Lecture 16 Topics

- Superconductivity
- Structure of nucleus
- Size of nucleus
- Strong force and potential
- Nuclear binding energy (theoretical/empirical)
  - Strong force
  - Electrostatic force-repulsive force
  - Exclusion principle

Superconductivity

- Perfect conductivity—critical temperature (Tc)
- Perfect diamagnetism- critical B field (Bc)

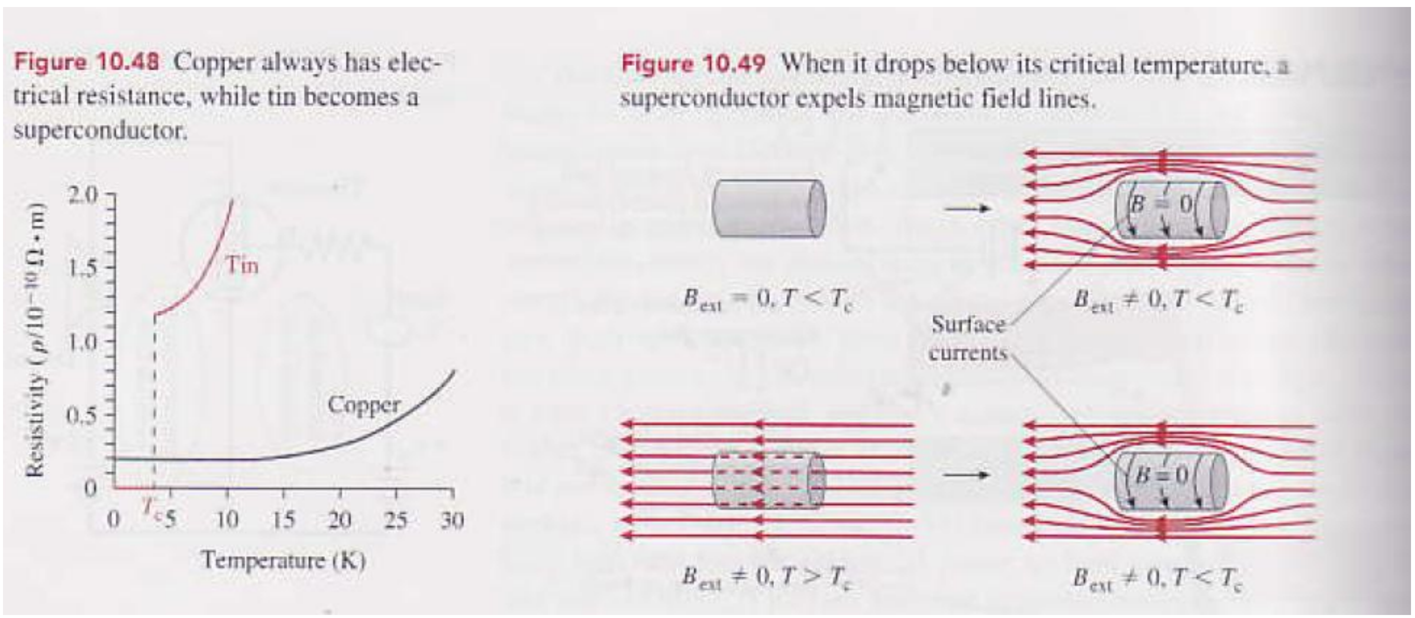
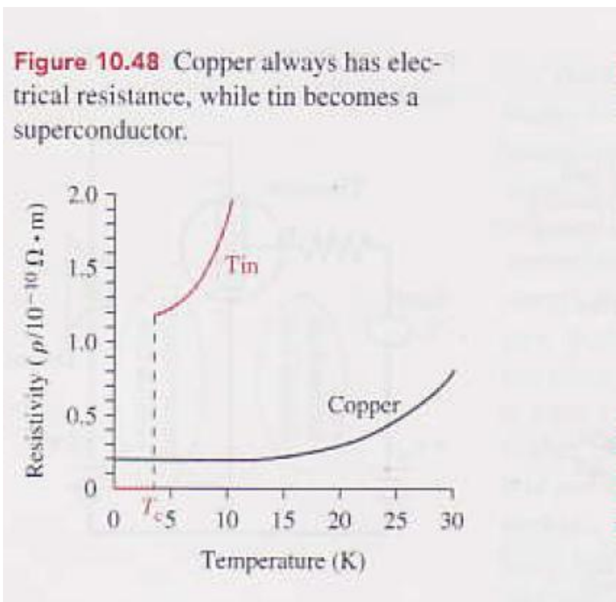


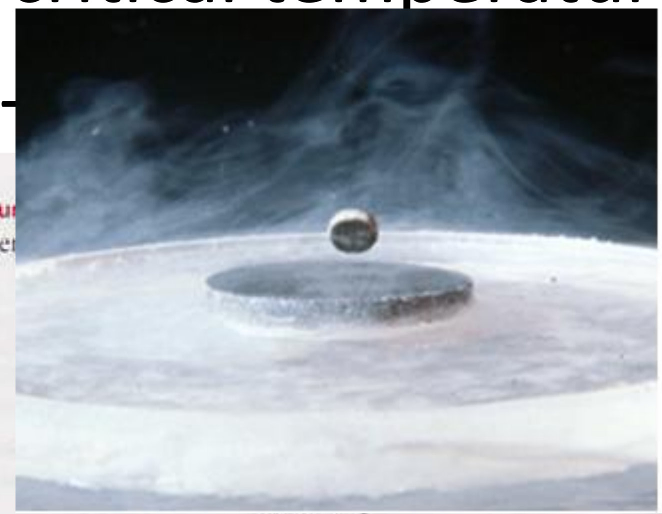
Figure 10.48 Copper always has electrical resistance, while tin becomes a superconductor.

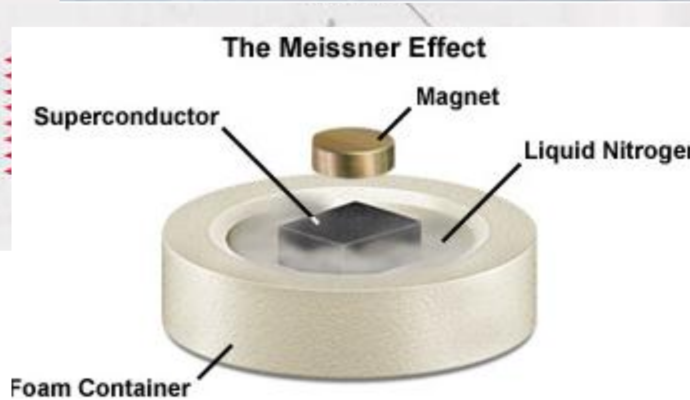
Figure 10.49 When it drops below its critical temperature, a superconductor expels magnetic field lines.

Superconductivity

- Perfect conductivity—critical temperature (Tc)
- Perfect diamagnetism–







Superconductors repel magnets

Type 1 Superconductors

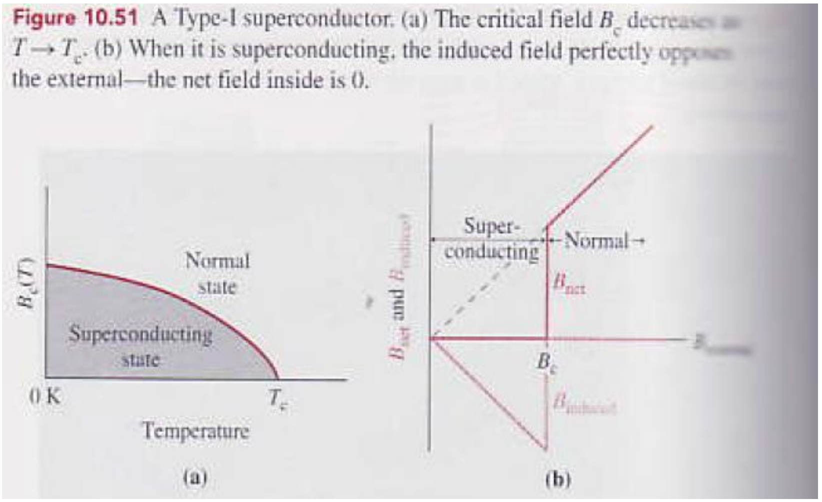


Figure 10.51 A Type-I superconductor. (a) The critical field B_c decreases as $T \to T_c$. (b) When it is superconducting, the induced field perfectly opposes the external—the net field inside is 0.

-Sharp transition at Bc
-No B field in the superconducting material because Bext is canceled by B-induced before Bc
-After Bc, the superconducting material is in a normal state allowing Bext to penetrate the material
-Superconducting elements tend to be Type 1
-Bc is relatively low, 0.01-0.1 T
-Tc is relatively low, 1-9 K

Type 1 vs. 2 Superconductors

Figure 10.51 A Type-I superconductor. (a) The critical field B_c decreases as $T \to T_c$. (b) When it is superconducting, the induced field perfectly opposes the external—the net field inside is 0.

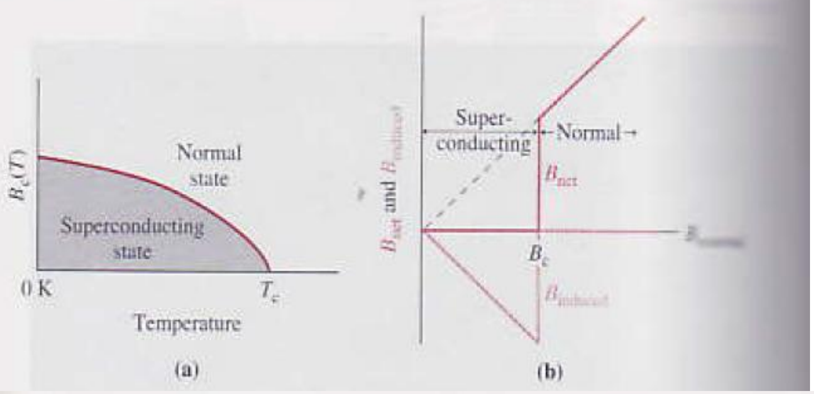
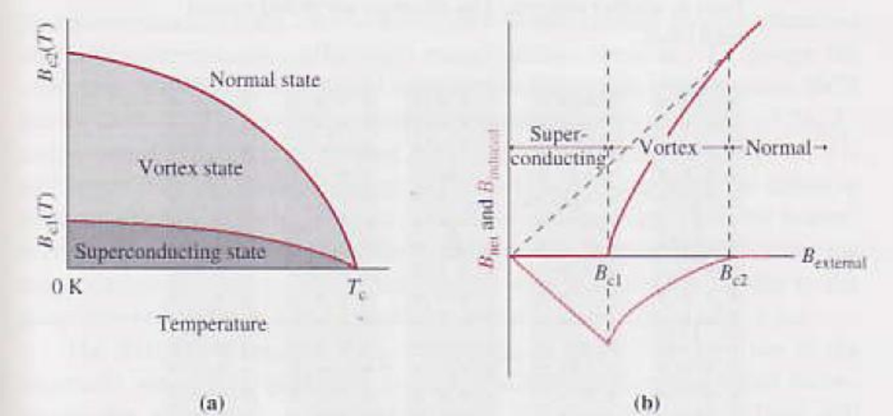


Figure 10.54 In a Type-II superconductor, an external field increasing from B_{c1} would penetrate through an increasing density of vortices, until at B_{c2} the material is normal.



Type 2 Superconductors

Figure 10.52 Magnetic field lines passing through vortices in a type-II superconductor.

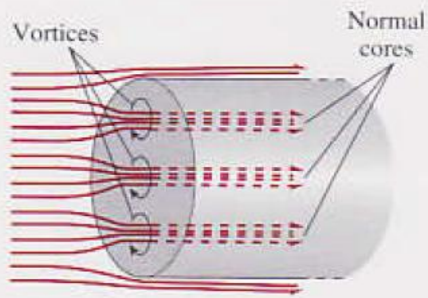
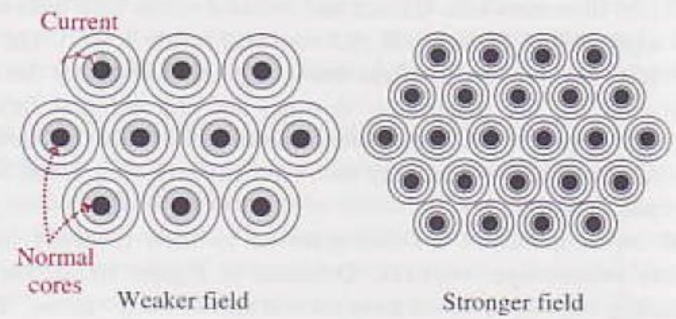


Figure 10.53 Vortices become more dense as field strength increases. (Both represent $B_{c1} < B_{external} < B_{c2}$. See Figure 10.54.)



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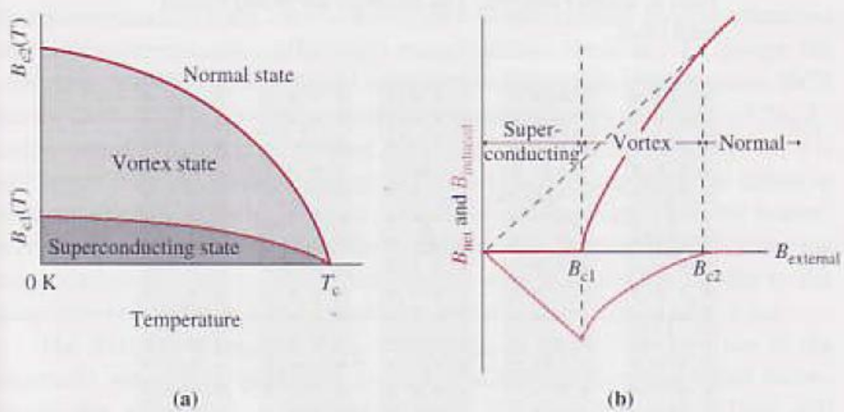


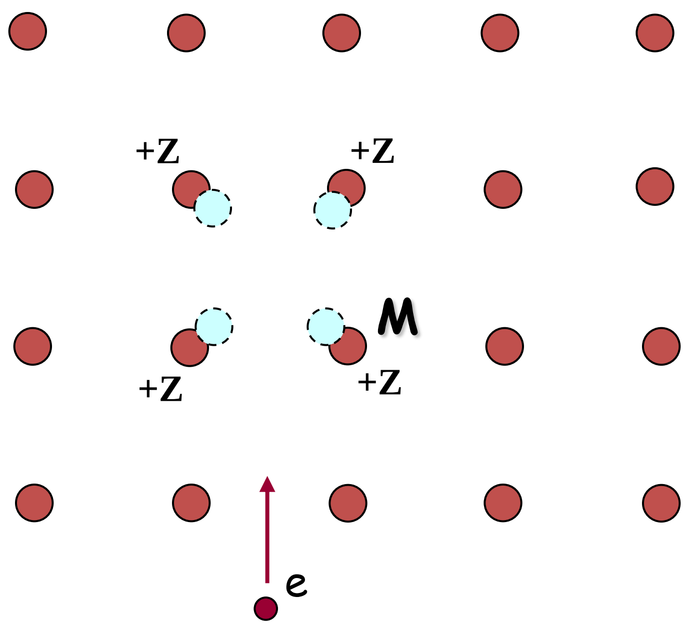
-Vortices: superconducting regions surrounding normal cores
-Stronger field makes vortices become more dense, allowing more field lines to penetrate.
-metallic compounds and alloys
-Bc are 2-3 orders of magnitude grater than type 1
-Tc is twice as high as type 1, around 20K

BCS Theory

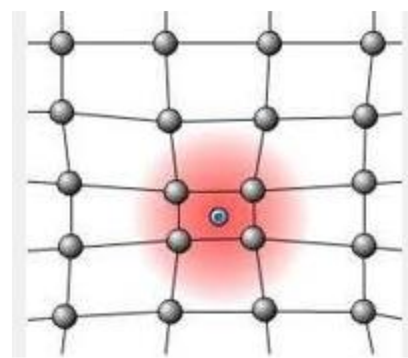
- Bardeen, Cooper, and Schrieffer (BCS)
- Explains Type 1 and Type 2 superconductors
- Lattice and electron interactions
- Coherent motion of the electrons in Cooper pairs
  - What is cooper pair?
  - How coherent motion is created?

BCS theory

PHONON MEDIATED PAIRING
(phonon = lattice vibration)



Pairs of electrons: Cooper pairs



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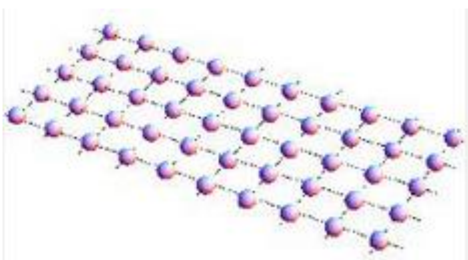

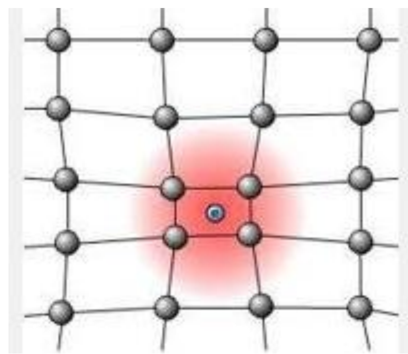

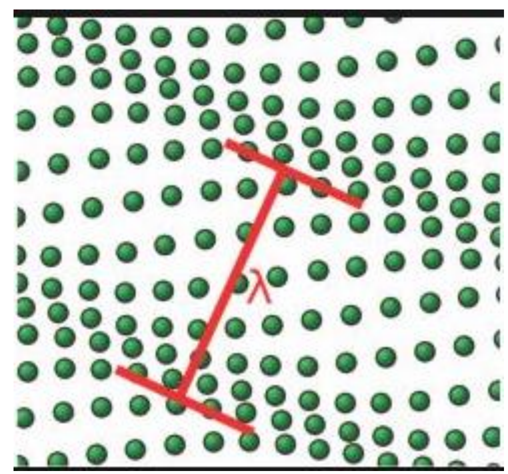

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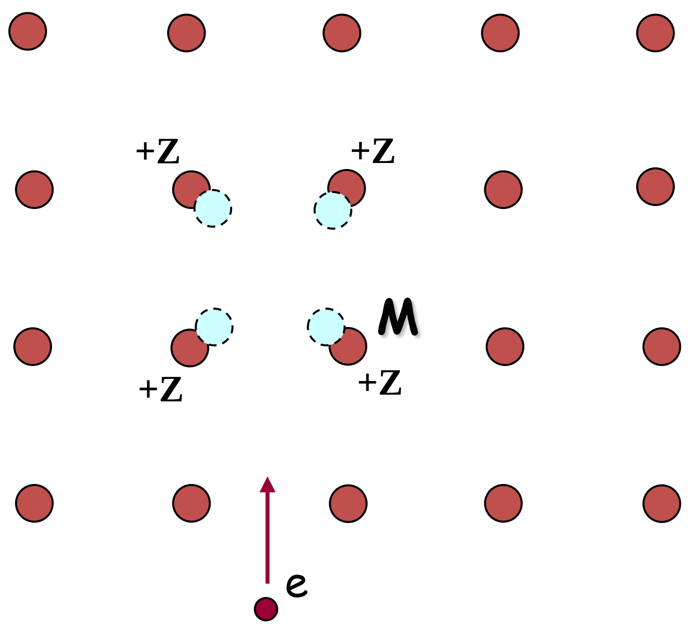

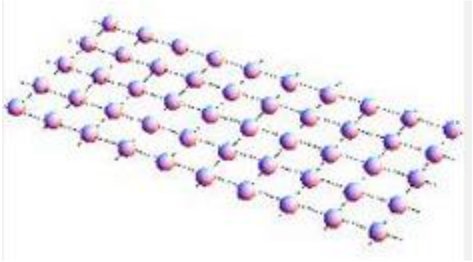

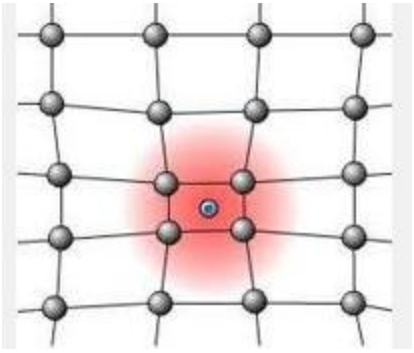

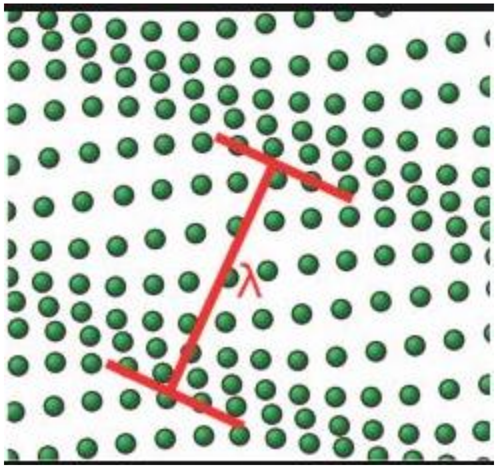

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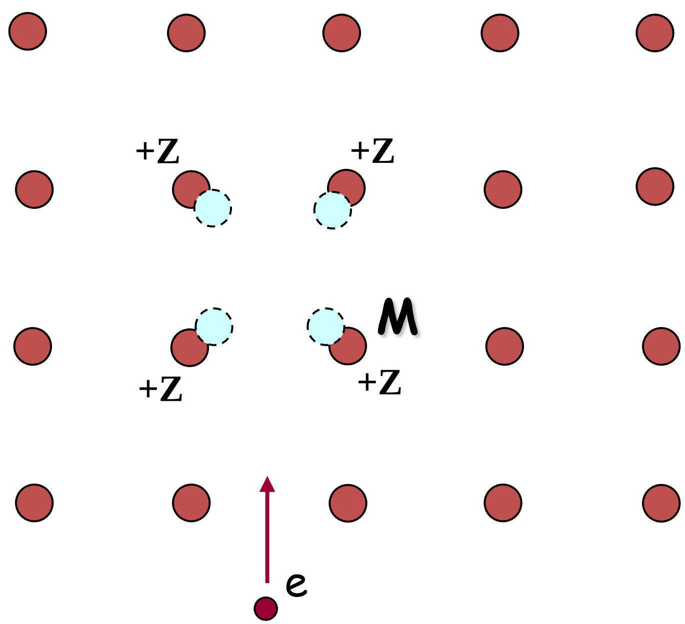
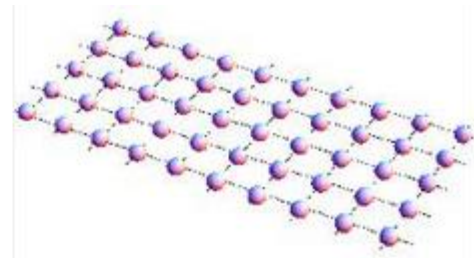
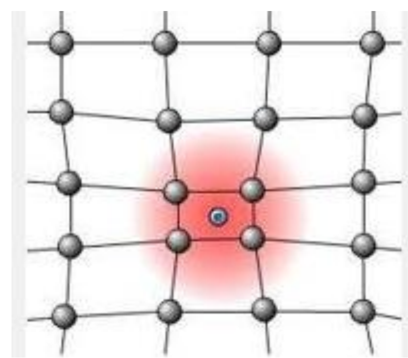
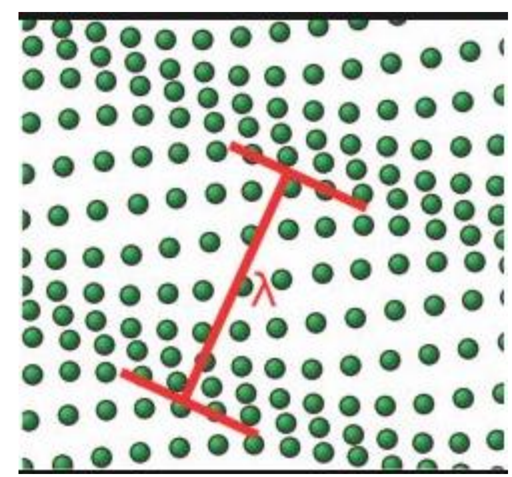
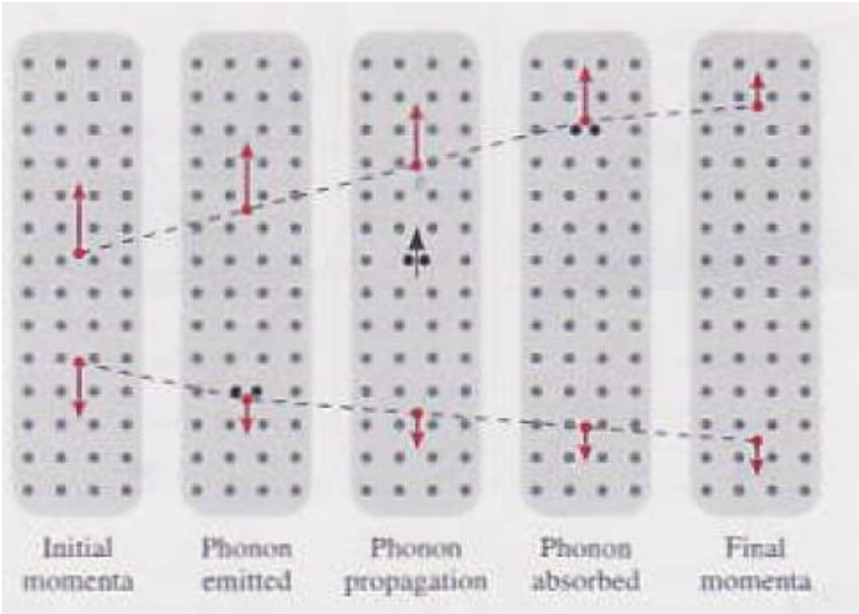






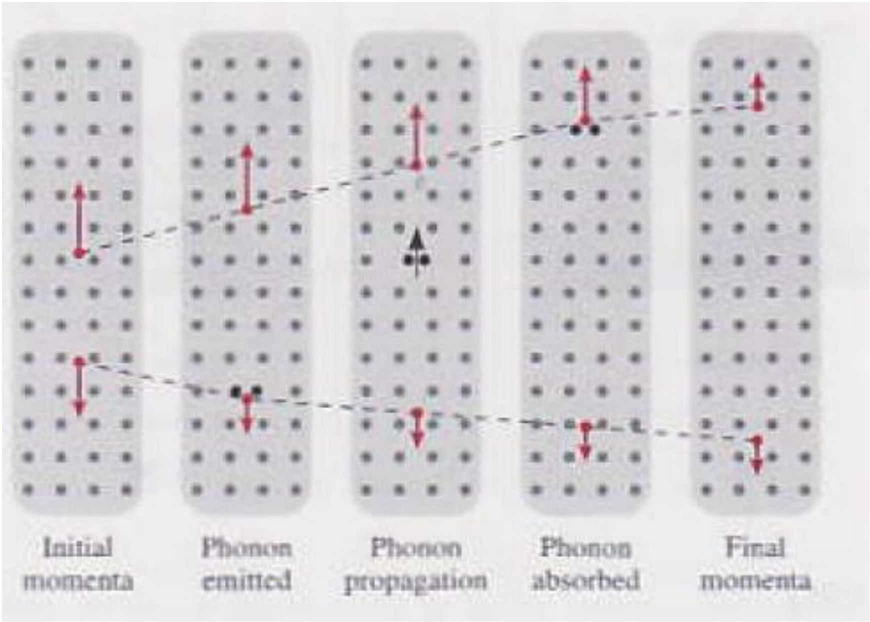


Cooper Pairs



Cooper paired electrons
-are bosons as a pair
-have opposite momenta
-opposite spin directions

Cooper Pairs



Cooper paired electrons
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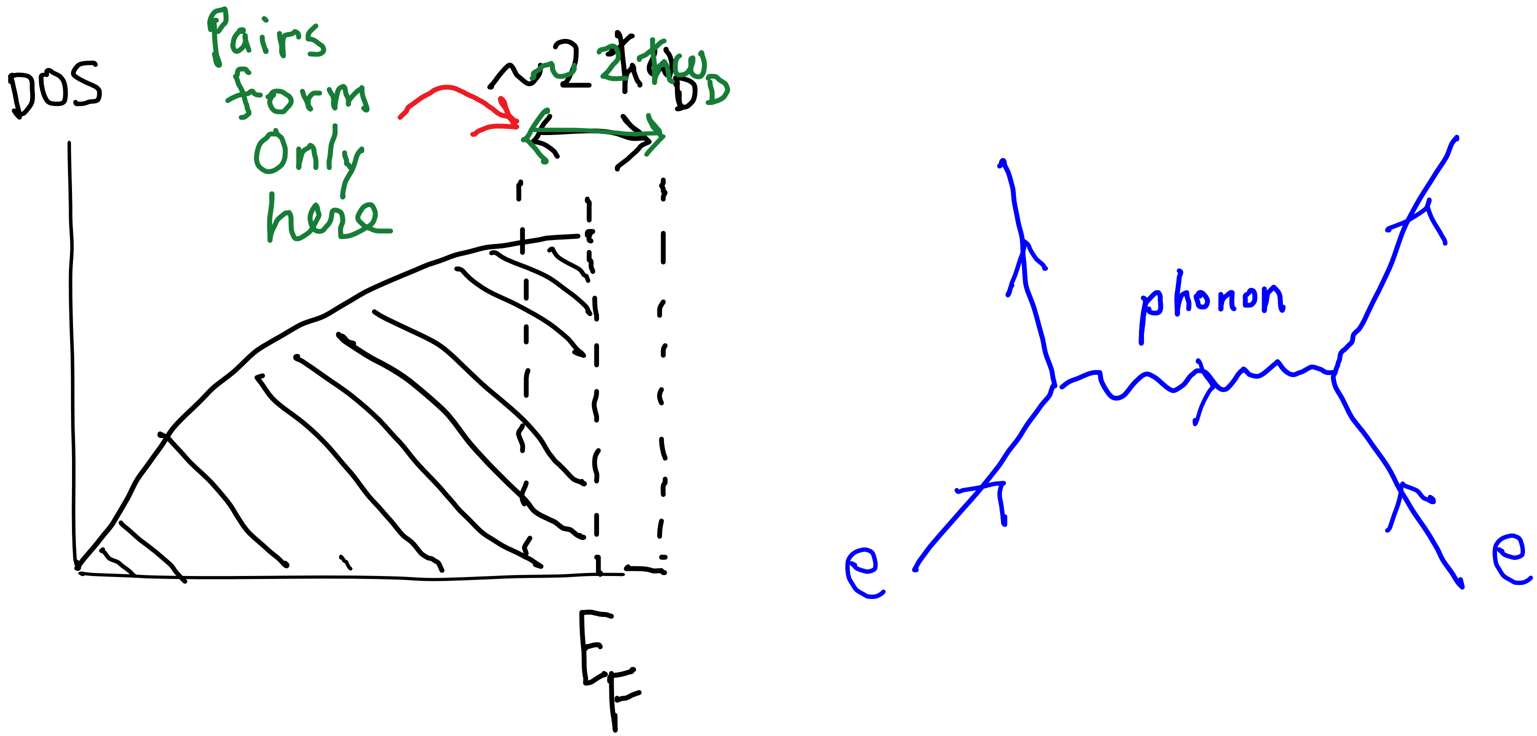
-Paired electrons experience a net attraction by exchanging a phonon
-Lattice is involved
-This attractive force is small at the order of 0.001 eV but is greater than electron –electron repulsion
-the separation between the two paired electrons is greater than atomic spacing in the lattice

How many electrons participate?



Only those electrons near E_F form Cooper pairs. These are the electrons with energies $E = E_F \pm \hbar\omega_D$ where ω_D is the Debye frequency (typical frequency for phonons). Why? Due to Pauli exclusion principle, electrons cannot scatter by absorbing or emitting a phonon when they are deep inside the Fermi sea ($E \ll E_F$). So, only a fraction, $O\left(\frac{\hbar\omega_D}{E_F}\right)$, of total electrons participate in Cooper pairs.

Coherent motion of Cooper pairs

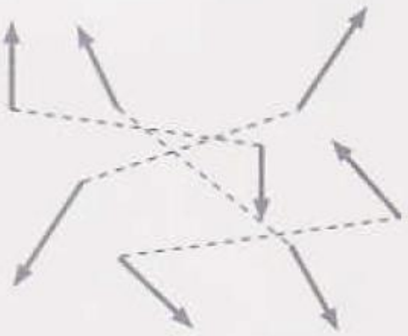


Figure 10.56 All Cooper pairs have the same momentum: zero when current is zero; nonzero otherwise.

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-binding energy between electrons in a Cooper pair is greater than typical thermal energy between electron and lattice ion at low temperatures

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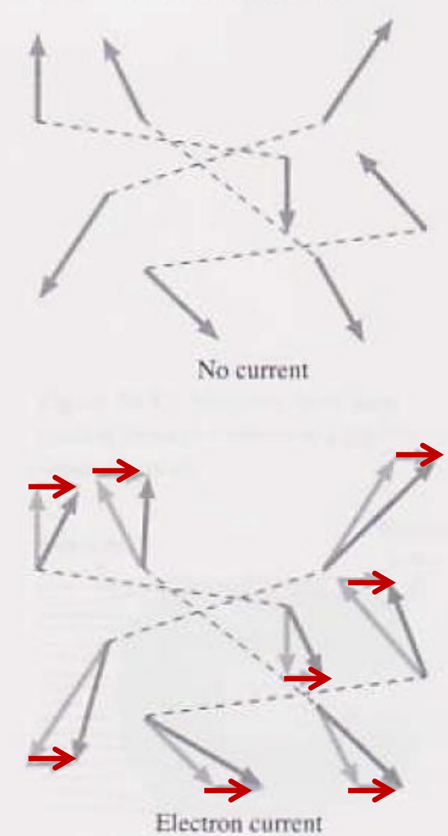

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-Type 1 superconductors have longer mean free paths than Type 2 superconductors

Coherent motion of Cooper pairs

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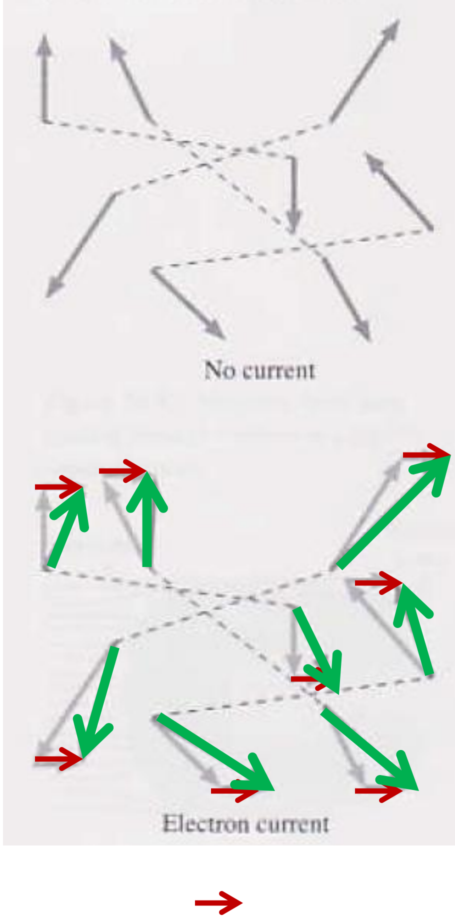

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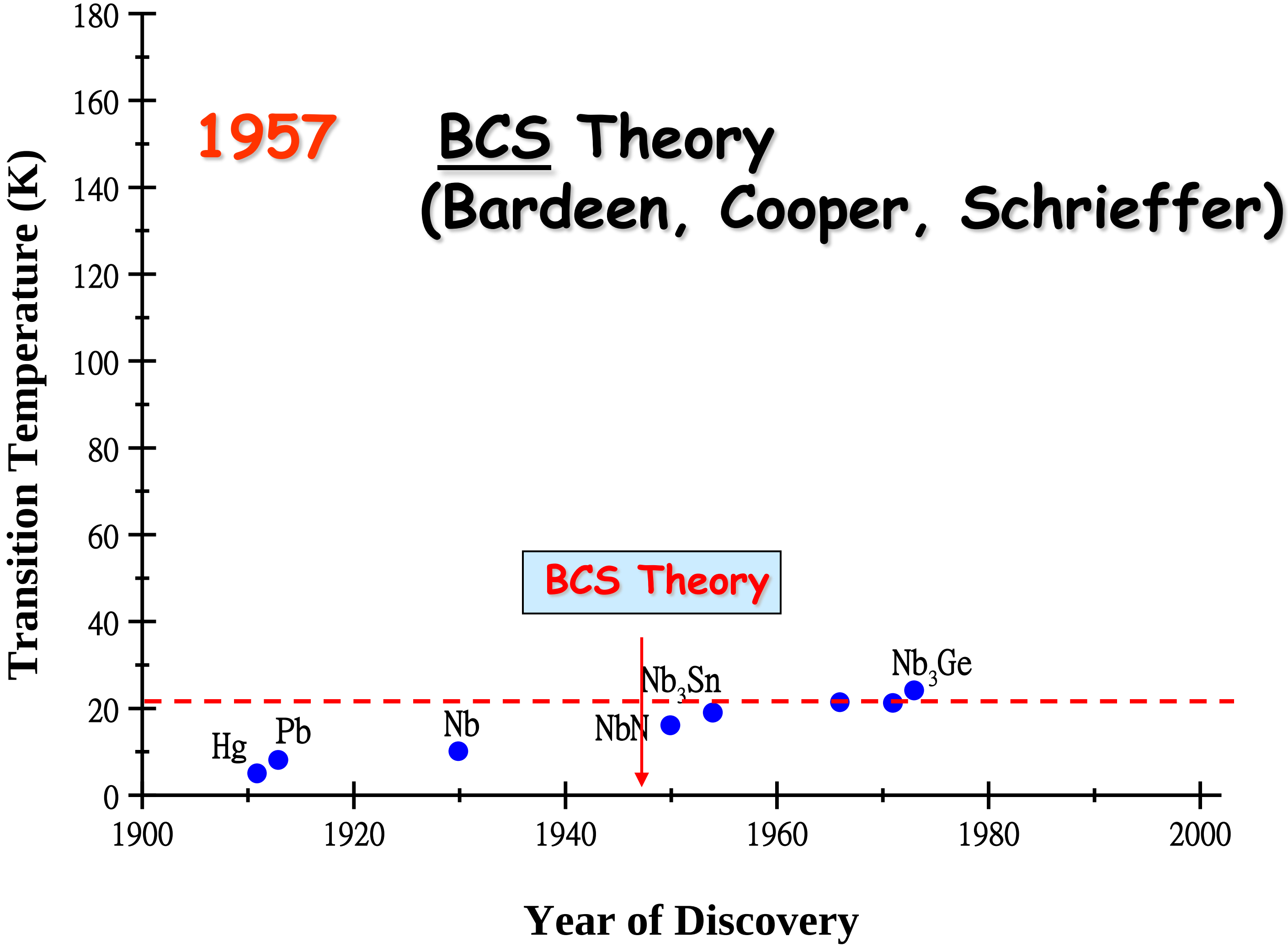
Understanding of Superconductivity



Nucleus

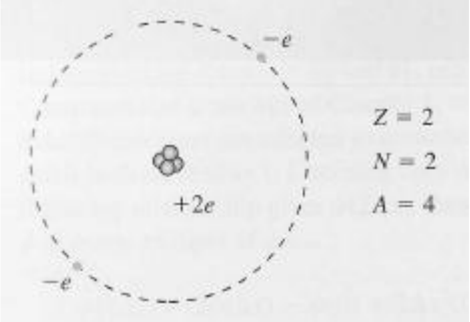


Figure 11.1 A crude picture of helium.

	charge	Mass in kg	Mass in u
Proton	+e	$1.6726217 \times 10^{-27}$	1.007276
Neutron	0	$1.6749273 \times 10^{-27}$	1.008665
Electron	-e	9.109×10^{-31}	0.0005486

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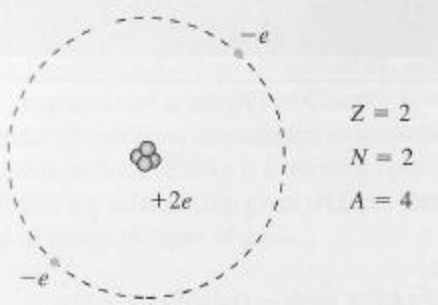

Figure 11.1 A crude picture of helium.

Z= number of protons
N=number of neutrons
A=mass number=number of nucleons=Z+N

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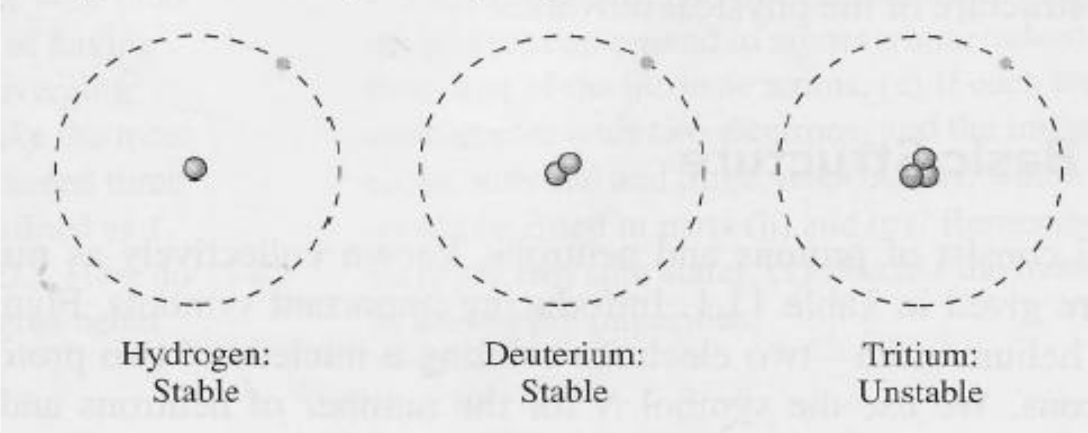
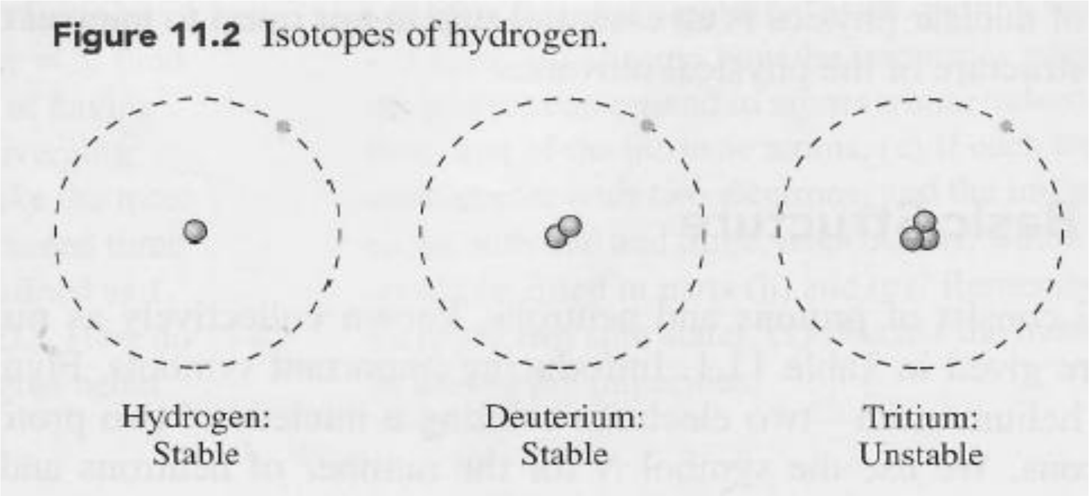
Isotopes

Figure 11.2 Isotopes of hydrogen.



Nuclei	Hydrogen	Deuterium	Tritium
Z			
N			
A			
Electron number			

Isotopes



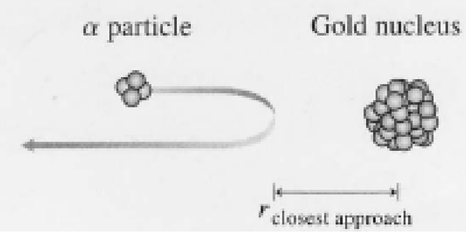
Nuclei	Hydrogen	Deuterium	Tritium
Z	1	1	1
N	0	1	2
A	1	2	3
Electron number	1	1	1

Stable

Nucleus size

Rutherford’s alpha particle experiment on Gold foil

Figure 11.3 Probing for the radius of the nucleus.



- →The nucleus is roughly spherical
- →The nucleus is centered at the middle of the atom
- →The nucleus occupies a relatively small space
- →The head-on approach would enable the alpha particles to get closest

$$r = A^{1/3} R_0 \qquad R_0 = 1.2\ x\ 10^{-15} m.$$

Nucleus Volume =

Nuclear Density=

Nucleon volume=

Nucleus size

Rutherford's alpha particle experiment on Gold foil

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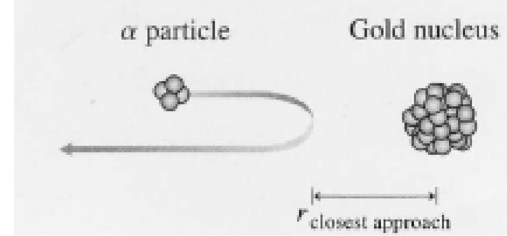

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$$r = A^{1/3} R_0 \qquad R_0 = 1.2\ x\ 10^{-15} m.$$

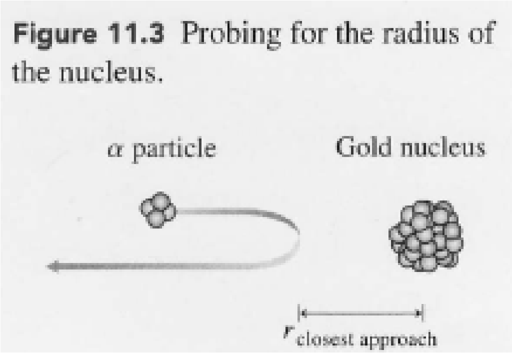
Nuclear Volume = $V = \frac{4}{3}\pi r^3 =$

Nuclear Density= $density = \frac{mass}{volume} =$

Nucleon volume= $\frac{V}{A} =$

Nucleus size

Rutherford's alpha particle experiment on Gold foil



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$$r = A^{1/3} R_0 \qquad R_0 = 1.2 \text{ x } 10^{-15} m.$$

Nuclear Volume = $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (A^{1/3} R_0)^3 = \frac{4}{3}\pi A R_o^3 \sim A$

Nuclear Density= $density = \frac{mass}{volume} = \frac{A \text{ x } mass\ of\ nucleon}{A \text{ x } \frac{4}{3}\pi R_o^3} \cong 10^{17} kg/m^3$

Nucleon volume= $\frac{V}{A} = \frac{4}{3}\pi R_o^3$

Strong force

→ Strong

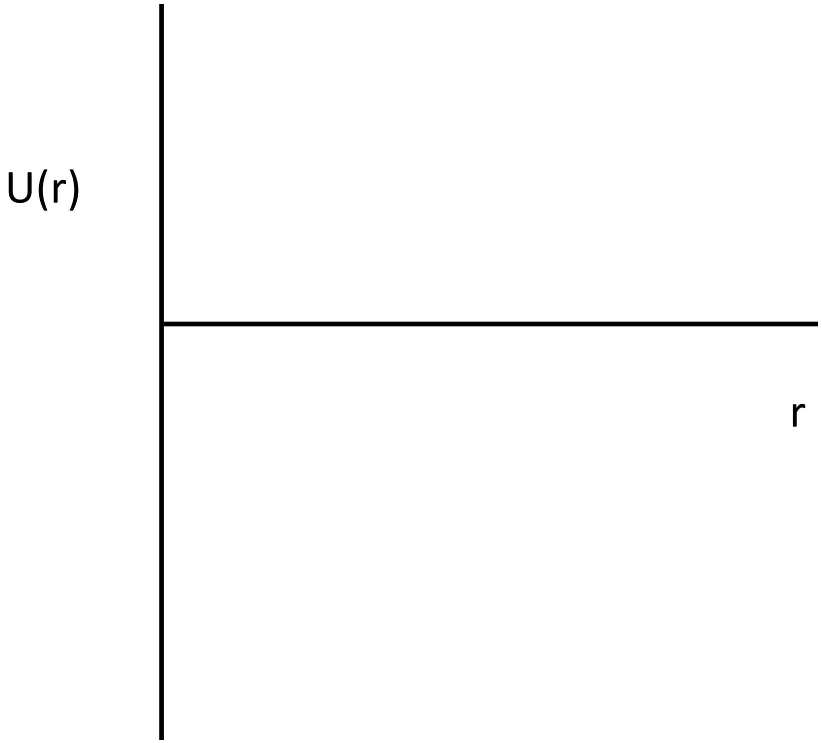
Strong force

→ Strong

Force	Applying to	Relative strength	Range
Strong	Between nucleons	1	$\sim 1\, fm$
Electromagnetic	Between charges	$\sim 10^{-2}$	$\propto 1/r^2$ (long range)
Weak	Related to radioactive decay	$\sim 10^{-6}$	$\sim 10^{-3}\, fm$
Gravitational	Between masses	$\sim 10^{-39}$	$\propto 1/r^2$ (long range)

Strong force

→Strong
→ attractive
→Short-ranged (about 2 fm)



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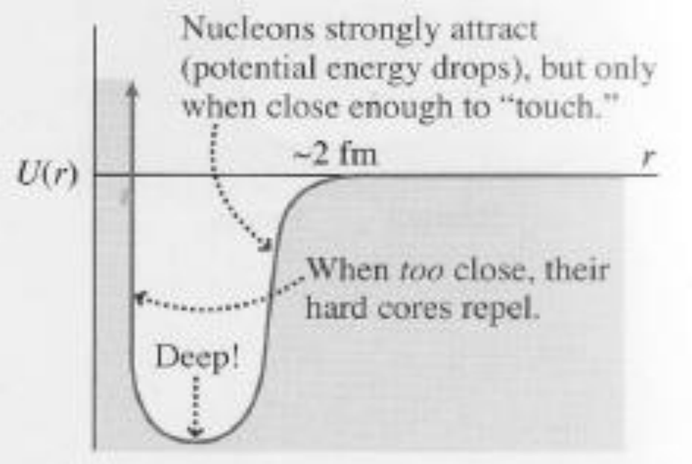
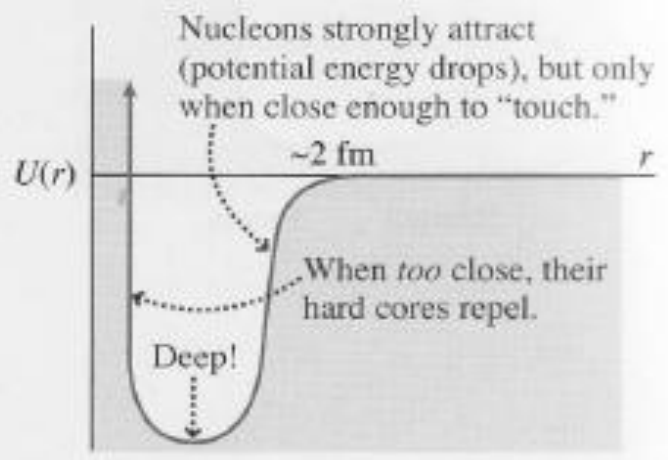


Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.

Strong force

→Strong
→ attractive
→Short-ranged (about 2 fm)
→Nearly identical between
    Proton-proton
    Neutron-neutron
    Proton-neutron

Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.



Nuclear binding: Two nucleons

- Protons and neutrons are subject to Exclusion Principle, independently
- Parallel spin arrangements in internucleon attraction are more stable

Nuclear binding: Two nucleons

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- Parallel spin arrangements are more stable

Example:

Use two nucleon arrangements and figure which arrangement is most stable:

1. Proton-proton
2. Neutron-neutron
3. Proton-neutron

Nuclear binding: Two nucleons

- Protons and neutrons are subject to Exclusion Principle, independently
- Parallel spin arrangements are more stable

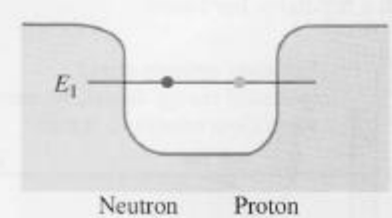
Example:

Proton-neutron pair can be most stably arranged

Experimental evidence:

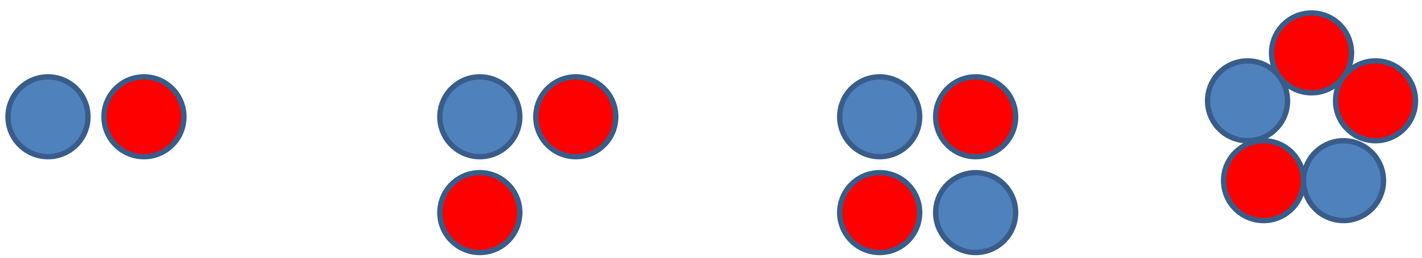
- Deuterium's nucleus' (deuteron) total spin is 1
- Deuteron has one single bound state.

Figure 11.5 The deuteron's neutron and proton bound in a well resulting from their attractive potential energy.



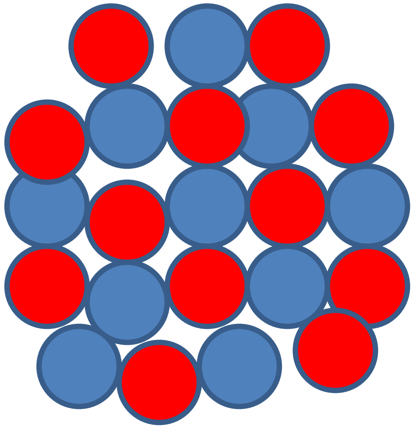
Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:



Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large



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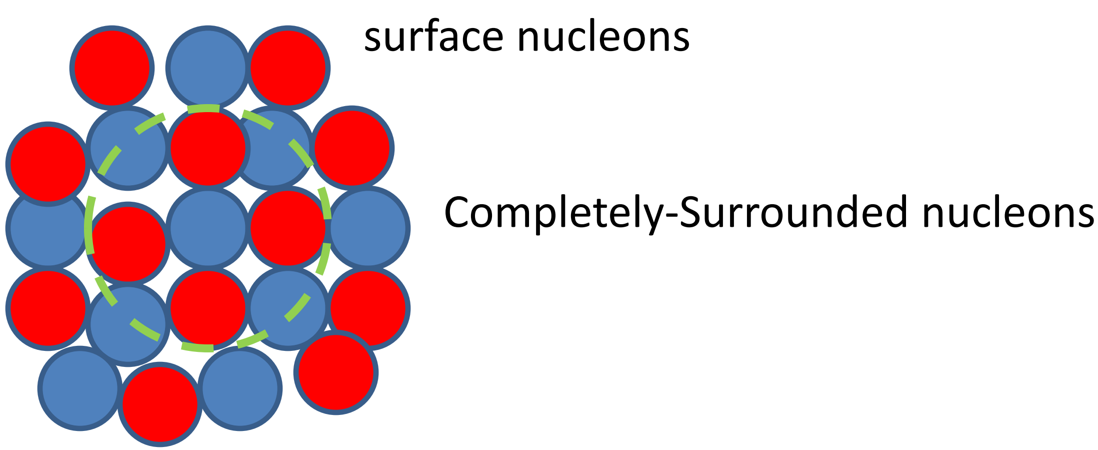
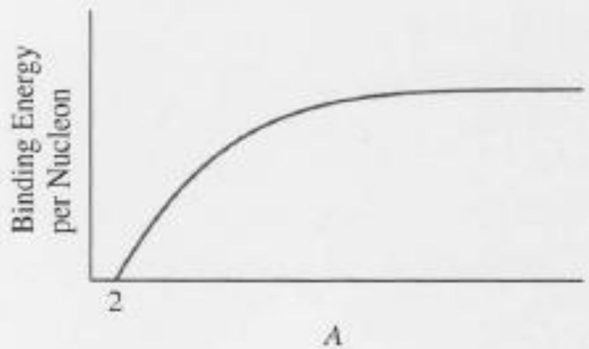


Multi-nucleon nuclei

- Strong force:
  - When the total number of nucleons is small:
    - The number of bonds each nucleon can have increases, thus binding energy per nucleon increases
  - When the total number of nucleons is large
    - Since strong force is short ranged, making each nucleon has the same number of surrounding nucleons.
    - The nucleons at the surface are not completely surrounded. The proportion of surface nucleons diminish by 1/r

Only strong force is considered

Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.



Coulomb repulsion

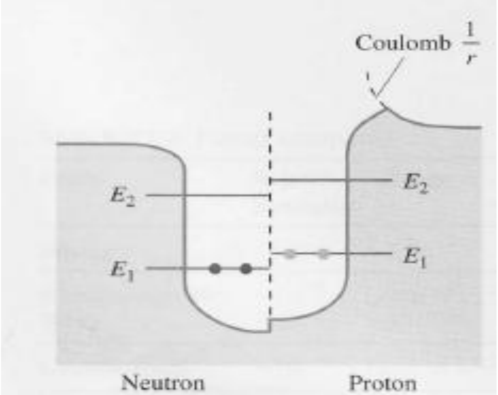
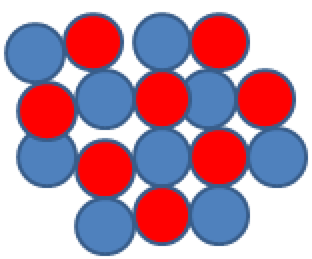


Figure 11.7 Coulomb repulsion raises proton energies.



Coulomb repulsion

Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.

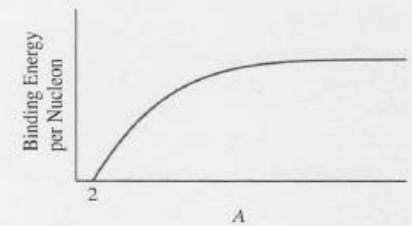


Figure 11.7 Coulomb repulsion raises proton energies.

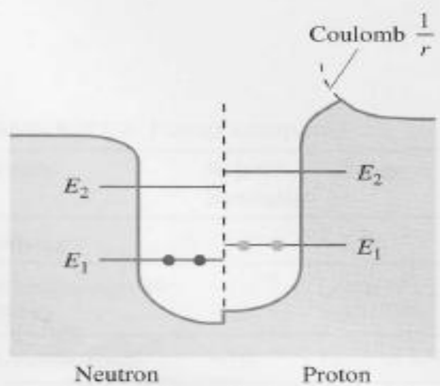
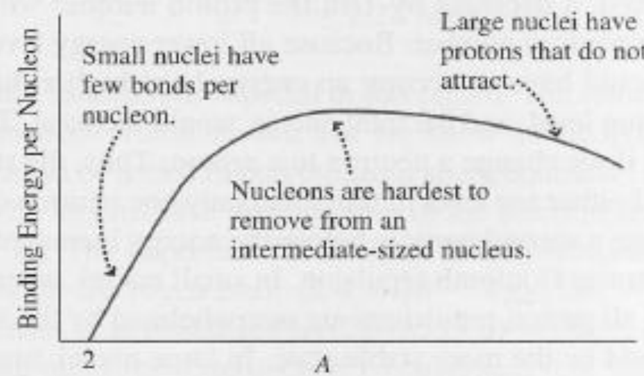


Figure 11.8 Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.



Exclusion principle

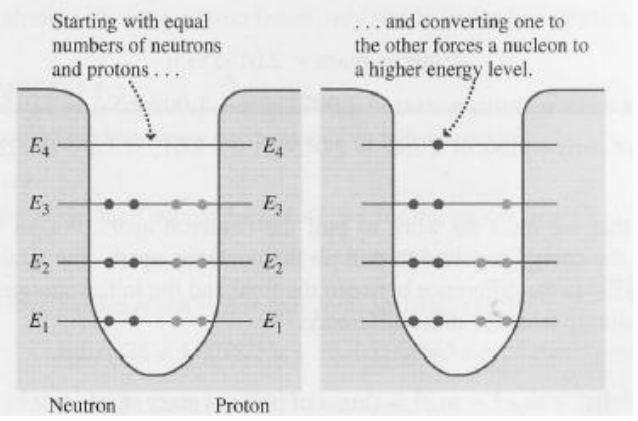


Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.

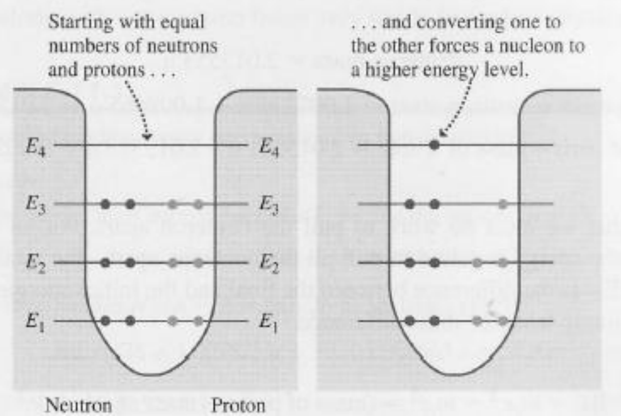
For small nuclei, repulsive coulomb interactions are less influential

When N=Z,

When N≠Z

Exclusion principle

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.



For small nuclei, repulsive coulomb interactions are less influential

When N=Z, more stable

When N≠Z, energy raised

Exclusion principle

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.

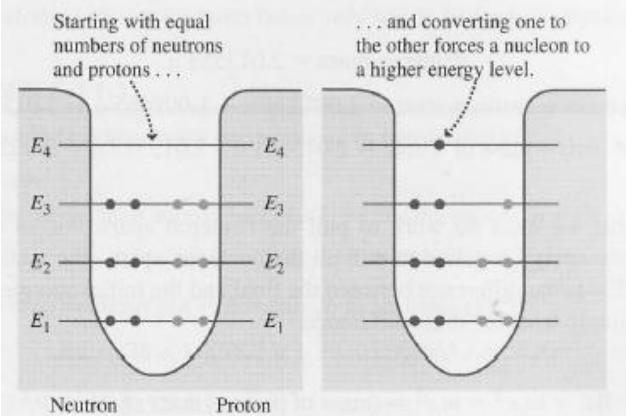
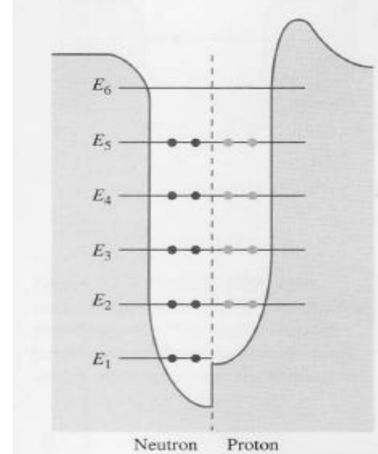


Figure 11.10 In large nuclei, when Coulomb repulsion becomes significant, the lowest energy should have $N > Z$.



For large nuclei, repulsive force changes

N>Z is more stable

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$$C_1 = 15.8$$
$$C_2 = 17.8$$
$$C_3 = 0.71$$
$$C_4 = 23.7$$

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

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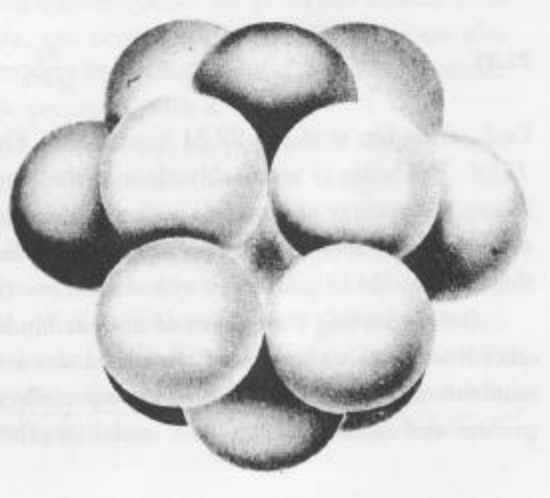


$C_1 = 15.8$
$C_2 = 17.8$
$C_3 = 0.71$
$C_4 = 23.7$

Strong force (C1 and C2)

→Strong
→ attractive
→Short-ranged (about 2 fm)
→Nearly identical between
  Proton-proton
  Neutron-neutron
  Proton-neutron

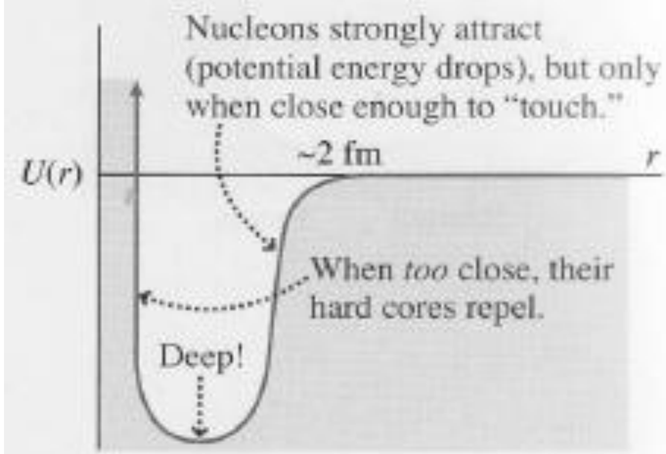
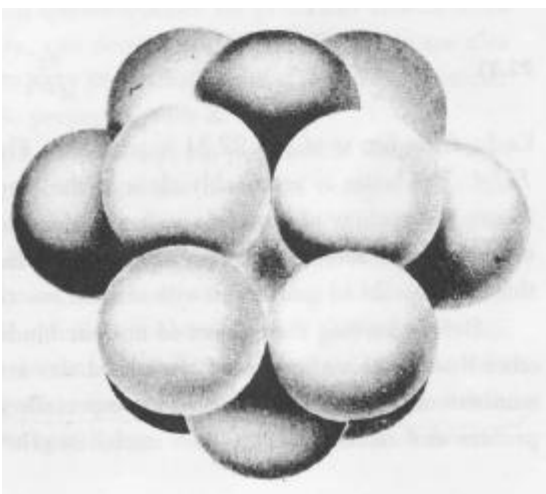


Figure 11.4 The basic elements of the internucleon (strong force) potential energy—a strong, short-range attraction with a repulsive hard core.

Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$



$C_1 = 15.8$
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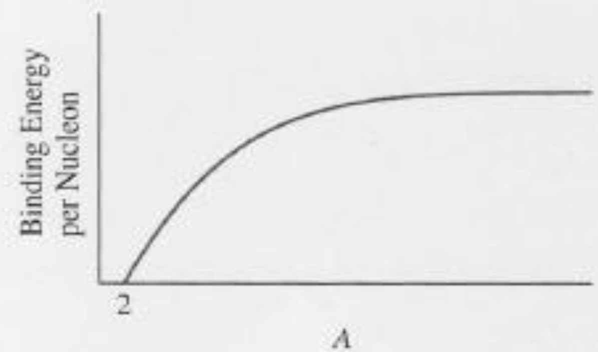
Figure 11.6 Binding energy per nucleon due to the strong internucleon attraction only. The smallest nuclei have few bonds per nucleon. In large nuclei, many nucleons are surrounded.

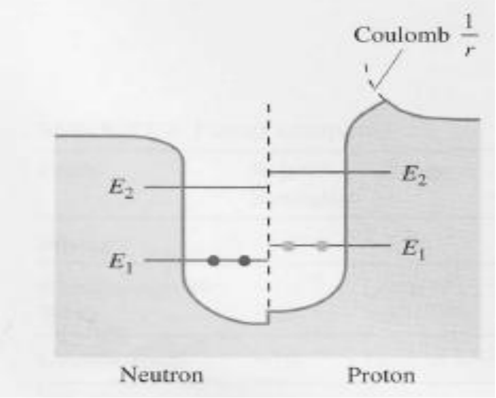


Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

Figure 11.7 Coulomb repulsion raises proton energies.



$C_1 = 15.8$
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Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

$C_1 = 15.8$
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Figure 11.7 Coulomb repulsion raises proton energies.

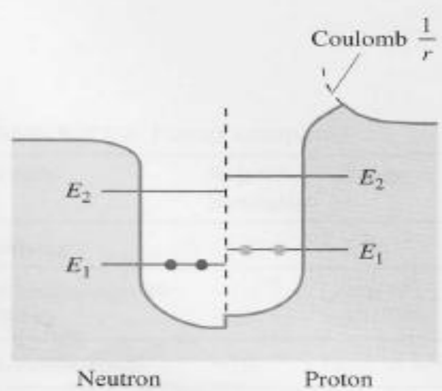
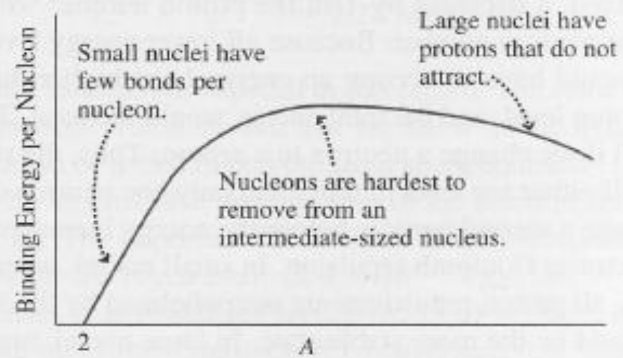


Figure 11.8 Binding energy per nucleon due to both the strong internucleon attraction and Coulomb repulsion.

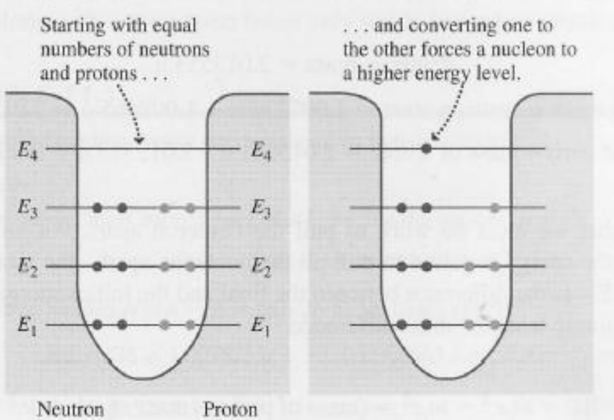


Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

Figure 11.9 Ignoring Coulomb repulsion, the exclusion principle argues that for a given number of nucleons, the lowest energy should have $N = Z$.



$$C_1 = 15.8$$
$$C_2 = 17.8$$
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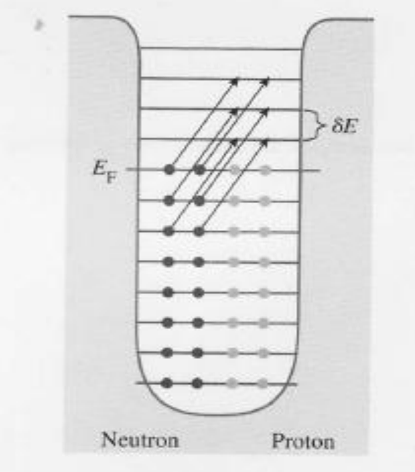
$$C_4 = 23.7$$

Figure 11.15 If j neutrons become protons, the energy increases by $\frac{1}{2} j^2 \delta E$.

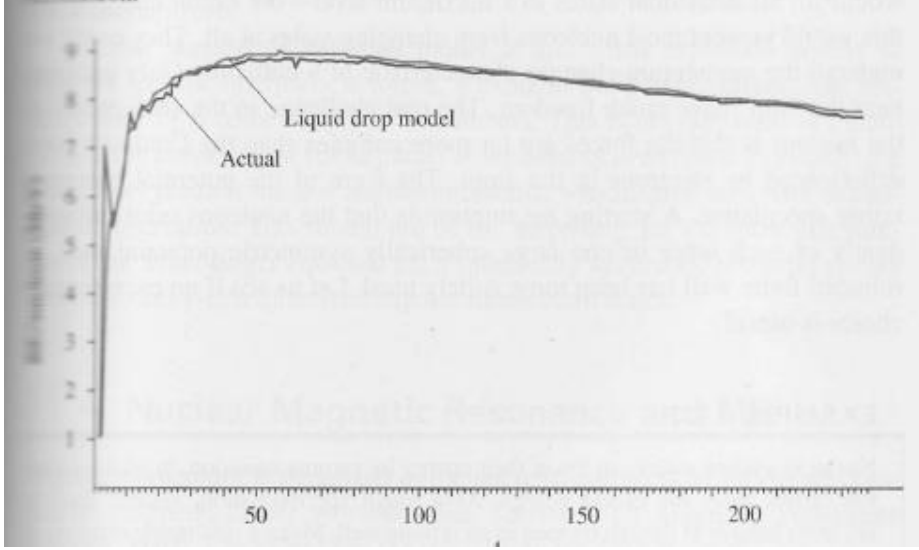


Liquid Drop Model

Binding energy = Volume term + Surface term + Coulomb term + Asymmetric term

$$= C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A}$$

Figure 11.16 Comparison of the semiempirical binding energy formula and actual ... nucleon.



Magic numbers= 2, 8, 20, 28, 50, 82, 126