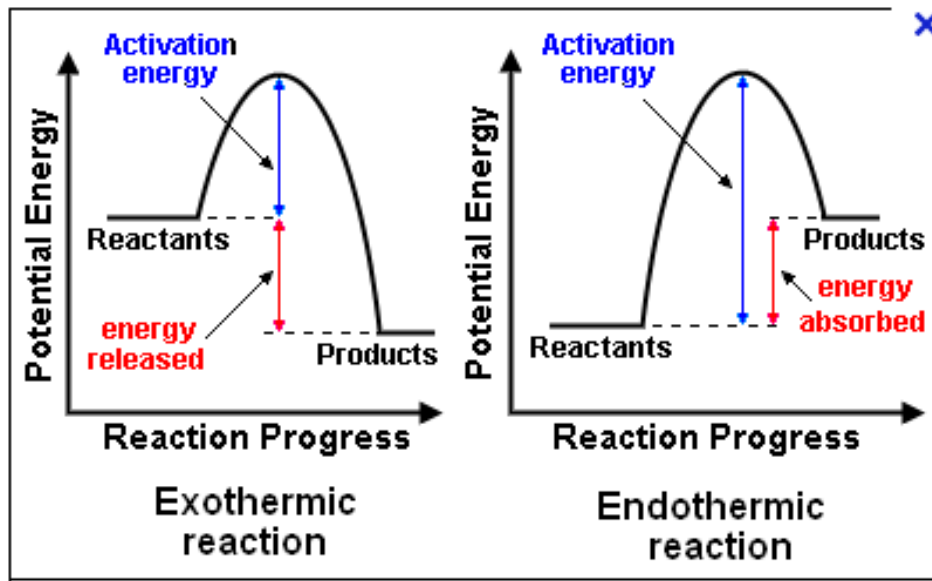


- Nuclear reactions



Nuclear Reactions

- Chemical reactions



Nuclear Reactions

- Chemical reactions



- Nuclear reactions

Exothermic



Endothermic



Nuclear Reactions

- Chemical reactions



- Nuclear reactions

Exothermic



Endothermic



Released kinetic energy (Q)

Nuclear Reactions

- Chemical reactions



- Nuclear reactions

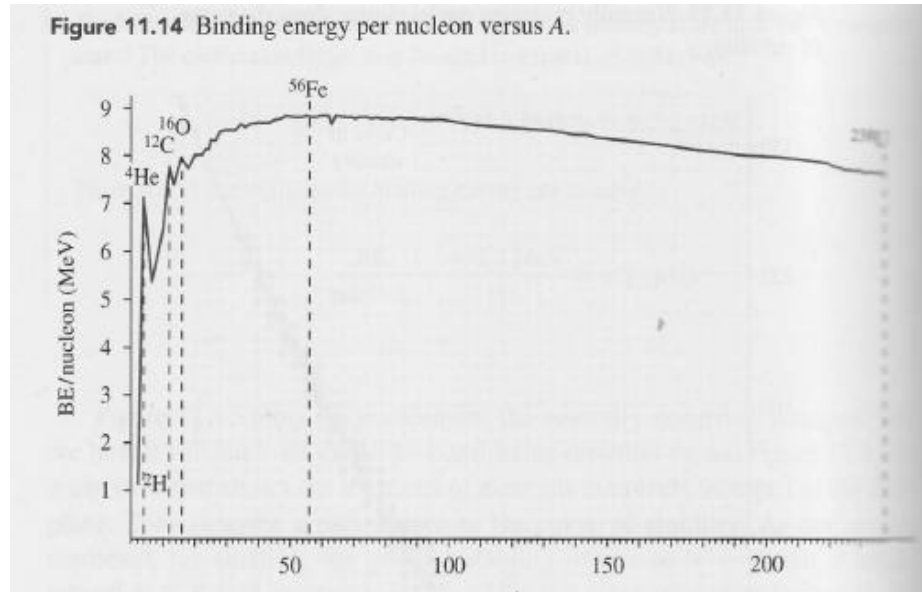


$$Q = (10.012937 + 1.008665 - 7.016003 - 4.002603) uc^2 = 2.79 \text{ MeV}$$



$$Q = (7.016003 + 1.007825 - 7.016928 - 1.008665) uc^2 = -1.64 \text{ MeV}$$

Binding energy/nucleon vs. A



Binding energy/nucleon vs. A

