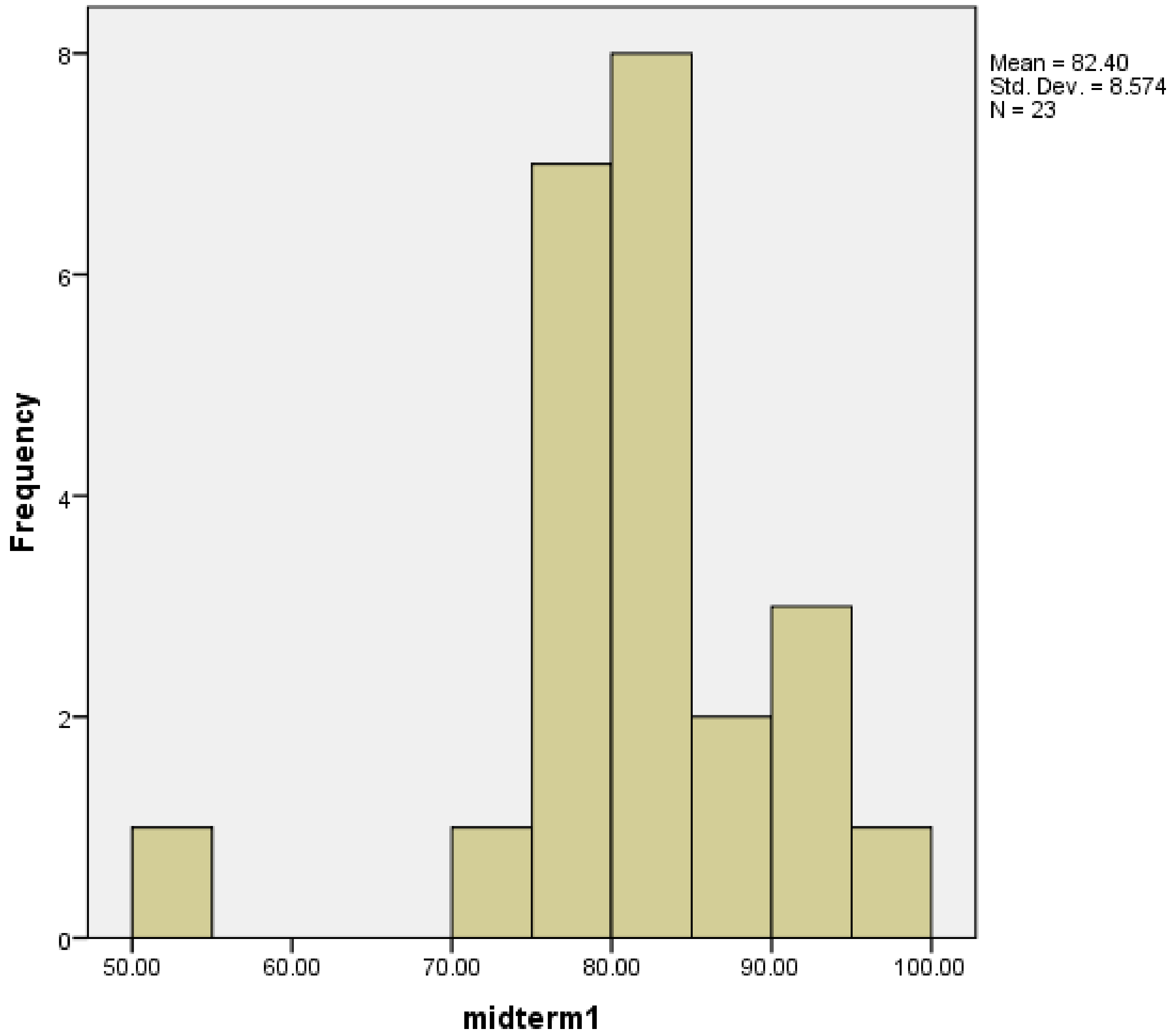
Announcements

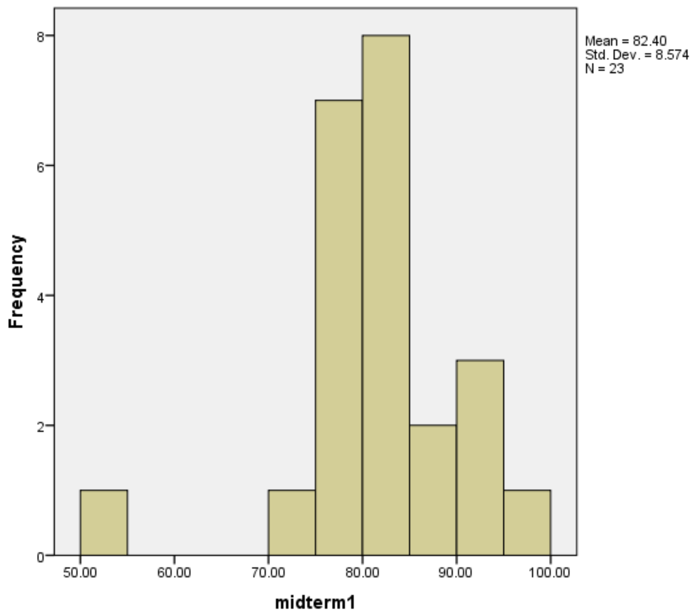
- HW5 and HW6
  - Both due on 03/17, Mon, 3 pm
  - Graded separately
- Final exam: 03/19, Wed, 8-11 am
- Grade:
    - 20%: Mid-term 1
    - 20%: Mid-term 2
    - 40%: Final
    - 15%: Homework (best five out of six HW sets)
    - 5%: Class participation

Mid term 1



Mean = 82.40
Std. Dev. = 8.574
N = 23
Frequency
0
2
4
6
8
50.00
60.00
70.00
80.00
90.00
100.00
midterm1

Mid term 2


Mean = 82.40
Std. Dev. = 8.574
N = 23
Frequency
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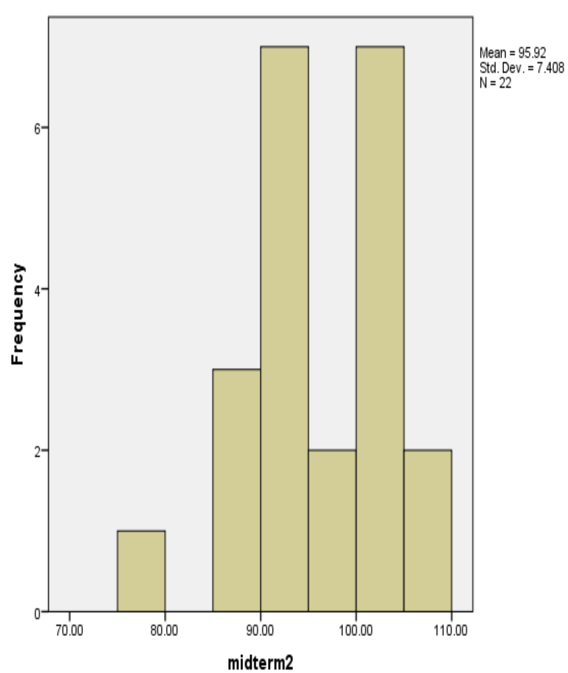
6
4
2
0
50.00
60.00
70.00
80.00
90.00
100.00
midterm1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Std. Dev. = 7.408
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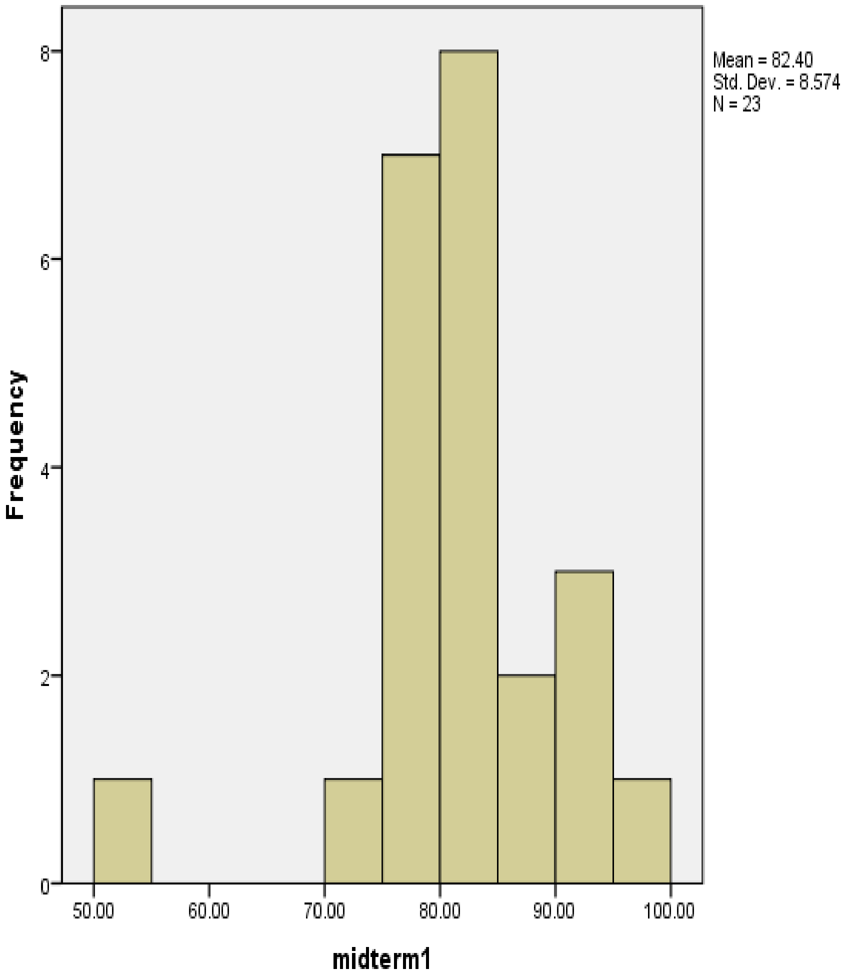
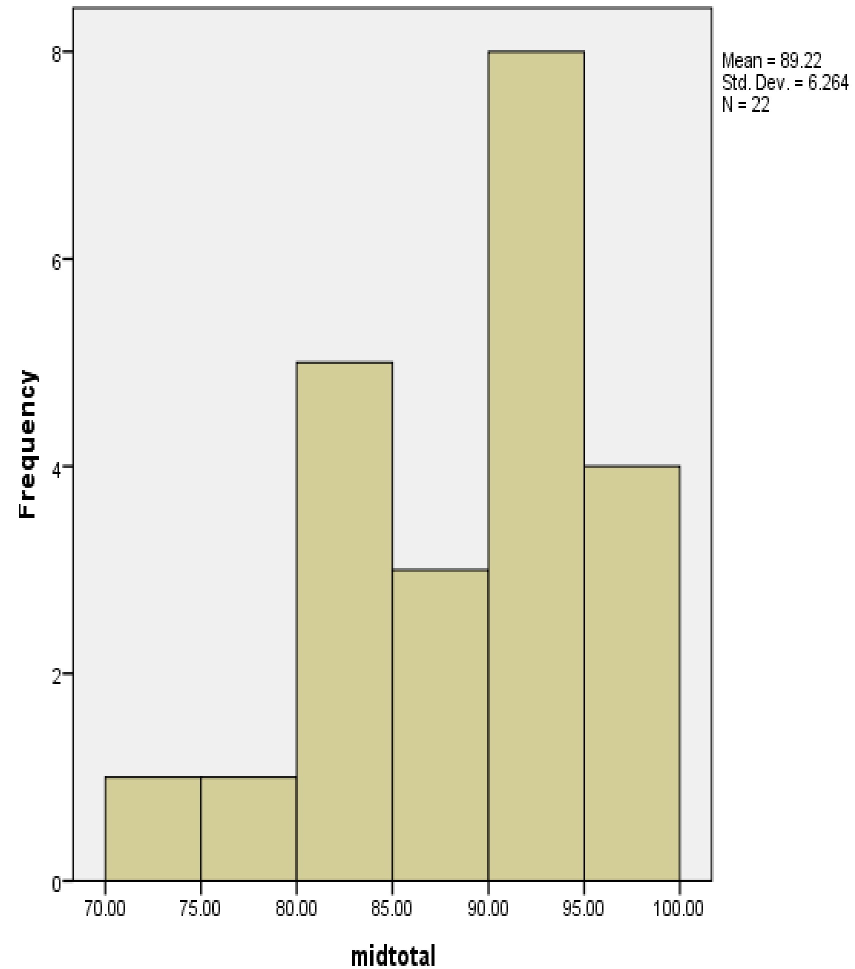
70.00
80.00
90.00
100.00
110.00
midterm2

Cumulative



Mean = 82.40
Std. Dev. = 8.574
N = 23
Frequency
0
2
4
6
8
50.00
60.00
70.00
80.00
90.00
100.00
midterm1



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N = 22
Frequency
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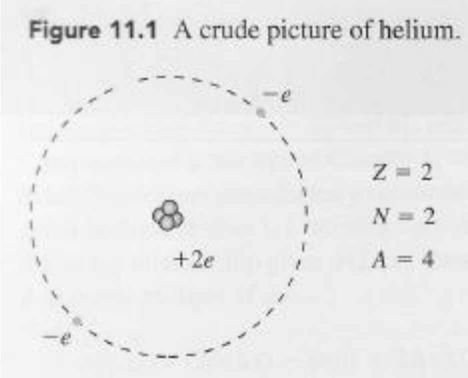
4
6
8
70.00
75.00
80.00
85.00
90.00
95.00
100.00
midtotal

Lecture 17 Topics

- Binding energy
- Nuclear models
  - Liquid Drop model
  - Shell model

Nucleus



Z= number of protons
N=number of neutrons
A=mass number=number of nucleons=Z+N

	charge	Mass in kg	Mass in u
Proton	+e	$1.6726217 \times 10^{-27}$	1.007276
Neutron	0	$1.6749273 \times 10^{-27}$	1.008665
Electron	-e	9.109×10^{-31}	0.0005486

Isotopes

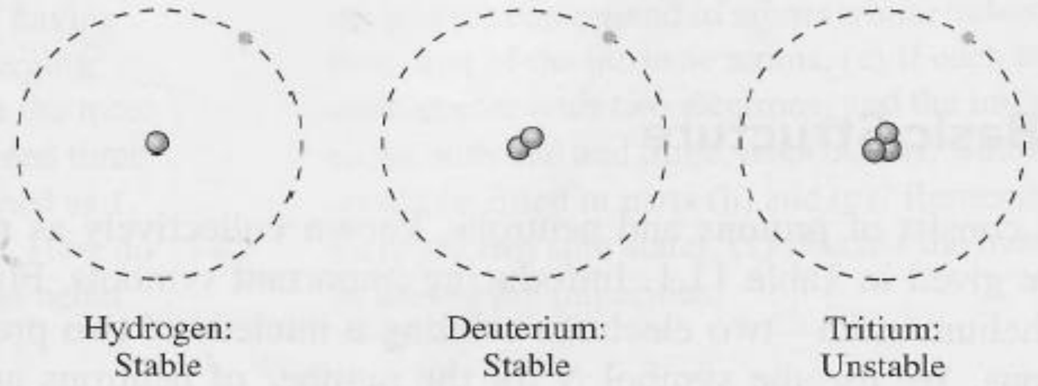


Figure 11.2 Isotopes of hydrogen.

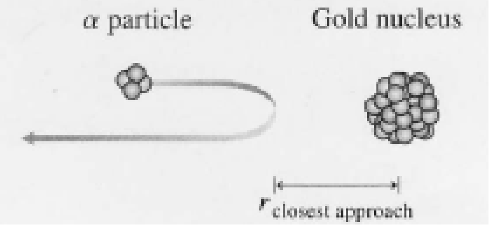
Nuclei	Hydrogen	Deuterium	Tritium
Z	1	1	1
N	0	1	2
A	1	2	3
Electron number	1	1	1

Stable (Hydrogen, Deuterium)

Nucleus size

Rutherford's alpha particle experiment on Gold foil

Figure 11.3 Probing for the radius of the nucleus.



→The nucleus is roughly spherical
→The nucleus is centered at the middle of the atom
→The nucleus occupies a relatively small space
→The head-on approach would enable the alpha particles to get closest

$$r = A^{1/3} R_0 \qquad R_0 = 1.2 \ x \ 10^{-15} m.$$

Nuclear Volume = $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (A^{1/3} R_0)^3 = \frac{4}{3}\pi A R_o^3 \sim A$

Nuclear Density= $density = \frac{mass}{volume} = \frac{A \text{ x } mass\ of\ nucleon}{A \text{ x } \frac{4}{3}\pi R_o^3} \cong 10^{17} kg/m^3$

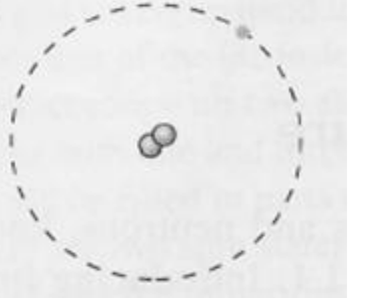
Nucleon volume= $\frac{V}{A} = \frac{4}{3}\pi R_o^3$

Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

Deuteron = 1 proton + 1 neutron

Binding Energy =



Deuterium:
Stable

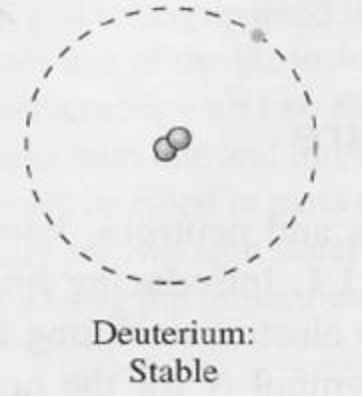
Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

$$\text{Deuteron} = 1\ \text{proton} + 1\ \text{neutron}$$

$$\text{Binding Energy} = (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2$$

m_P =1.007276 u
m_N=1.008665 u
m_D=2.013553 u
uc^2=931.5 MeV



Binding energy

$$\text{Binding energy} = (\text{mass of individual nucleons} - \text{mass of nucleus})c^2$$

$$\text{Deuteron} = 1 \text{ proton} + 1 \text{ neutron}$$

$$\text{Binding Energy} = (\text{proton mass} + \text{neutron mass} - \text{Deuteron mass})c^2$$

$$= (1.007276\,u + 1.008665\,u - 2.013553\,u)c^2 = 0.002388\,uc^2$$

$$= 0.002388\,x\,1.661\,x\,10^{-27}kg\,x\,(3x10^8 m/sec)^2 = 2.22\,MeV$$

uc^2=931.5 MeV

Binding energy

$$Binding\ Energy = \left(Zm_H + Nm_n - M_{^A_ZX}\right)c^2$$

Where m_H= atomic mass of hydrogen

m_n= neutron mass

$M_{^A_ZX}$ = atomic mass of the nucleus

Binding energy

$$Binding\ Energy = \left(Zm_H + Nm_n - M_{^A_ZX}\right)c^2$$

Where m_H= atomic mass of hydrogen

m_n= neutron mass

$M_{^A_ZX}$ = atomic mass of the nucleus

Example: $^{56}_{26}Fe = 53.934939\,u$

m_H =1.007825 u

m_N=1.008665 u

uc^2=931.5 MeV

Binding energy

$$Binding\ Energy = \left(Zm_H + Nm_n - M_{^A_ZX}\right)c^2$$

Where m_H= atomic mass of hydrogen

m_n= neutron mass

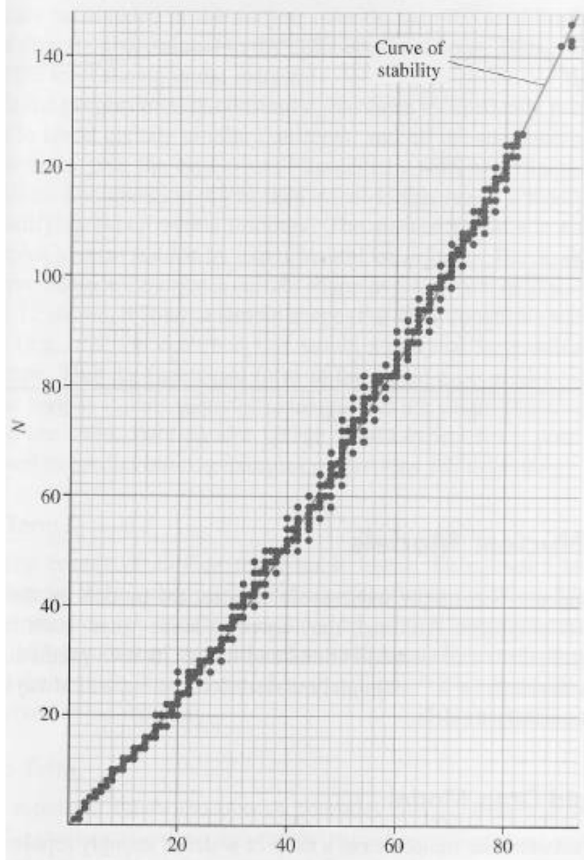
$M_{^A_ZX}$ = atomic mass of the nucleus

Example: $^{56}_{26}Fe = 53.934939\,u$

Binding energy = (26 x 1.007825 u +30 x 1.008665 u – 55.934939 u) c^2=0.528461 uc^2 =492.3 MeV (consider $c^2 = 931.5\ MeV/u$).

Binding energy/nucleon over Z

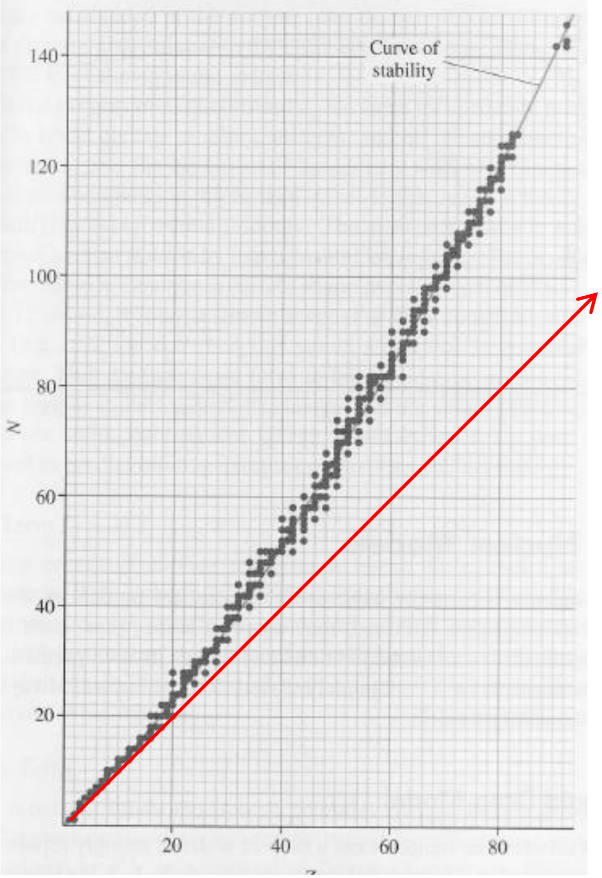
Figure 11.13 Naturally occurring nuclei cluster along the curve of stability.



Curve of stability

Figure 11.13 Naturally occurring nuclei cluster along the curve of stability.

N



Z

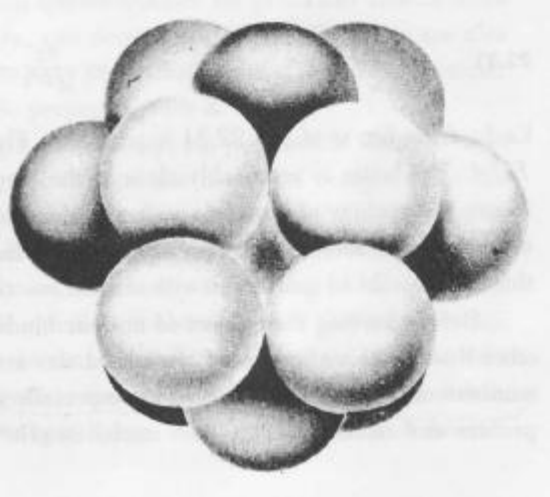
For smaller nuclei, N=Z to be stable

As nuclei get larger, N > Z to be stable

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

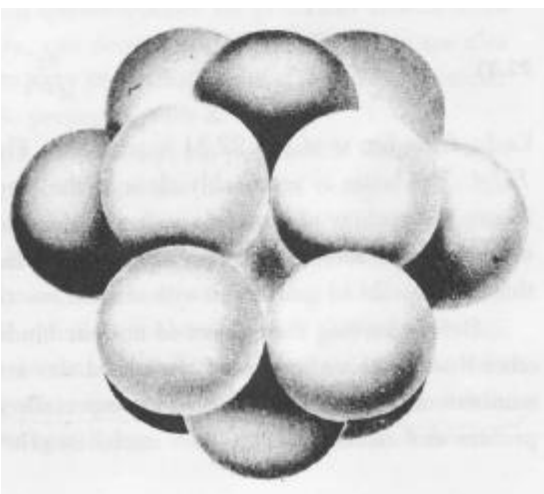


$C_1 = 15.8$
$C_2 = 17.8$
$C_3 = 0.71$
$C_4 = 23.7$

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$



$C_1 = 15.8$
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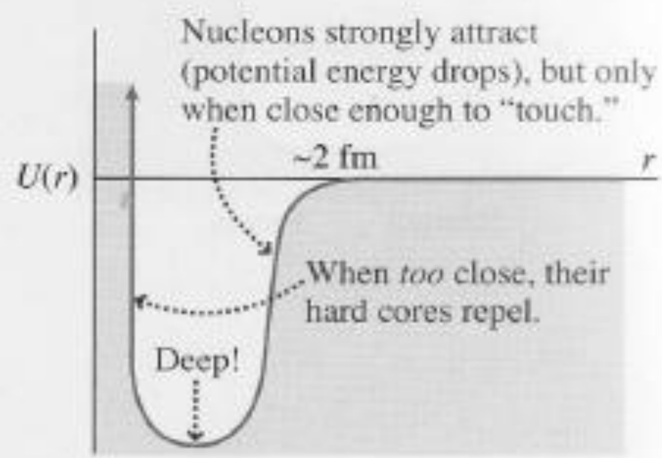
$C_3 = 0.71$
$C_4 = 23.7$

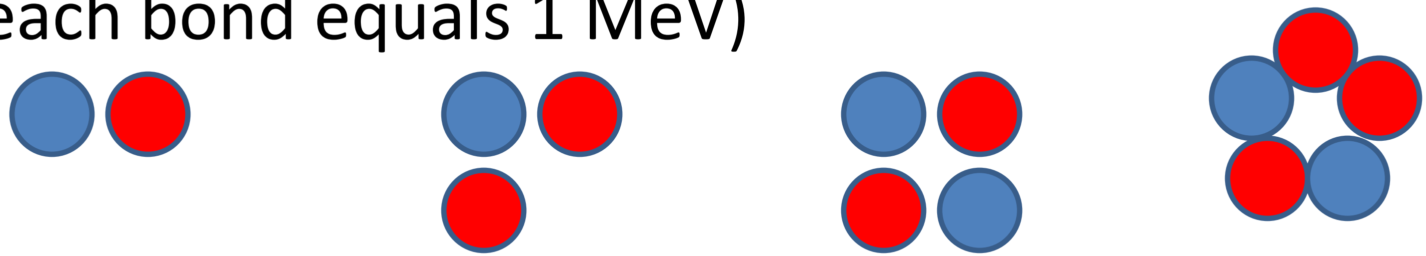
Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.



Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:

(Calculate bonding energy due to strong force when each bond equals 1 MeV)



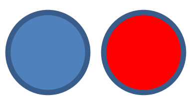
Total energy

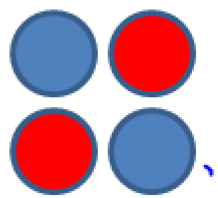
Energy per nucleon

Multi-nucleon nuclei

- Strong force:
  – When the total number of nucleons is small:

  (Calculate bonding energy due to strong force when each bond equals 1 MeV)



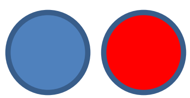


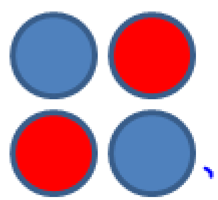
Total energy	1 MeV	6 MeV
Energy per Nucleon	1 MeV	1.5 MeV

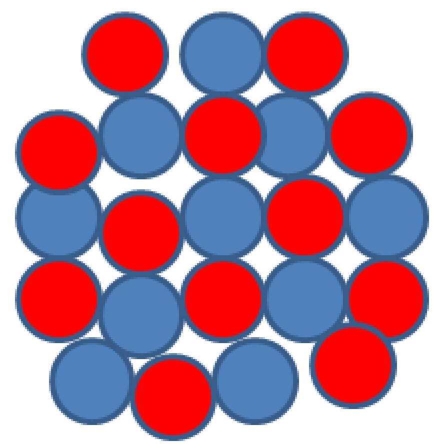
Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:

(Calculate bonding energy due to strong force when each bond equals 1 MeV)



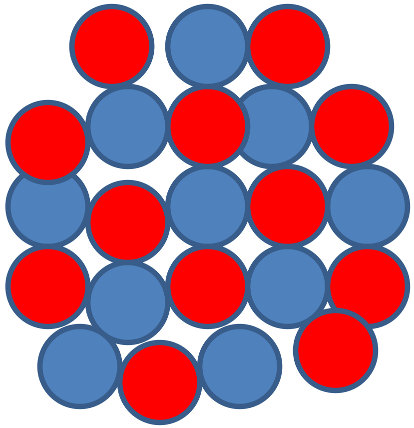




Total energy	1 MeV	6 MeV
Energy per Nucleon	1 MeV	1.5 MeV

Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large



Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
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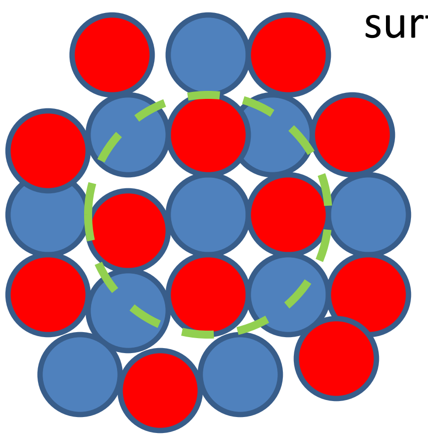


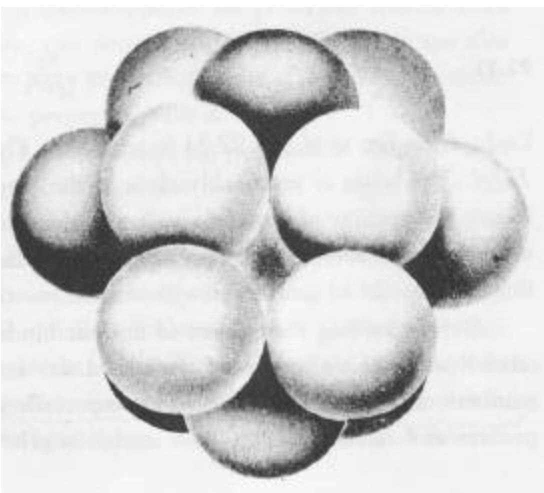
Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large
    - Since strong force is short ranged, making each nucleon has the same number of surrounding nucleons.
    - The nucleons at the surface are not completely surrounded. The proportion of surface nucleons diminish by 1/r

Liquid Drop Model

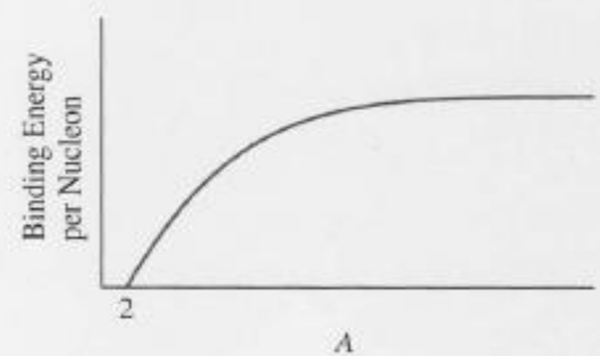
Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$



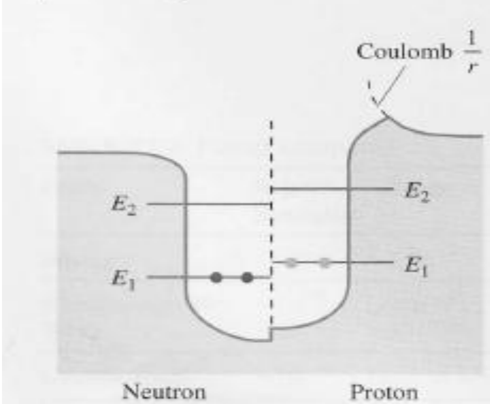
$C_1 = 15.8$
$C_2 = 17.8$
$C_3 = 0.71$
$C_4 = 23.7$

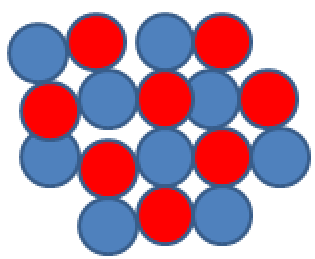
Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.



Coulomb repulsion

Figure 11.7 Coulomb repulsion raises proton energies.





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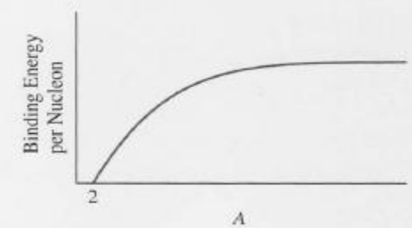


Figure 11.7 Coulomb repulsion raises proton energies.

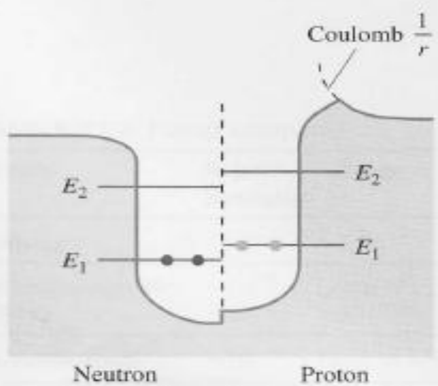
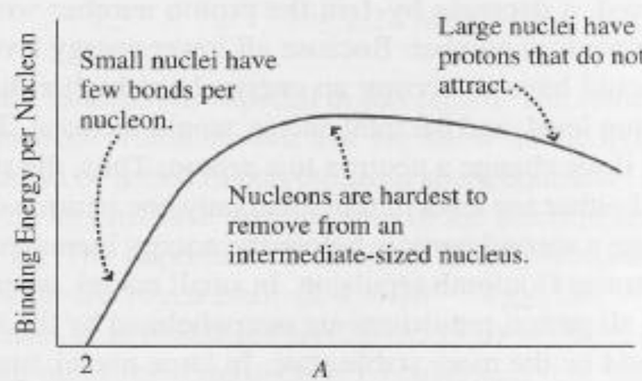


Figure 11.8 Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.



Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

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$$C_1 = 15.8$$
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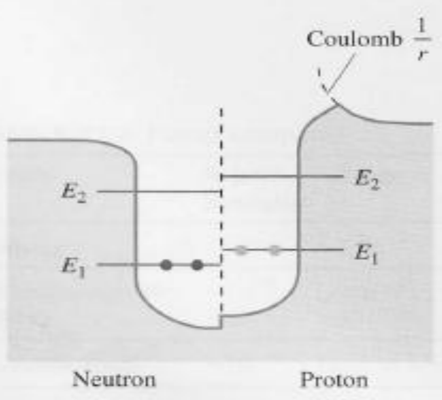
Figure 11.7 Coulomb repulsion raises proton energies.



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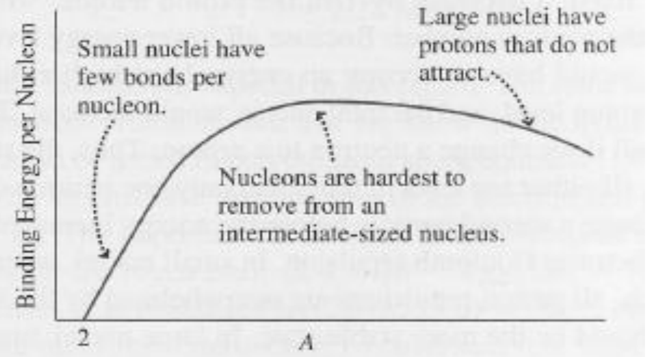
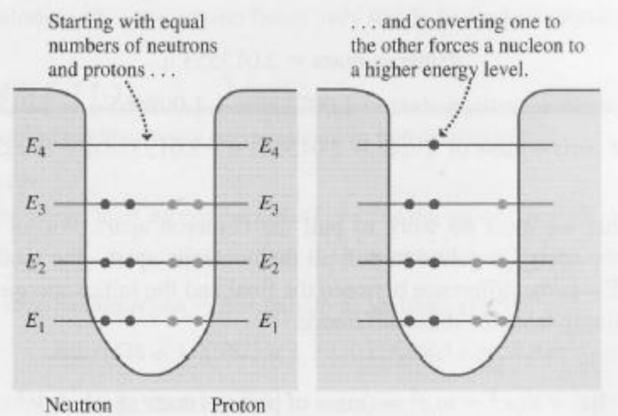


Exclusion principle

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.



For small nuclei, repulsive coulomb interactions are less influential

When N=Z,

When N≠Z

Exclusion principle

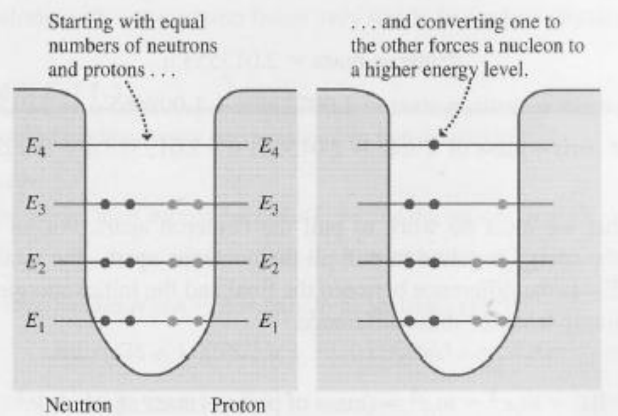


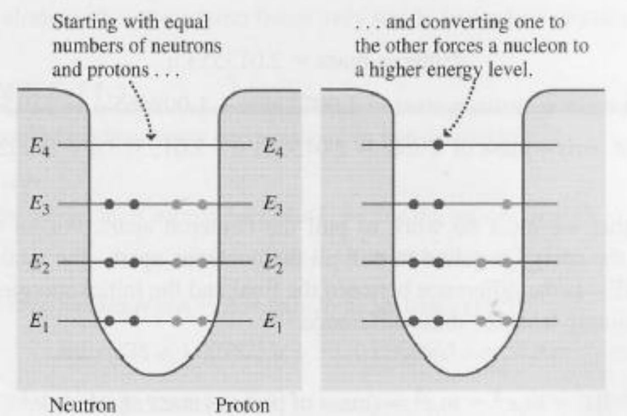
Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.

For small nuclei, repulsive coulomb interactions are less influential

When N=Z, more stable

When N≠Z, energy raised

Exclusion principle



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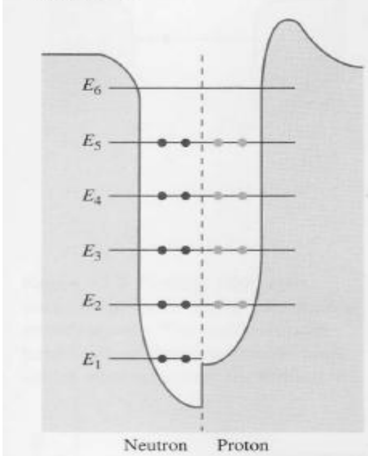

Figure 11.10 In large nuclei, when Coulomb repulsion becomes significant, the lowest energy should have $N > Z$.

For large nuclei, repulsive force changes

N>Z is more stable

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$C_1 = 15.8$
$C_2 = 17.8$
$C_3 = 0.71$
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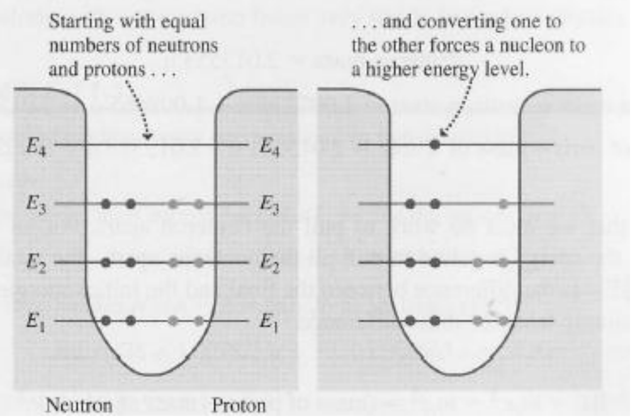
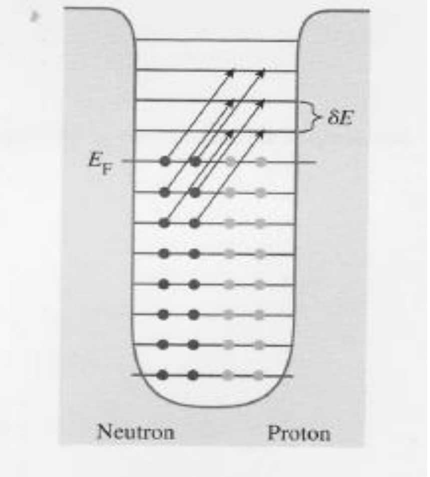


Figure 11.15 If j neutrons become protons, the energy increases by $\frac{1}{2} j^2 \delta E$.



Liquid Drop Model

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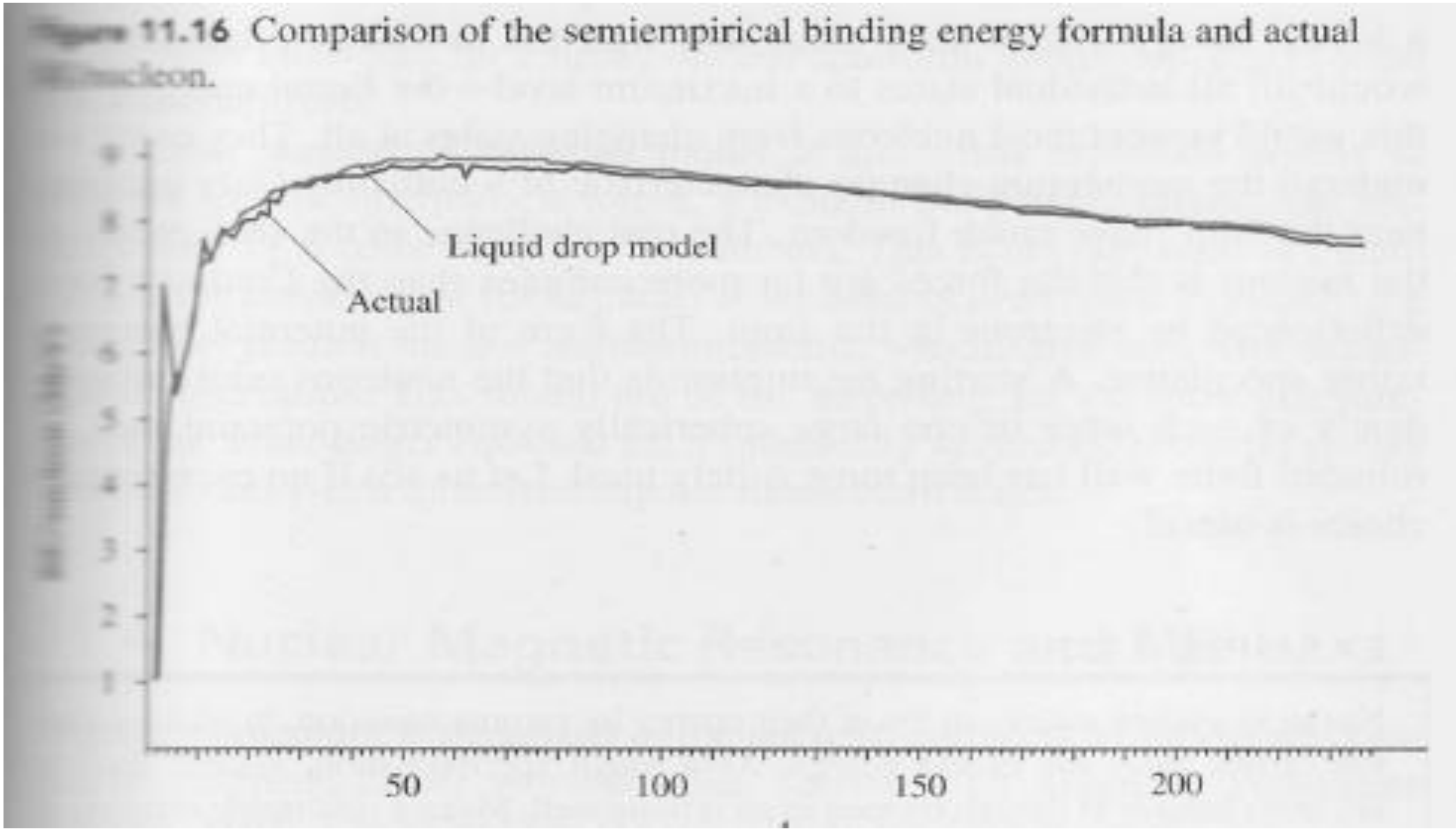
Liquid Drop Model



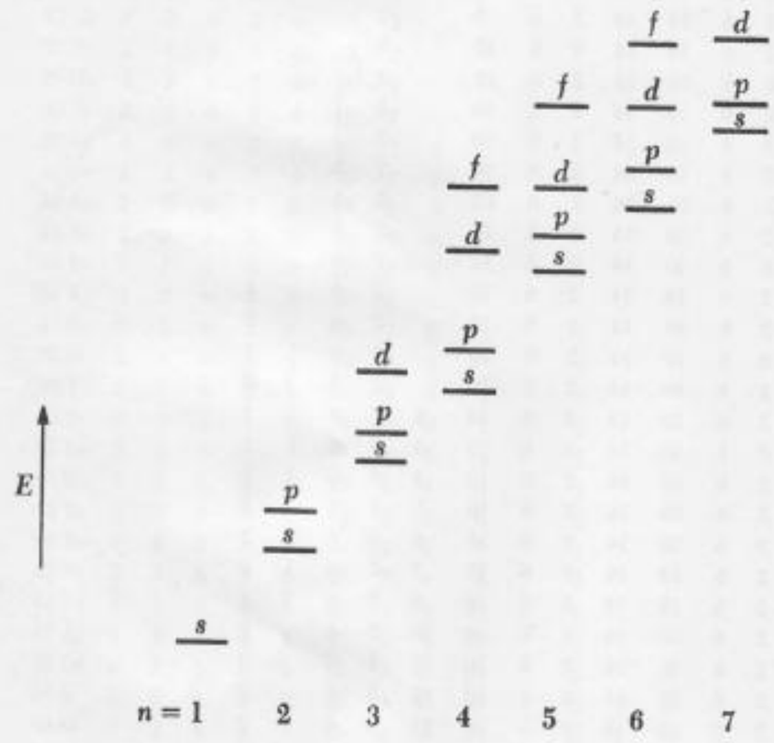
Magic numbers= 2, 8, 20, 28, 50, 82, 126

Shell Model

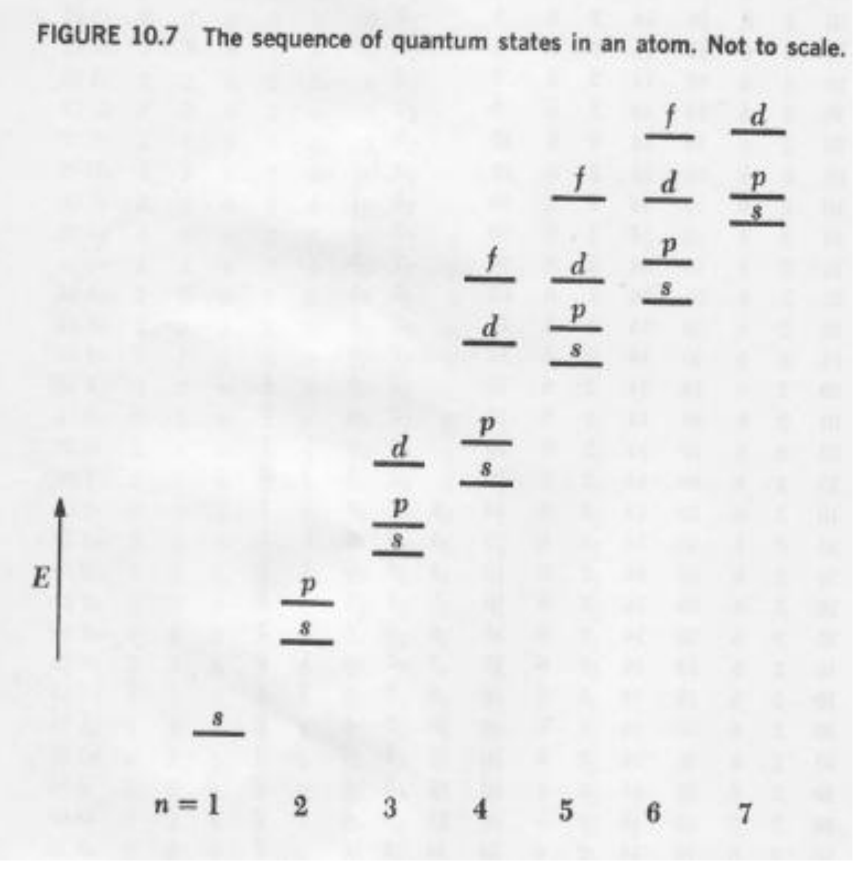
	Atomic Shell Model	Nuclear Shell Model
Potential	Electrostatic between nucleus and electrons	Net effect of all the forces nucleons experience in a nucleus
Magic numbers	Atoms with closed electronic shells are stable such as He (2 electrons), Ne (10), Ar (18), Kr (36), Xe (54), Rn (86).	Nuclei are particularly stable when the number of nucleons is 2, 8, 20, 28, 50, 82, and 126.
Exclusion principle	Electrons follow the Exclusion principle	Protons and neutrons separately follow the Exclusion Principle
Movement	Electrons move in orbitals	Nucleons do not move like electrons because most nucleons fill states to a maximum level, preventing them from changing momentum.

Electrons: Atomic Orbital

FIGURE 10.7 The sequence of quantum states in an atom. Not to scale.



Electrons: Atomic Orbital



86: Rn

54: Xe

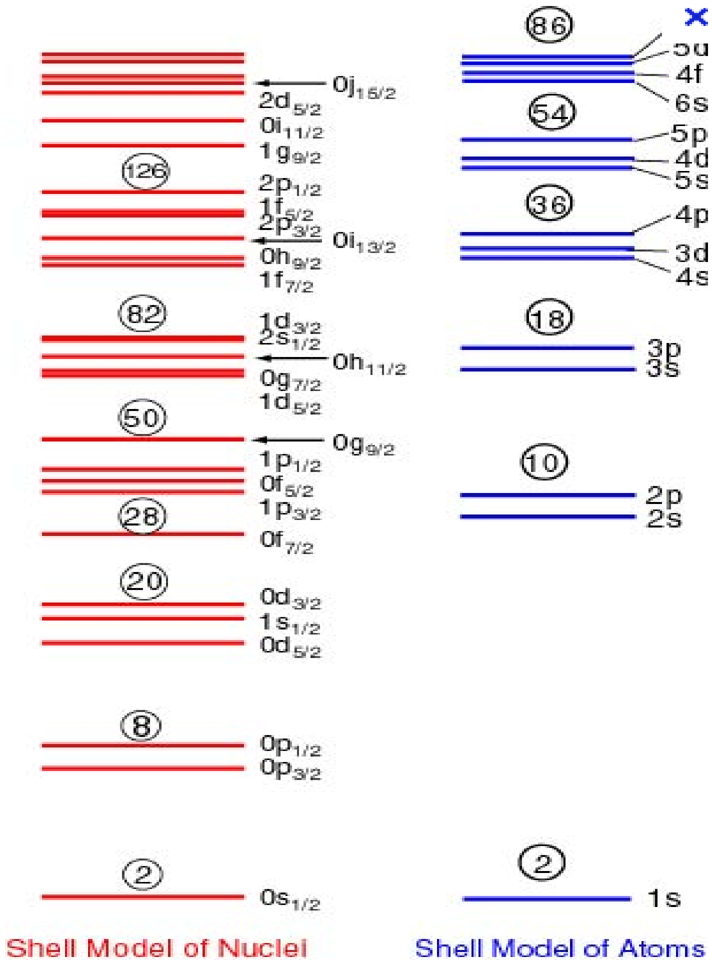
36: Kr

18: Ar

10: Ne

2: He

Electrons: Atomic Orbital
Nucleons: Nuclei Orbital



86: Rn

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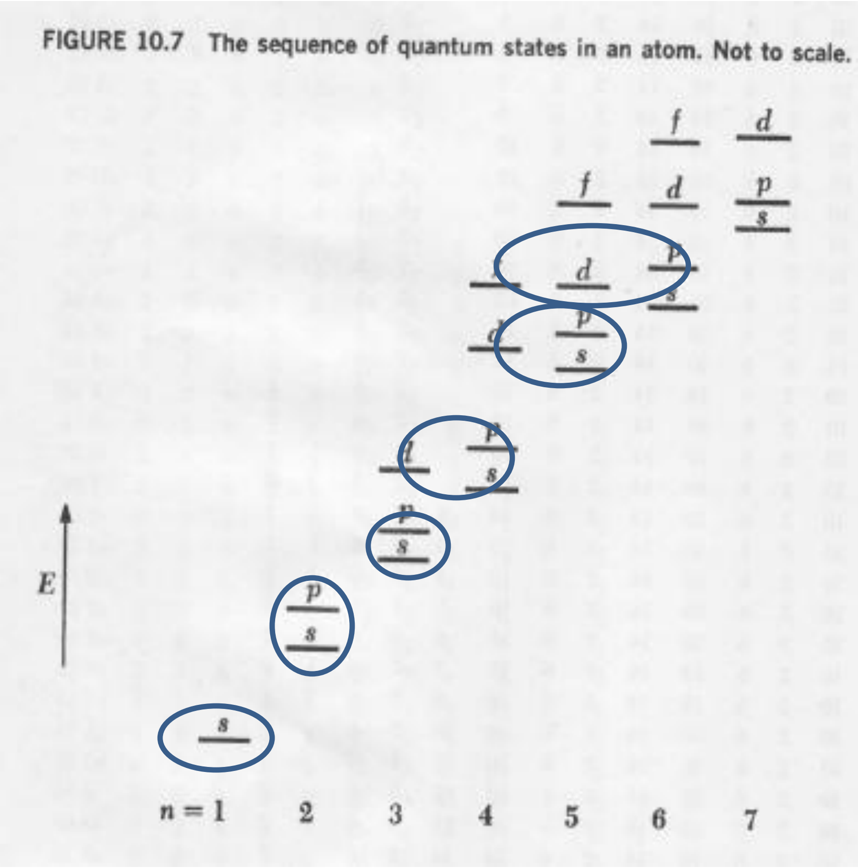
2: He

Electrons: Atomic Orbital



86: Rn

54: Xe

36: Kr

18: Ar

10: Ne

2: He

3 dimensional harmonic oscillator

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + \frac{1}{2}\mu\omega^2 x^2\,\psi(x) = \mathrm{E}\,\psi(x)$$

3 dimensional harmonic oscillator

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$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$n \equiv 2k + l$

- For every even $n, l\ =\ 0, 2, \ldots n-2, n$
- For every odd $n, l =\ 1, 3, \ldots n-2, n$
- $-l \leq m \leq l$
- Every n and l, there are $2l+1$ energy degeneracies, which can accommodate $2(2l+1)$ nucleons

3 dimensional harmonic oscillator

$$n \equiv 2k + l$$

even $n, l = 0, 2, \ldots n-2, n$

odd $n, l = 1, 3, \ldots n-2, n$

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

n	k	l	Energy
0	0	0	$\frac{3}{2}\hbar\omega$

3 dimensional harmonic oscillator

$$n \equiv 2k + l$$

even $n, l \;=\; 0, 2, \ldots n-2, n$

odd $n, l = \; 1, 3, \ldots n-2, n$

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

n	k	l	Energy
0	0	0	$\frac{3}{2}\hbar\omega$
1	0	1	$\frac{5}{2}\hbar\omega$
2	0	2	$\frac{7}{2}\hbar\omega$
	1	0	

3 dimensional harmonic oscillator

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$$n \equiv 2k + l$$

2(2l+1)

n	k	l	No. of nucleons in (n, k, l)	Energy
0	0	0	2	$\frac{3}{2}\hbar\omega$
1	0	1		$\frac{5}{2}\hbar\omega$
2	0	2		$\frac{7}{2}\hbar\omega$
	1	0		
3	0	3		$\frac{9}{2}\hbar\omega$
	1	1		
4	0	4		$\frac{11}{2}\hbar\omega$
	1	2		
	2	0		
5	0	5		$\frac{13}{2}\hbar\omega$
	1	3		
	2	1		

3 dimensional harmonic oscillator

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$$n \equiv 2k + l$$

2(2l+1)

n	k	l	No. of nucleons in (n, k, l)	Energy
0	0	0	2	$\frac{3}{2}\hbar\omega$
1	0	1	6	$\frac{5}{2}\hbar\omega$
2	0	2	10	$\frac{7}{2}\hbar\omega$
	1	0	2	
3	0	3	14	$\frac{9}{2}\hbar\omega$
	1	1	6	
4	0	4	18	$\frac{11}{2}\hbar\omega$
	1	2	10	
	2	0	2	
5	0	5	22	$\frac{13}{2}\hbar\omega$
	1	3	14	
	2	1	6	

Not Magic Numbers!!!

3 dimensional harmonic oscillator

$$E = \hbar\omega\left(2k + l + \frac{3}{2}\right)$$

$$n \equiv 2k + l$$

2(2l+1)

n	k	l	No. of nucleons in (n, k, l)	No. of nucleons in n	Total nucleons	Energy
0	0	0	2	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	6	6	8	$\frac{5}{2}\hbar\omega$
2	0	2	10	12	20	$\frac{7}{2}\hbar\omega$
	1	0	2			
3	0	3	14	20	40	$\frac{9}{2}\hbar\omega$
	1	1	6			
4	0	4	18	30	70	$\frac{11}{2}\hbar\omega$
	1	2	10			
	2	0	2			
5	0	5	22	42	112	$\frac{13}{2}\hbar\omega$
	1	3	14			
	2	1	6			

Not Magic Numbers!!!

LS coupling

n	k	l	j			Energy
0	0	0	½			$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2			$\frac{5}{2}\hbar\omega$
2	0	2				$\frac{7}{2}\hbar\omega$
	1	0				
3	0	3				$\frac{9}{2}\hbar\omega$
	1	1				
4	0	4				$\frac{11}{2}\hbar\omega$
	1	2				
	2	0				
5	0	5				$\frac{13}{2}\hbar\omega$
	1	3				
	2	1				

LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in $(n, \|\vec{j}\|)$	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2				$\frac{7}{2}\hbar\omega$
	1	0				
3	0	3				$\frac{9}{2}\hbar\omega$
	1	1				
4	0	4				$\frac{11}{2}\hbar\omega$
	1	2				
	2	0				
5	0	5				$\frac{13}{2}\hbar\omega$
	1	3				
	2	1				

LS coupling

2j+1

n	k	l	j	No. of nucleons in (n, j)		Energy
0	0	0	½	2		$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2		$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4		$\frac{7}{2}\hbar\omega$
	1	0	½	2		
3	0	3	7/2 5/2	8 6		$\frac{9}{2}\hbar\omega$
	1	1	3/2 1/2	4 2		
4	0	4	9/2 7/2	10 8		$\frac{11}{2}\hbar\omega$
	1	2	5/2 3/2	6 4		
	2	0	½	2		
5	0	5	11/2 9/2	12 10		$\frac{13}{2}\hbar\omega$
	1	3	7/2 5/2	8 6		
	2	1	3/2 1/2	4 2		

LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in $(n,\|\vec{j}\|)$	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
	1	0	½	2	2	
3	0	3	7/2 5/2	8 6	14	$\frac{9}{2}\hbar\omega$
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	1	3	7/2 5/2	8 6	14	
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LS coupling

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, $\|\vec{J}\|$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
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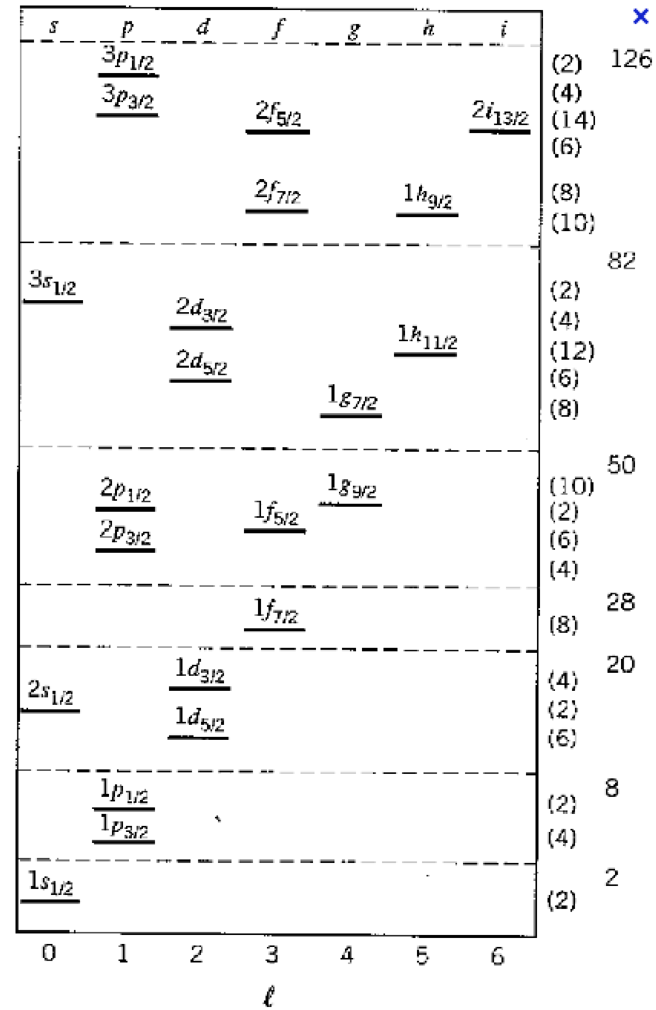

LS coupling

K+1 is the name of energy level

n	k	l	j	No. of nucleons in (n, j)	No. of nucleons in (n, $|\vec{J}|$)	Energy
0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
1	0	1	3/2 1/2	4 2	6	$\frac{5}{2}\hbar\omega$
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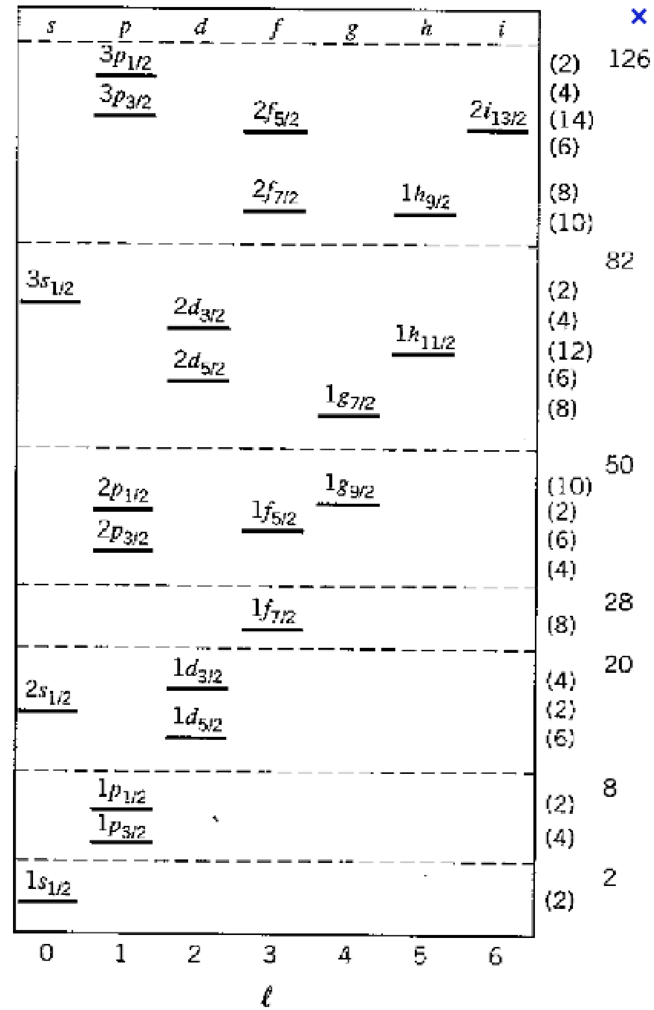

LS coupling

K+1 is the name of energy level

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0	0	0	½	2	2	$\frac{3}{2}\hbar\omega$
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2	0	2	5/2 3/2	6 4	10	$\frac{7}{2}\hbar\omega$
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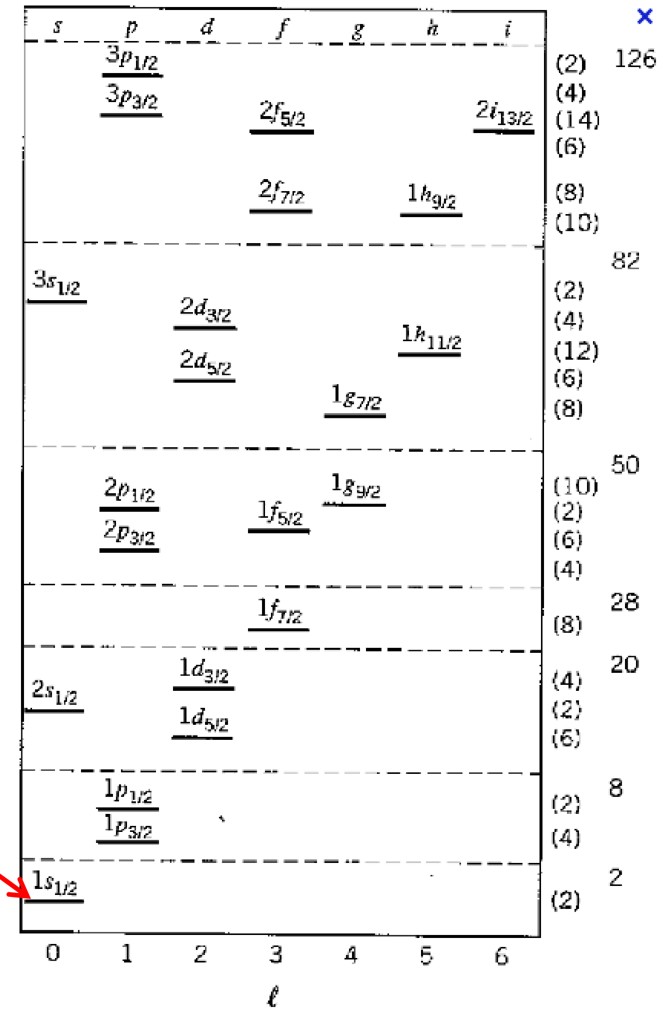

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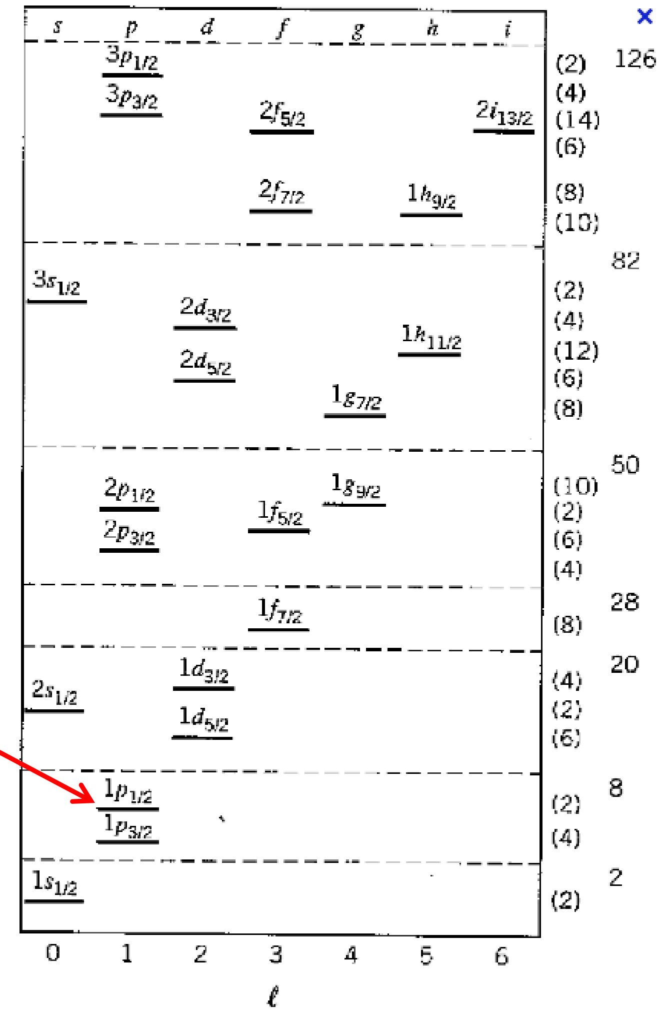

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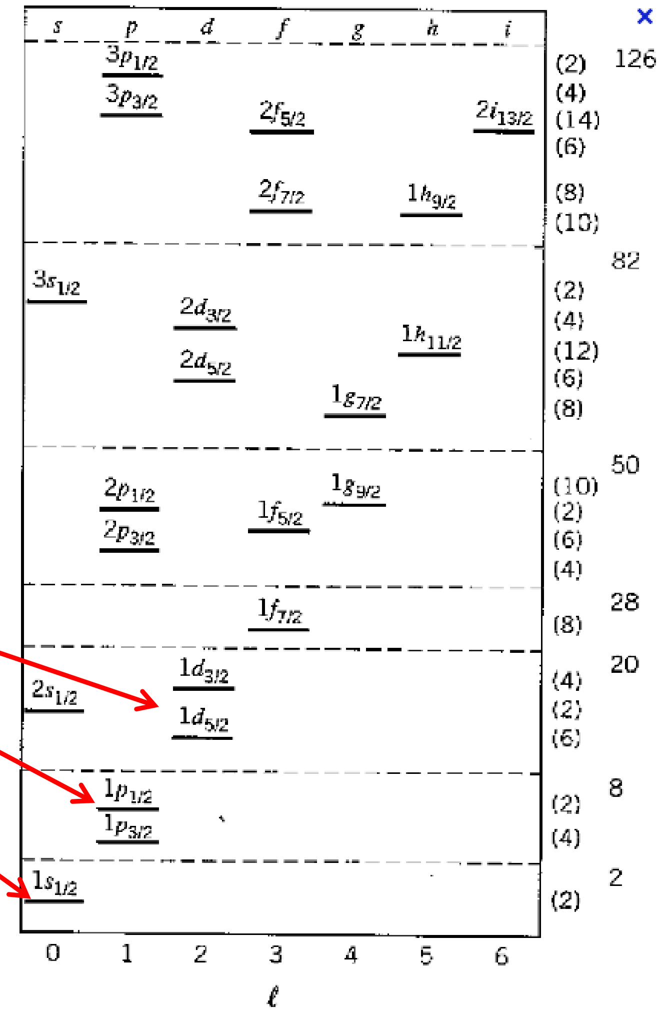

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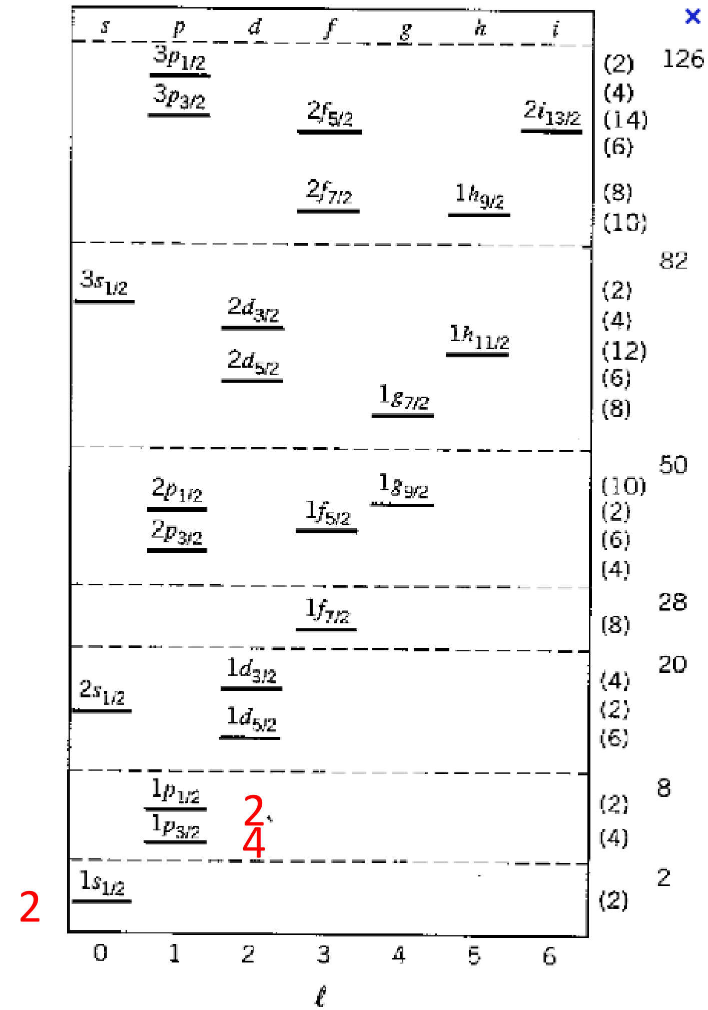

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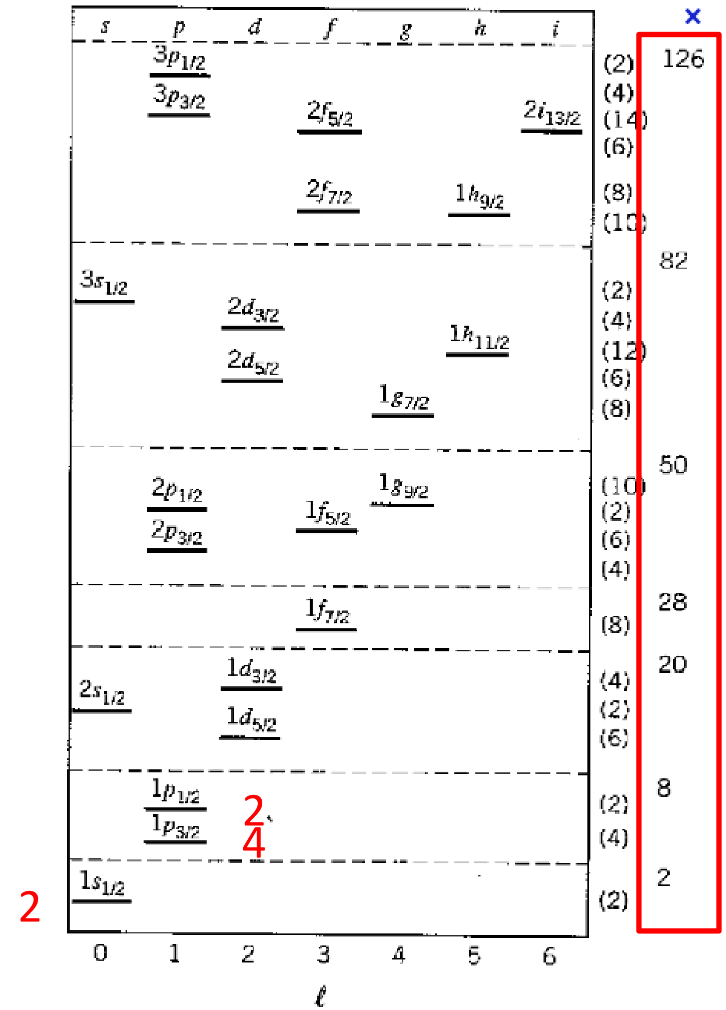
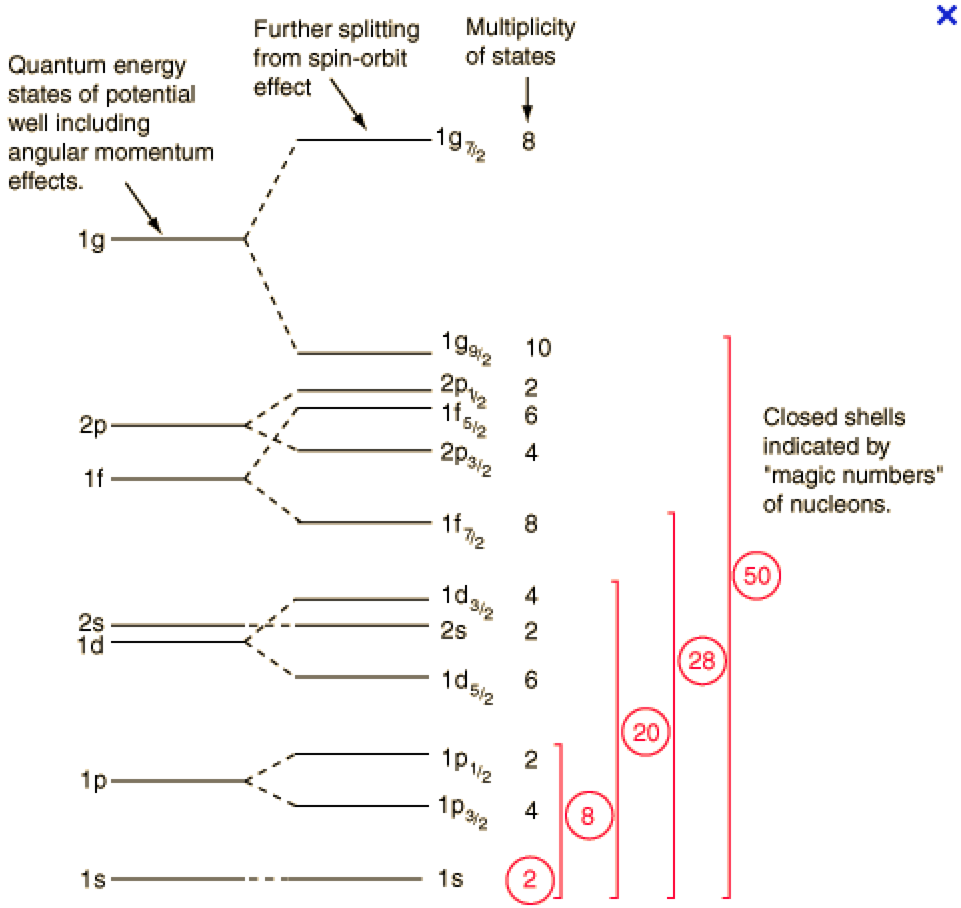
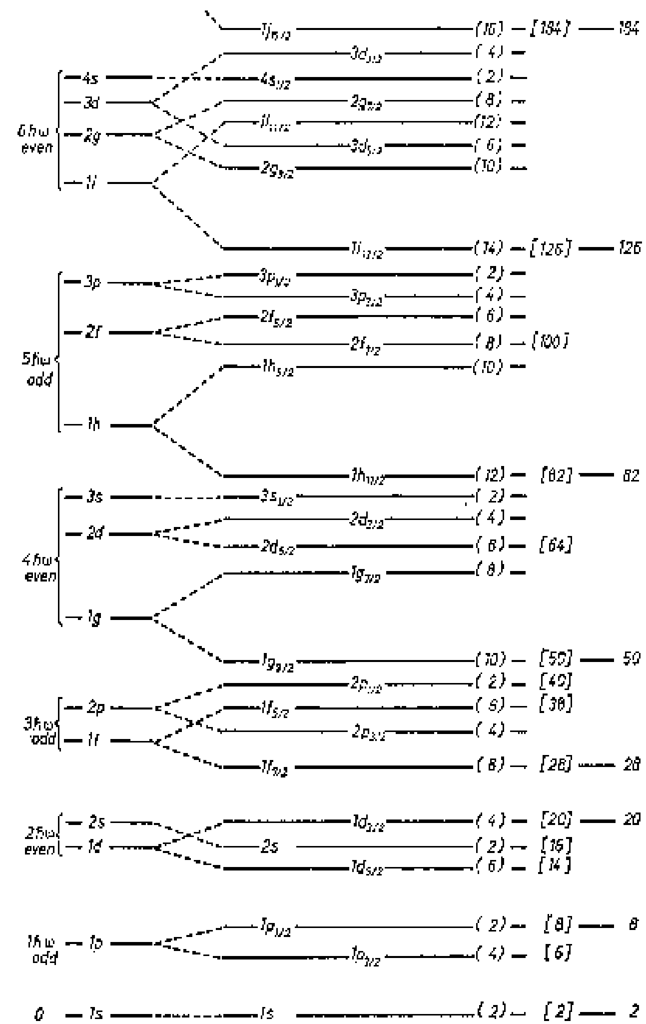
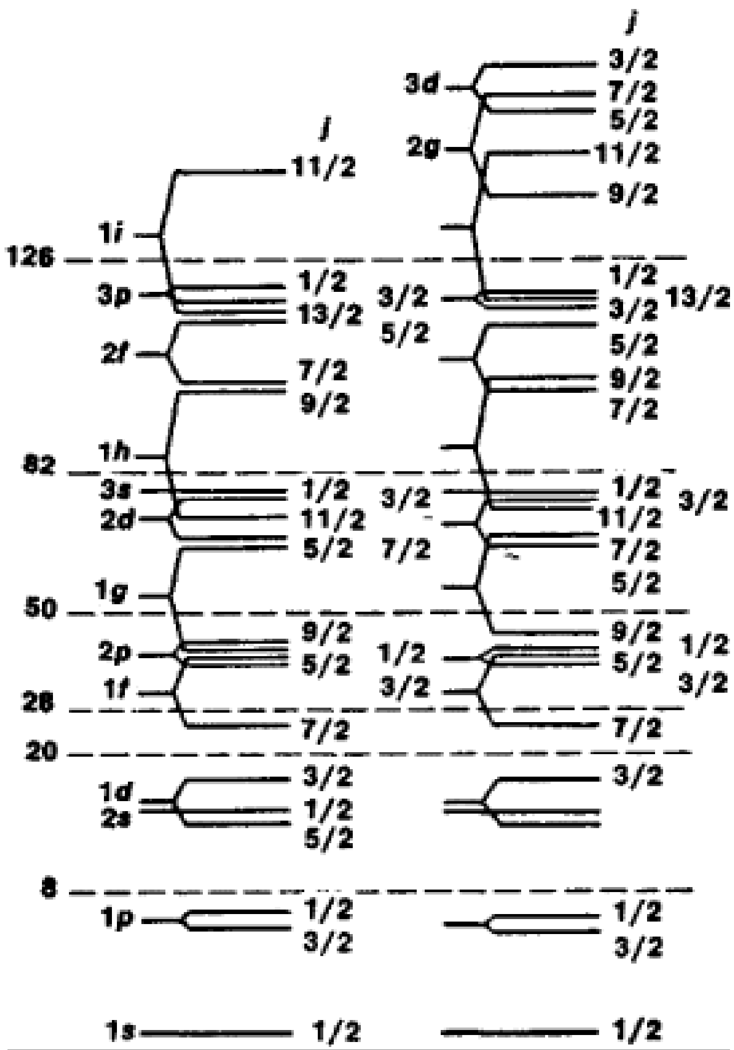




Quantum energy states of potential well including angular momentum effects.
Further splitting from spin-orbit effect
Multiplicity of states
1g
2p
1f
2s
1d
1p
1s
$1g_{7/2}$ 8
$1g_{9/2}$ 10
$2p_{1/2}$ 2
$1f_{5/2}$ 6
$2p_{3/2}$ 4
$1f_{7/2}$ 8
$1d_{3/2}$ 4
2s 2
$1d_{5/2}$ 6
$1p_{1/2}$ 2
$1p_{3/2}$ 4
1s
2
8
20
28
50
Closed shells indicated by "magic numbers" of nucleons.

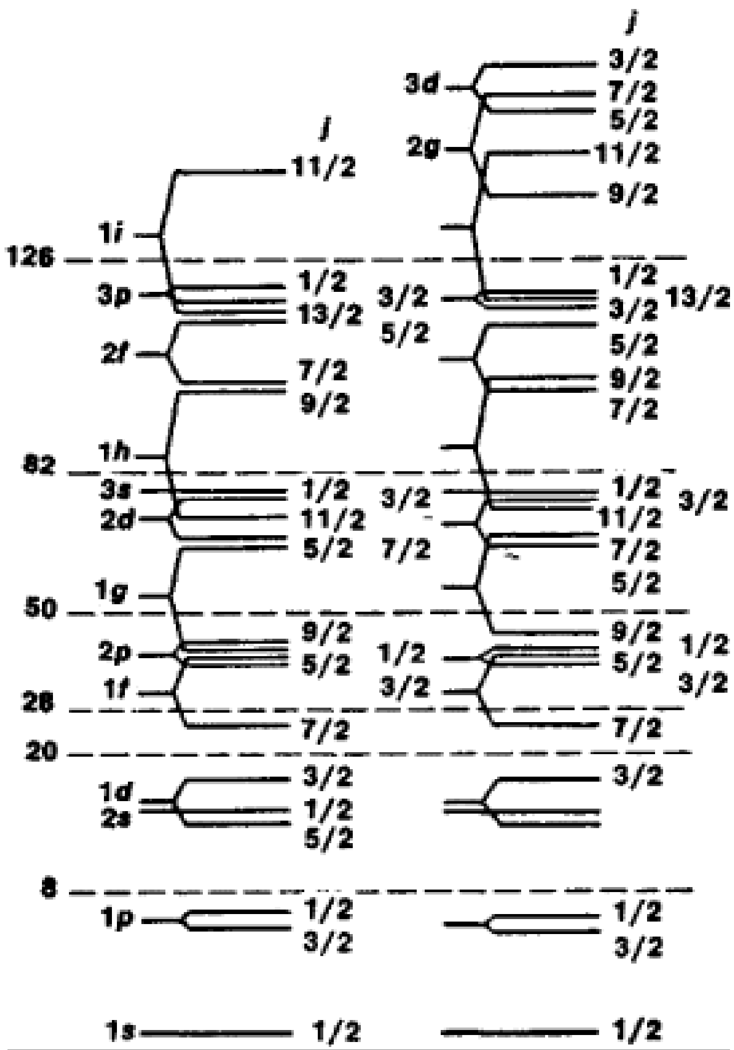


184
126
82
50
28
20
8
2



Proton Neutron

Oxygen Nucleus



Proton Neutron