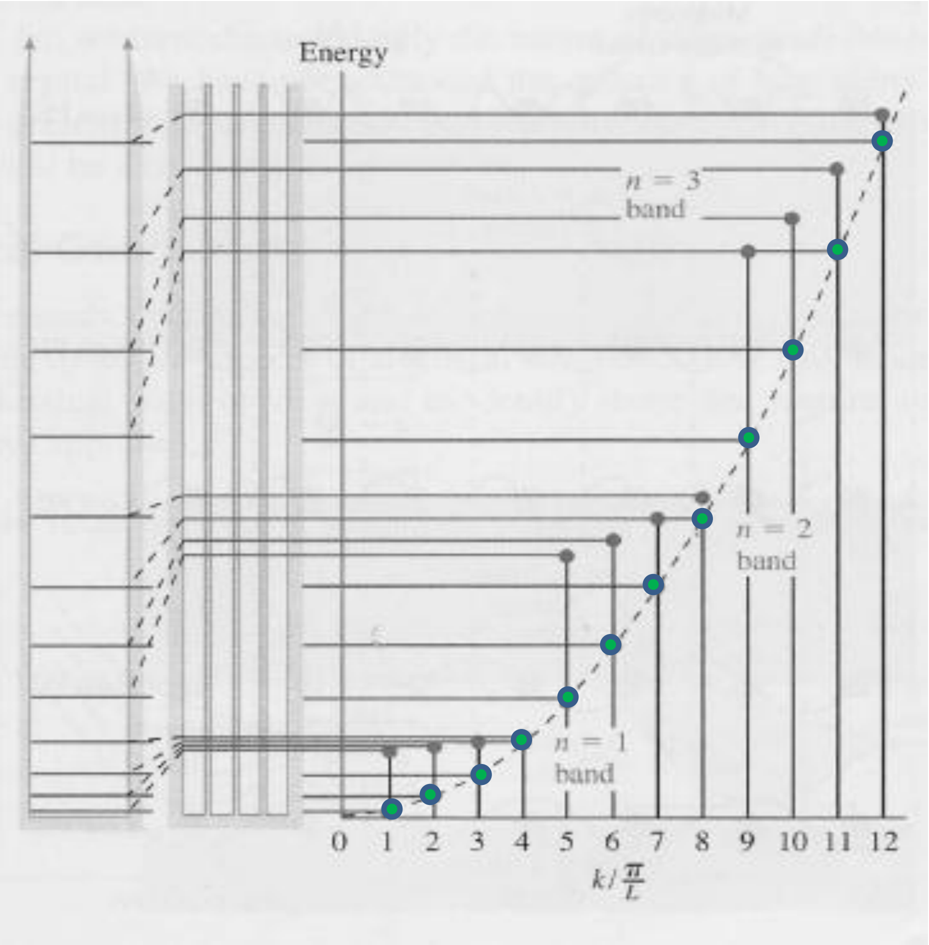
Lecture 13 Topics

- Energy bands
  - In a 4-atom crystal
  - in an N-atom crystal
  - Conductors, insulators, and semiconductors
- Charge carriers
  - Conduction electrons
  - Valence holes
  - Effective mass

Comparing energy bands vs. free particle solution

Free particle solution:

$$E = \frac{\hbar^2}{2m} k^2$$

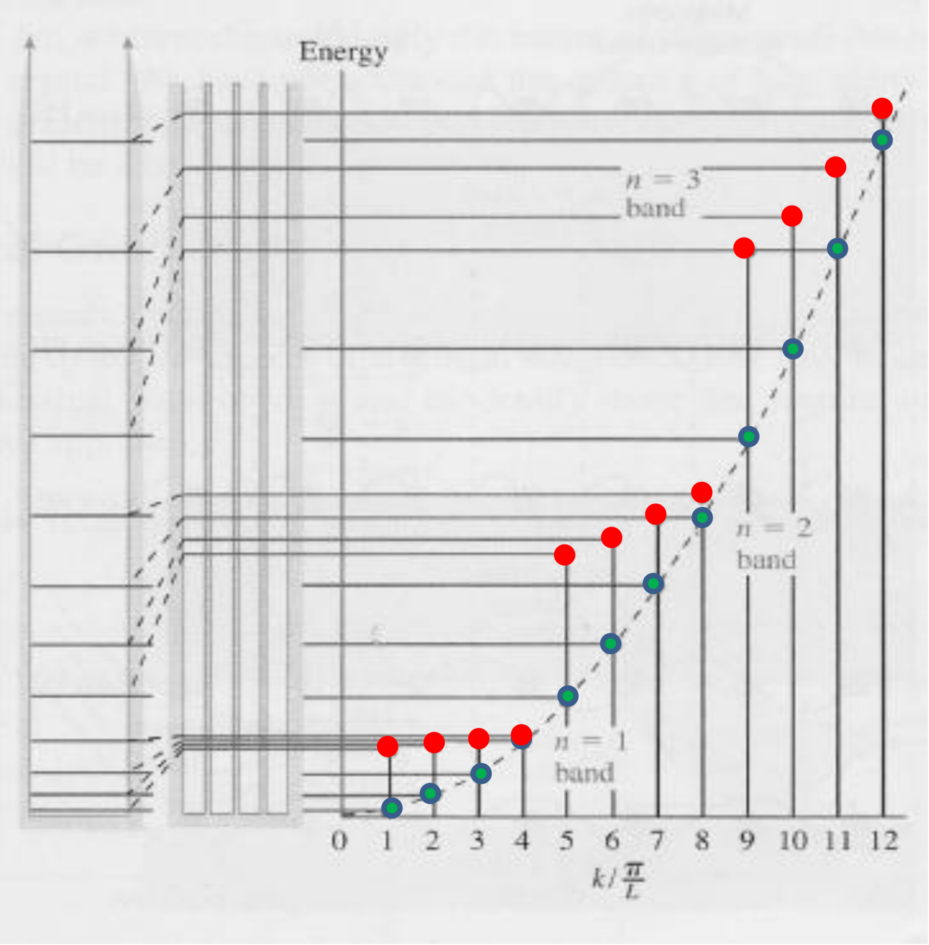


Large well   4-atom well

Comparing energy bands vs. free particle solution

Free particle solution:

$$E = \frac{\hbar^2}{2m} k^2$$

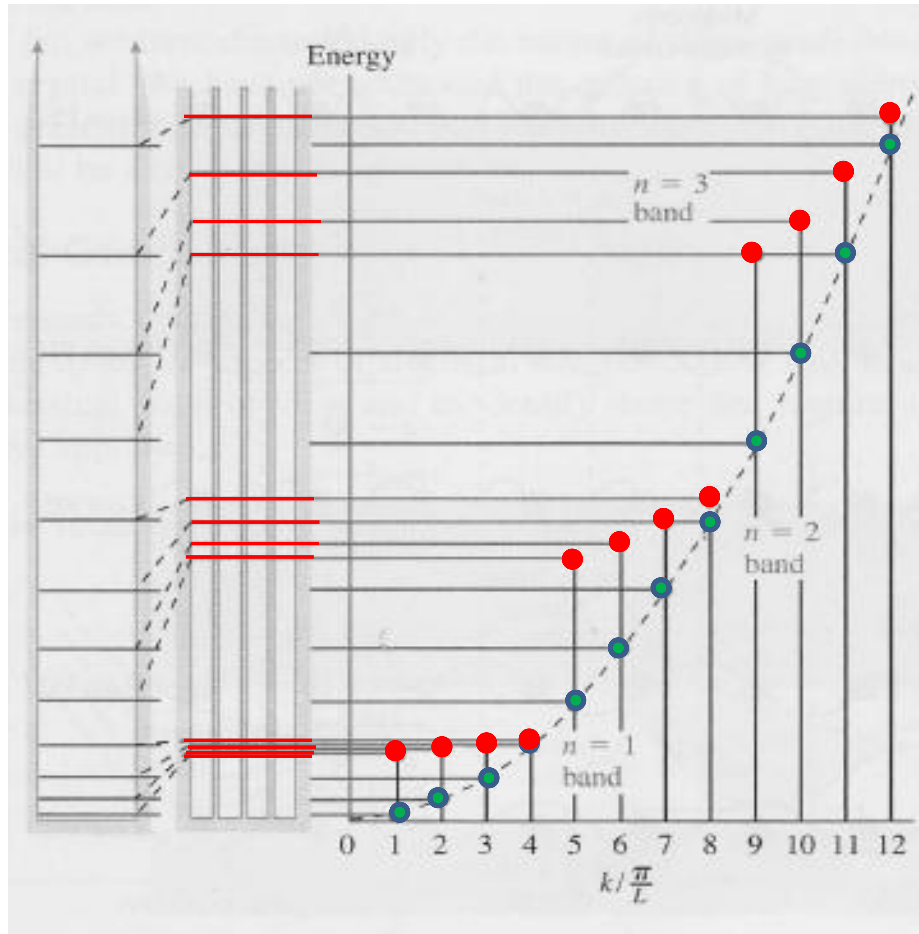


Large well 4-atom well

Comparing energy bands vs. free particle solution

Free particle solution:

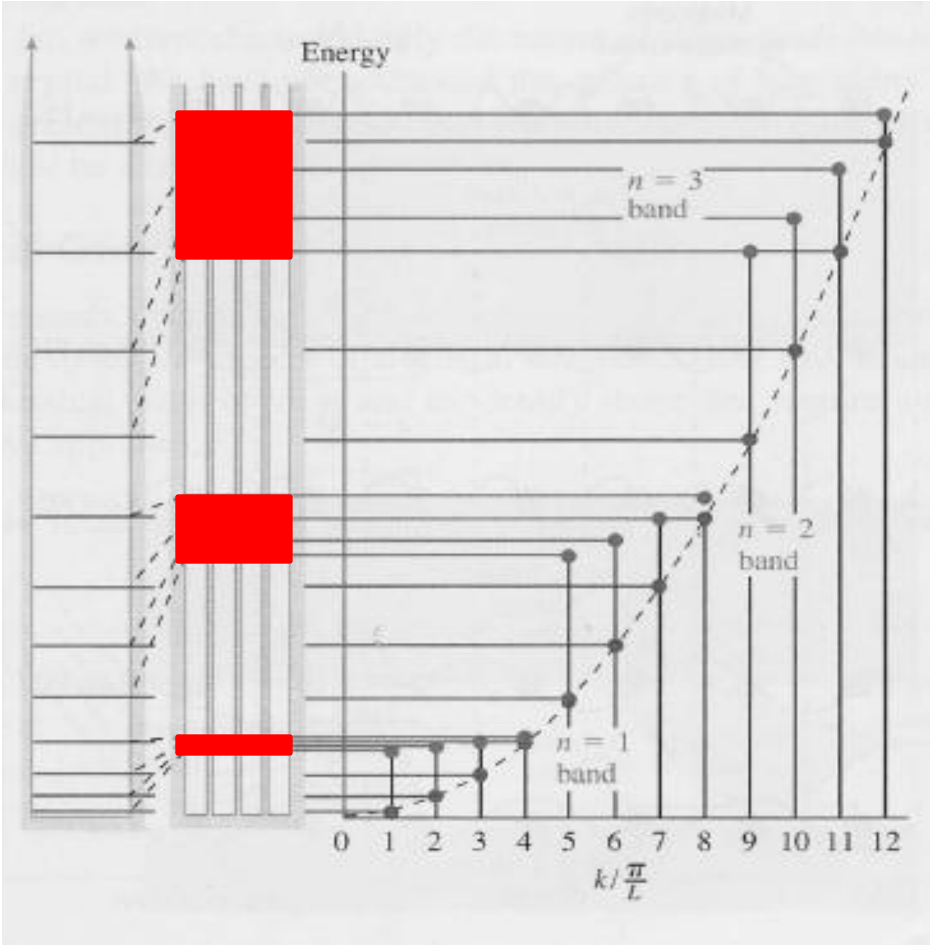
$$E = \frac{\hbar^2}{2m} k^2$$



Large well   4-atom well

Comparing energy bands vs. free particle solution

$$E = \frac{\hbar^2}{2m} k^2$$



Large well   4-atom well

Energy band gap

Three differences between free particle and electron in crystal:

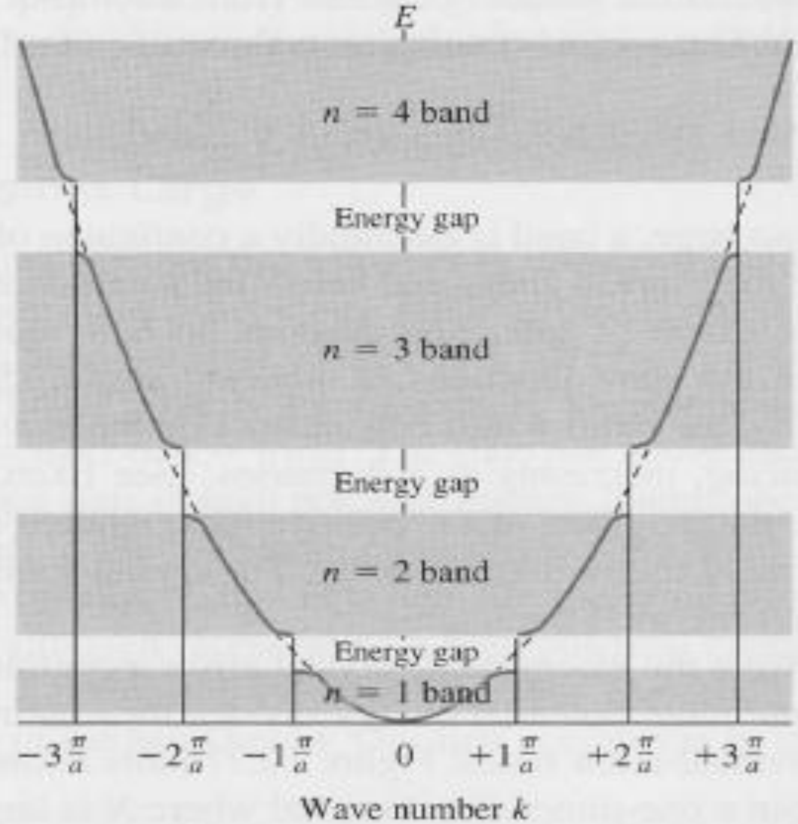


Figure 10.27 Bands and gaps in a one-dimensional crystal.

Energy band gap

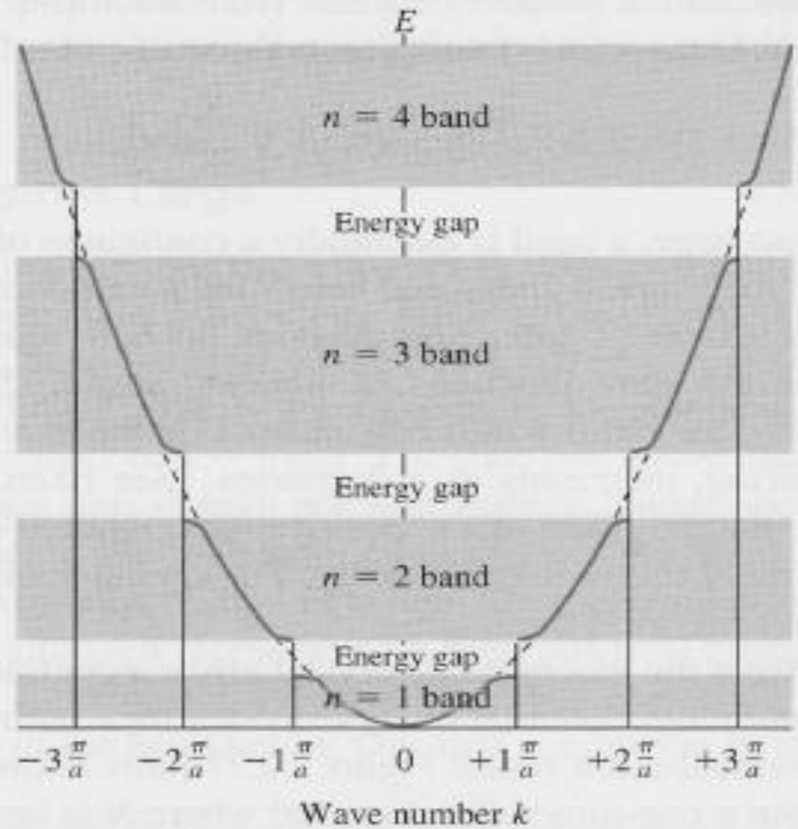
Three differences between free particle and electron in crystal:

1. Energy level continuity

2. Energy band existence

3. The curvature of energy values allowed in the system

Figure 10.27 Bands and gaps in a one-dimensional crystal.



Conductors, Insulators, and Semiconductors

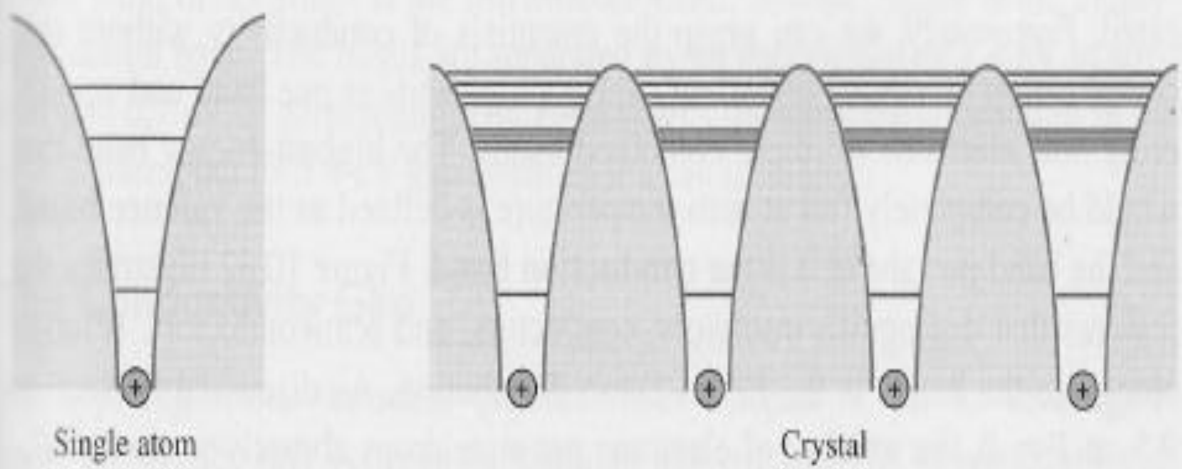


Figure 10.29 When atoms form a crystal, low-lying nonvalance levels still belong to each atom, while higher levels become bands.

Band structure

Conductors, Insulators, and Semiconductors

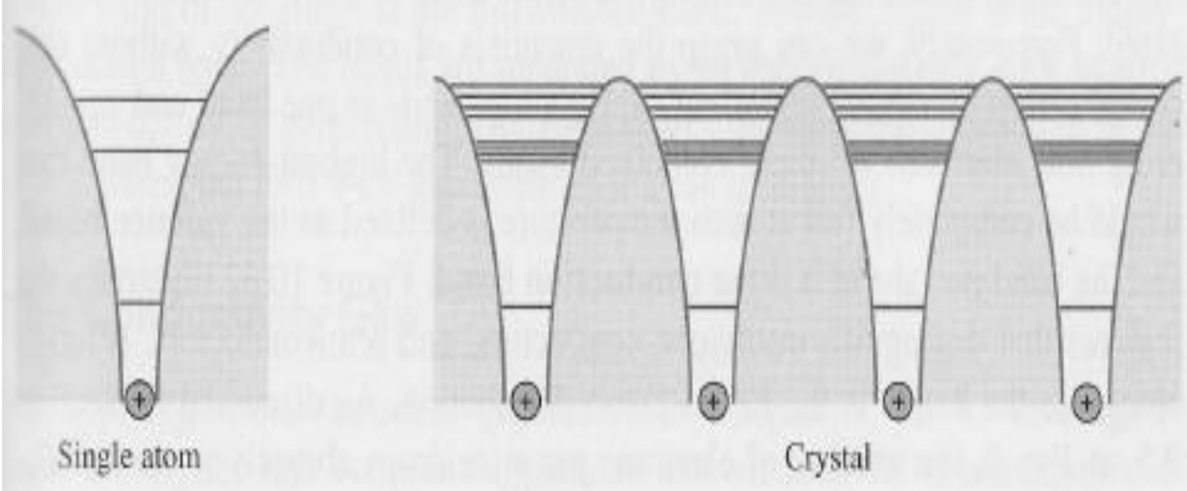
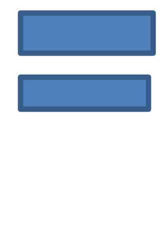


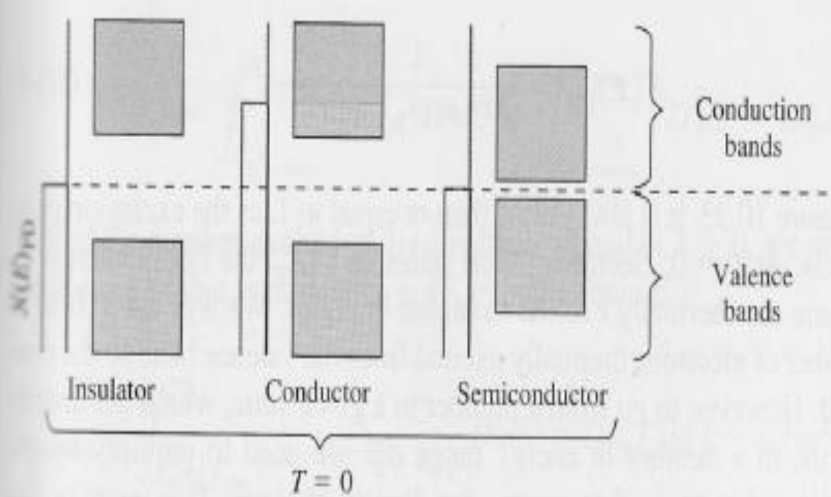
Figure 10.29 When atoms form a crystal, low-lying nonvalance levels still belong to each atom, while higher levels become bands.

Band structure



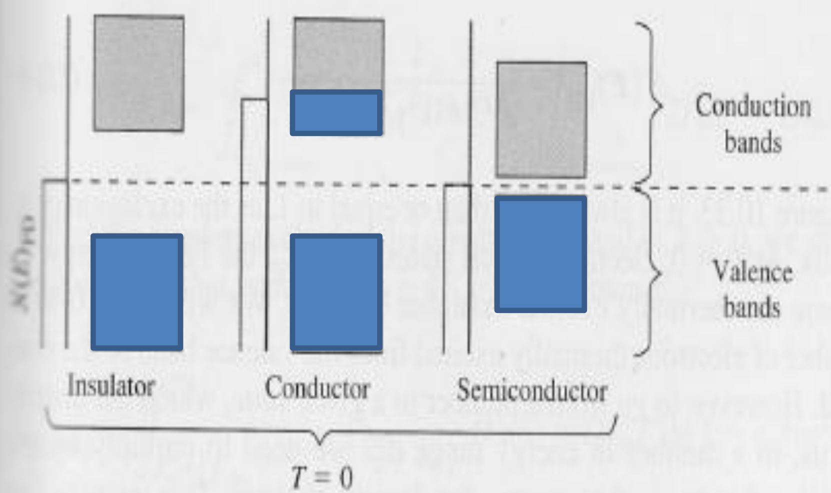
Insulator, Conductor, Semiconductor

Figure 10.32 Band filling for an insulator, a conductor, and a semiconductor at zero and nonzero temperature. When $T > 0$, a semiconductor will have some electrons in the conduction band.



Insulator, Conductor, Semiconductor

Figure 10.32 Band filling for an insulator, a conductor, and a semiconductor at zero and nonzero temperature. When $T > 0$, a semiconductor will have some electrons in the conduction band.



Valence band: The band that is completely filled at T= 0 K
Conduction band: The band that is just above the valence band

Figure 10.32 Band filling for an insulator, a conductor, and a semiconductor at zero and nonzero temperature. When $T > 0$, a semiconductor will have some electrons in the conduction band.

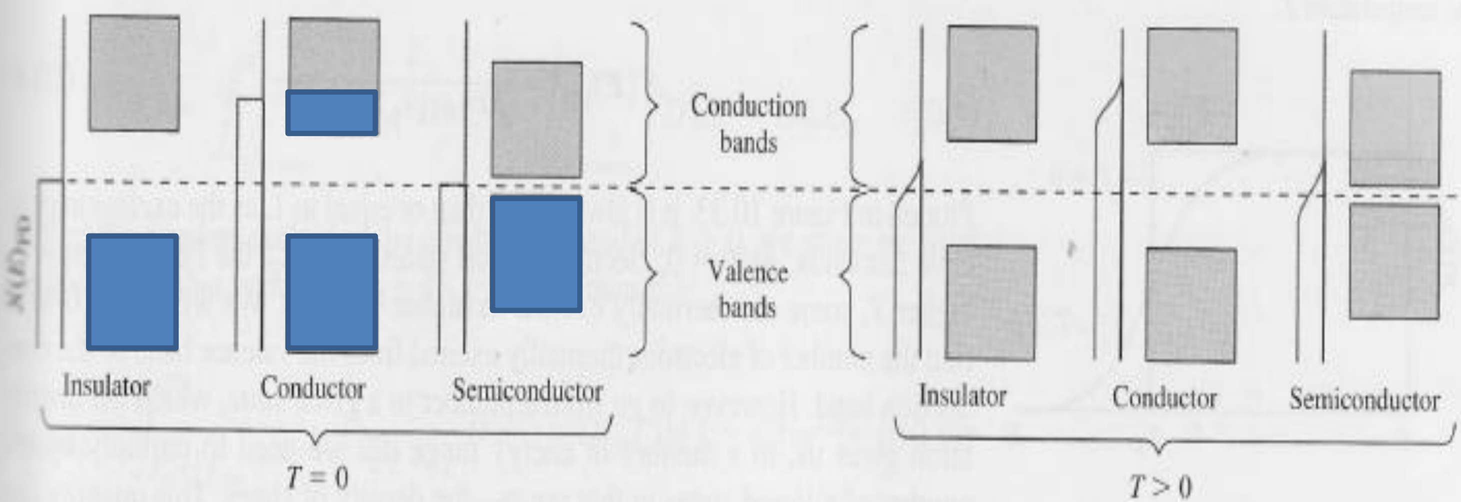
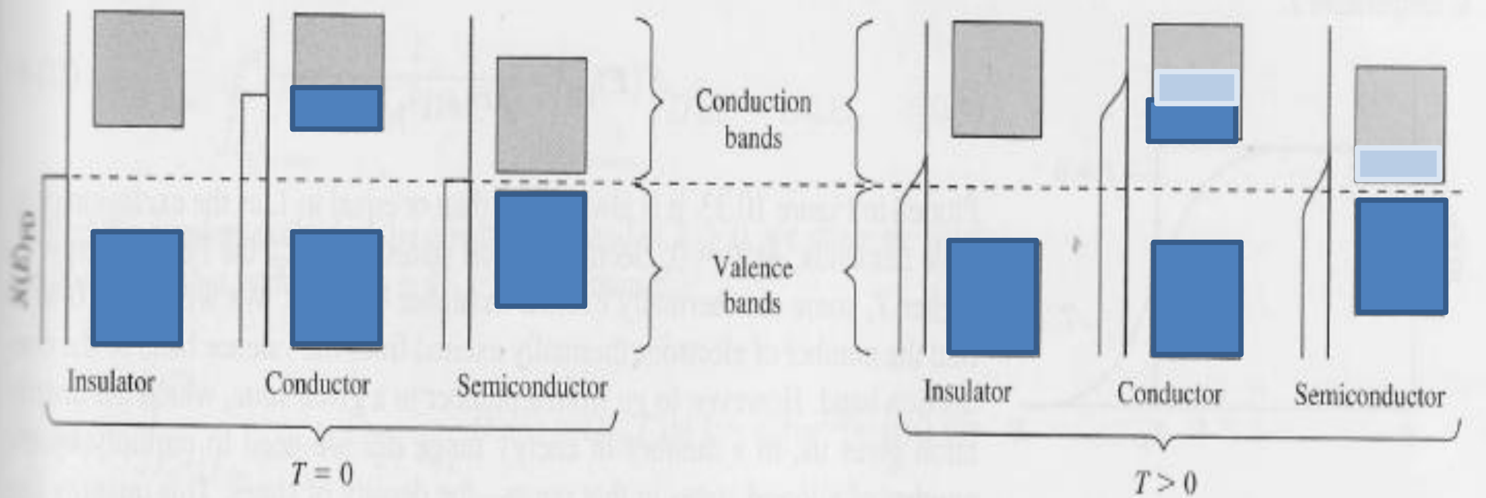
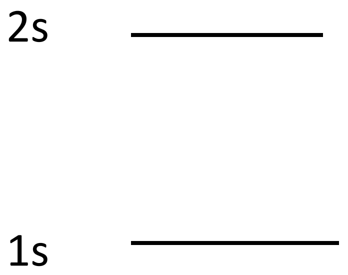


Figure 10.32 Band filling for an insulator, a conductor, and a semiconductor at zero and nonzero temperature. When $T > 0$, a semiconductor will have some electrons in the conduction band.



Lithium (Li)



Single Li Atom Energy Levels

Li Solid Energy Levels

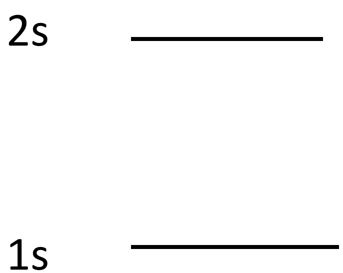
Lithium (Li): $1s_2 2s_1$

Single Li Atom Energy Levels

Li Solid Energy Levels

Conductor

Beryllium (Be): $1s_2 2s_2$

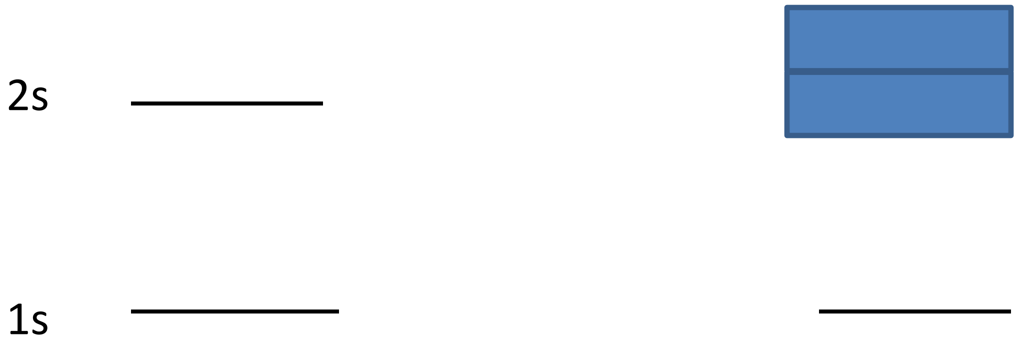


Single Be Atom Energy Levels

Be Solid Energy Levels

Conductor

Beryllium (Be): $1s_2 2s_2$

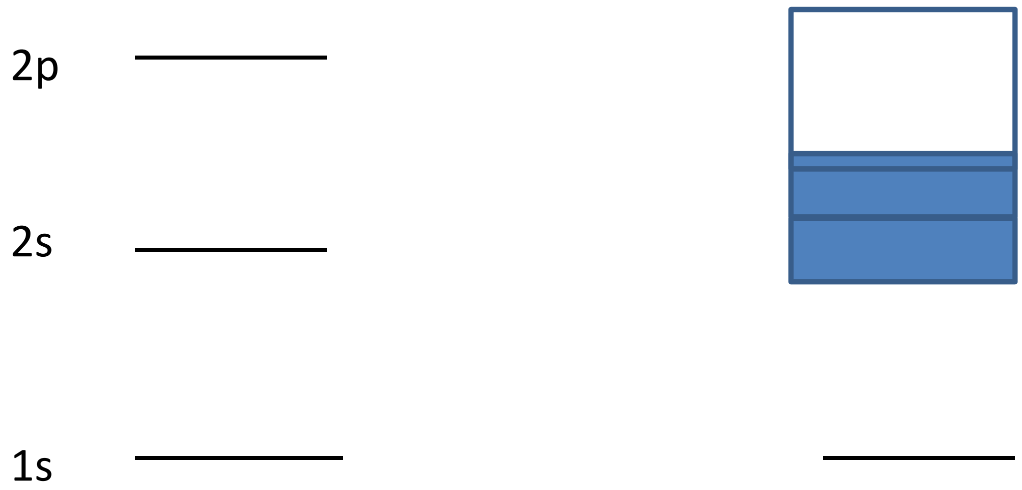


Single Be Atom Energy Levels

Be Solid Energy Levels

Conductor

Beryllium (Be): $1s_2 2s_2$

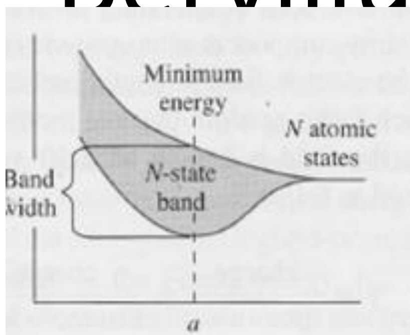


Single Be Atom Energy Levels

Be Solid Energy Levels

Conductor

Berylliur

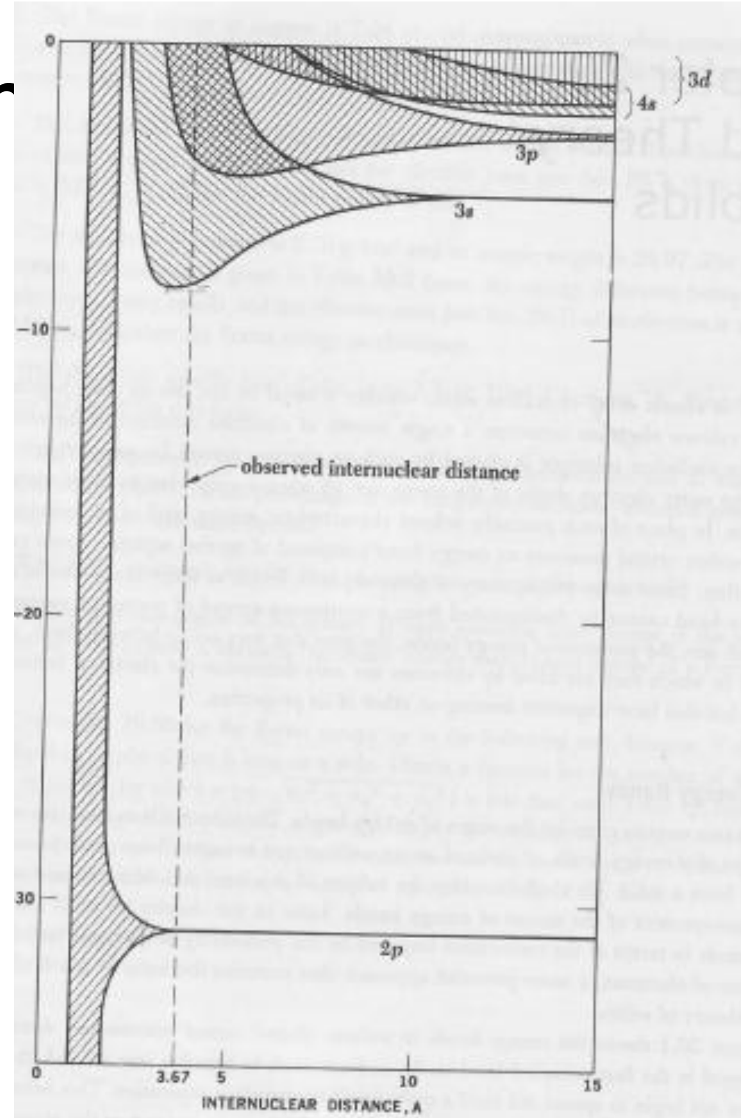


2p

2s

1s

Single Be Atom Energy Levels

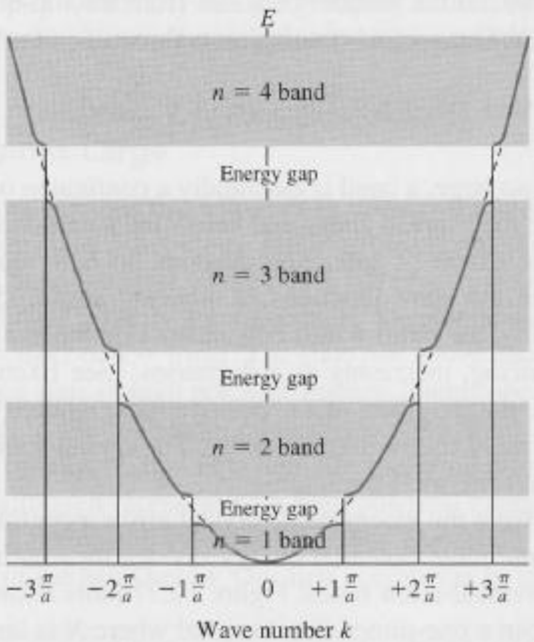


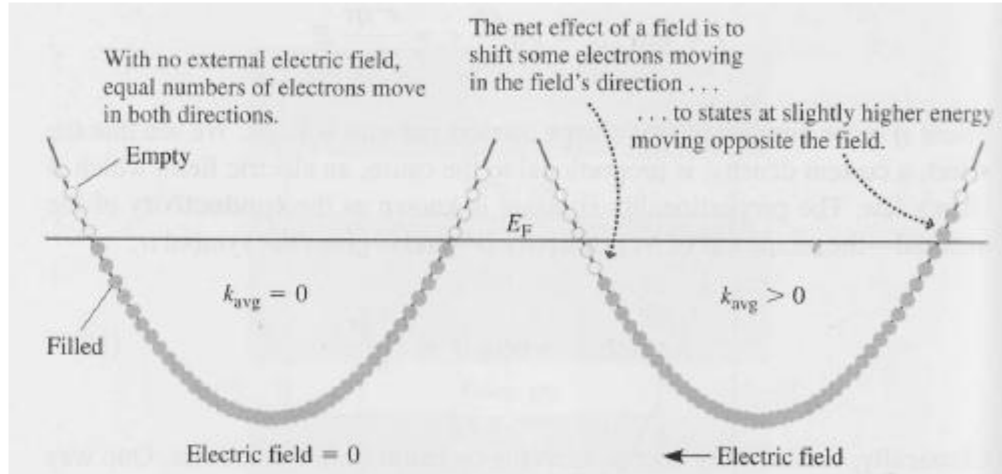
Be Solid Energy Levels

Conductor

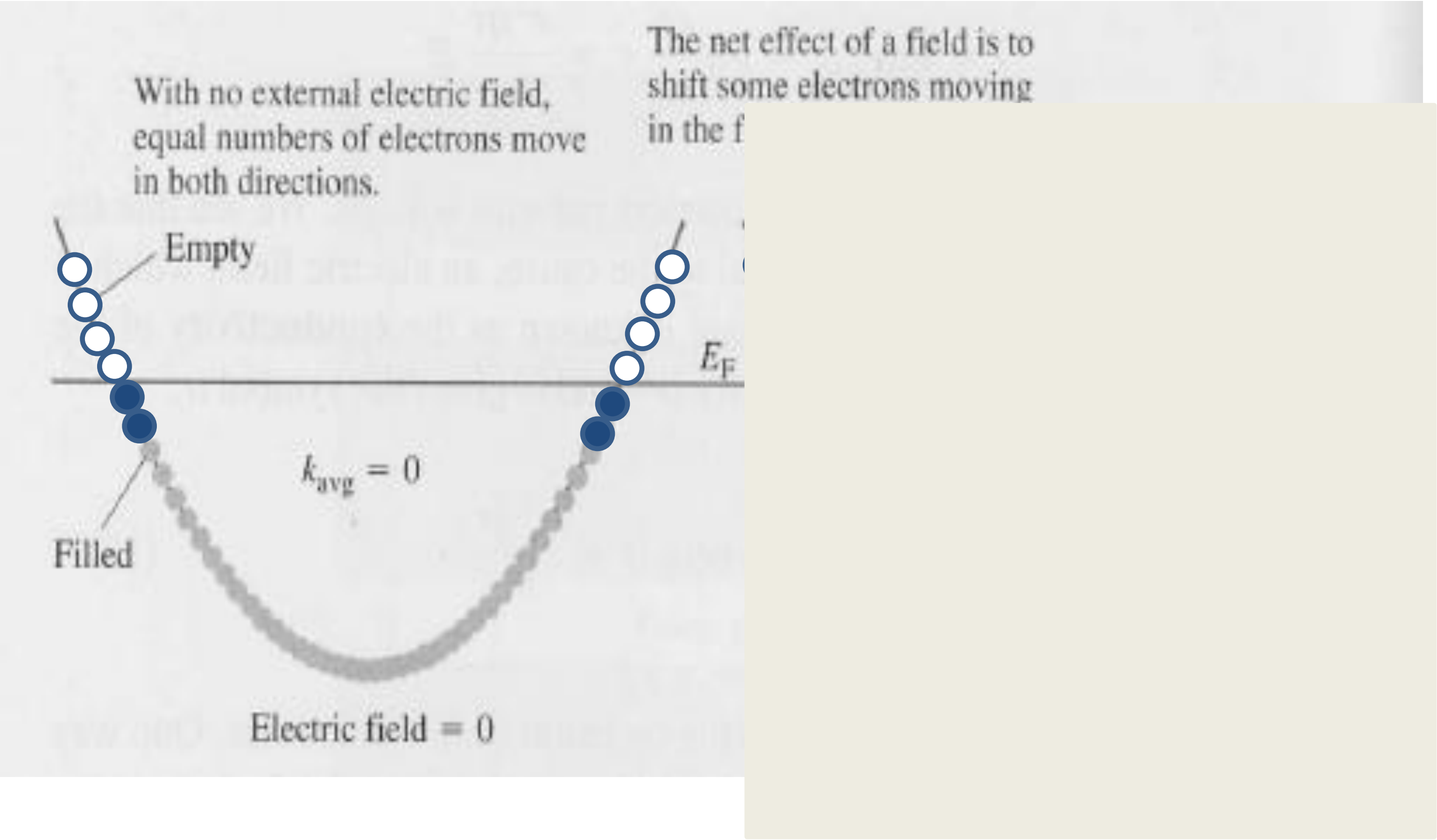
Conduction

Figure 10.27 Bands and gaps in a one-dimensional crystal.



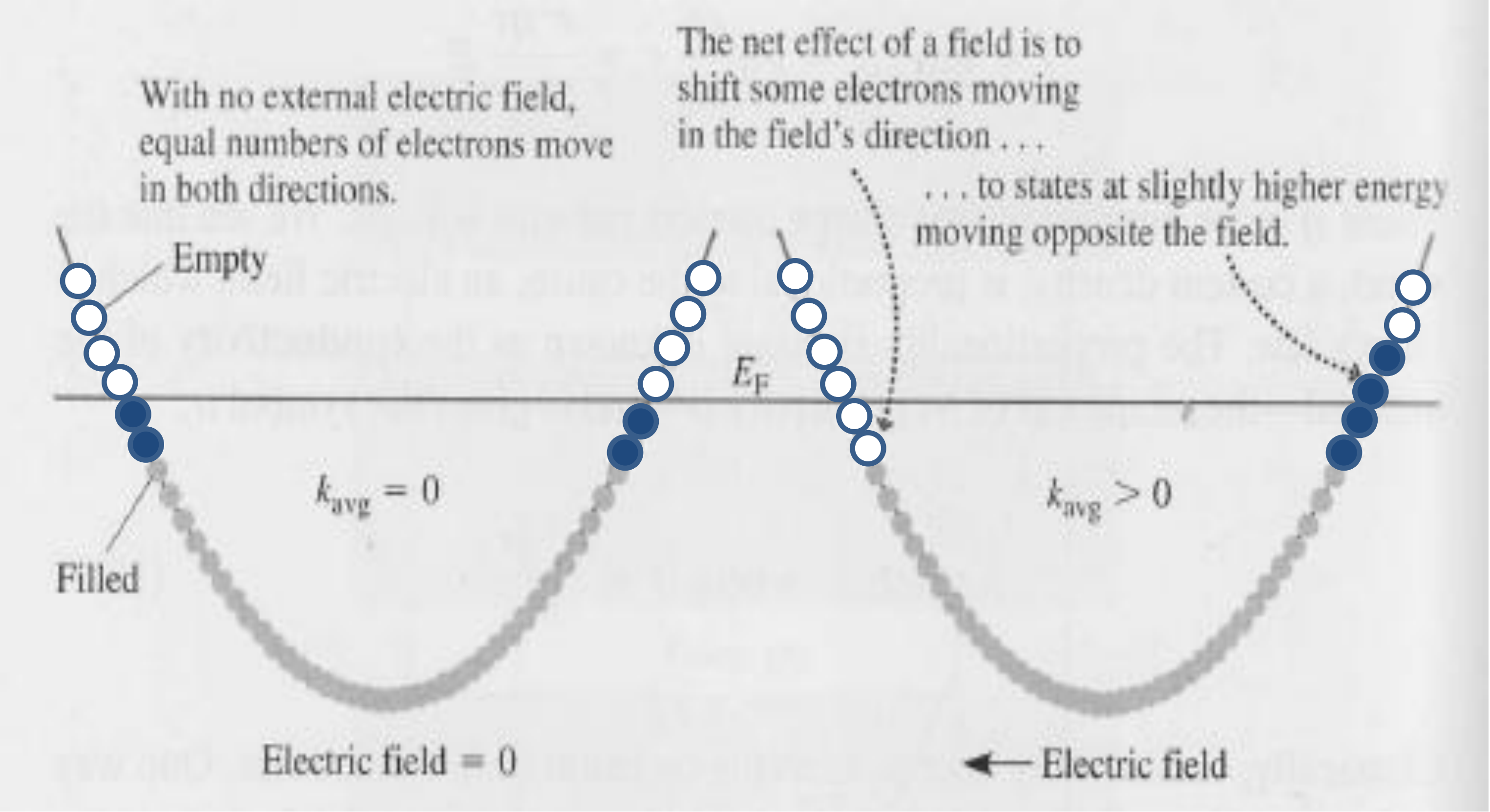


Conduction



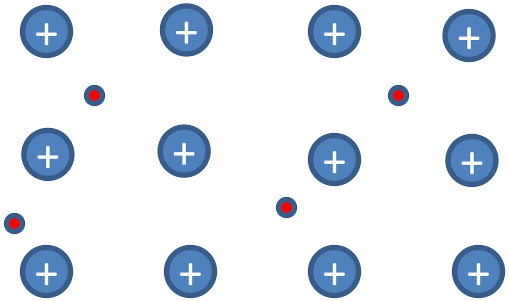
The net effect of a field is to
shift some electrons moving
in the f
With no external electric field,
equal numbers of electrons move
in both directions.
Empty
Filled
E_F
$k_{avg} = 0$
Electric field = 0

Conduction



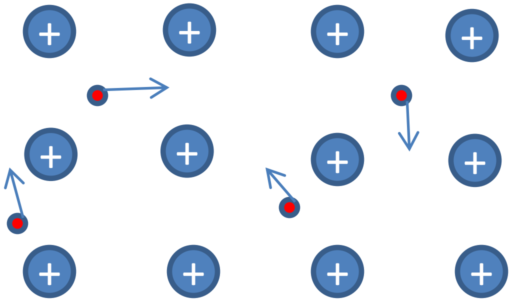
With no external electric field, equal numbers of electrons move in both directions.
The net effect of a field is to shift some electrons moving in the field's direction . . .
. . . to states at slightly higher energy moving opposite the field.
Empty
E_F
$k_{avg} = 0$
$k_{avg} > 0$
Filled
Electric field = 0
Electric field

Drift Velocity



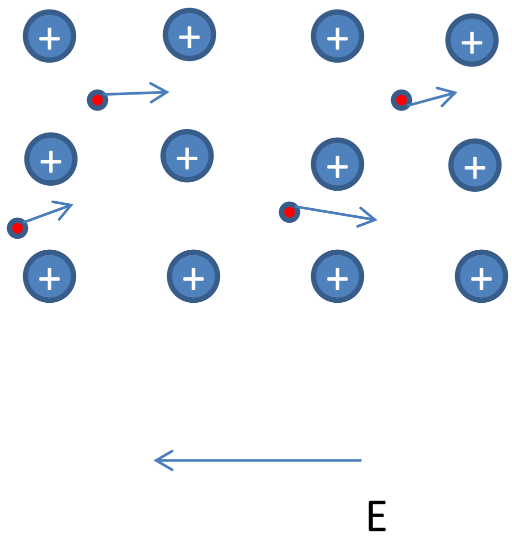
Drift Velocity

No electric field: No net velocity among free electrons



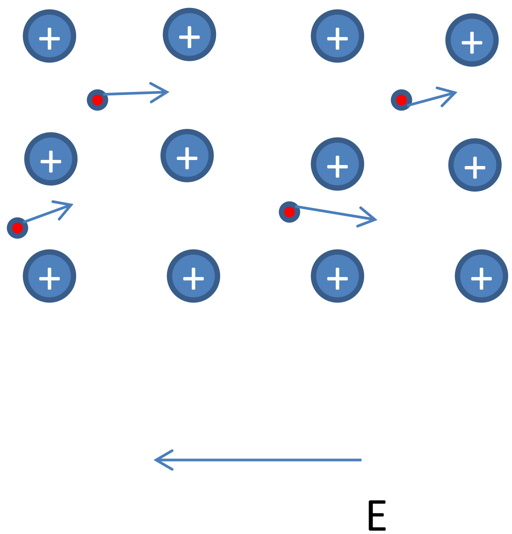
Drift Velocity

Yes electric field: Net velocity (=drift velocity) will emerge.



$$v_{drift} = \frac{eE}{m_e}\tau$$

Drift Velocity



Yes electric field: Net velocity (=drift velocity) will emerge.

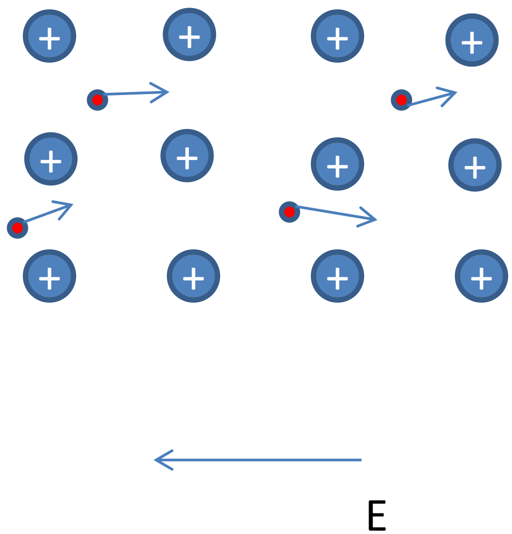
$$v_{drift} = \frac{eE}{m_e}\tau$$

Current density (j)

$$j \equiv \frac{charge}{time \text{ x } area} = \frac{charge}{distance \text{ x } area}\,\frac{distance}{time}$$

Drift Velocity

Yes electric field: Net velocity (=drift velocity) will emerge.



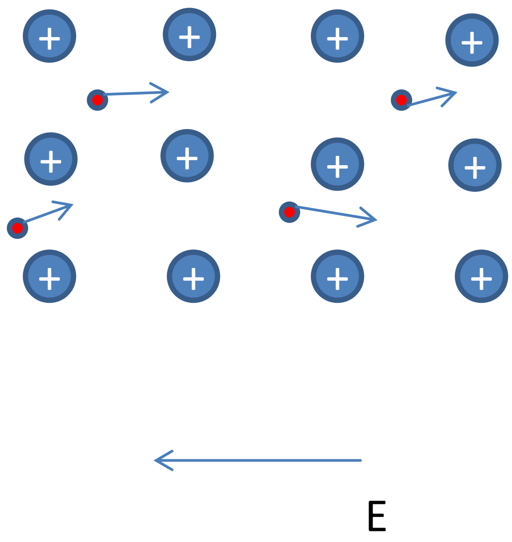
$$v_{drift} = \frac{eE}{m_e}\tau$$

Current density (j)

$$j \equiv \frac{charge}{time \text{ x } area} = \frac{charge}{distance \text{ x } area}\frac{distance}{time}$$

$$= \frac{e \text{ x } No.of\ electrons}{volume} v_{drift} = eNv_{drift}$$

Drift Velocity



Yes electric field: Net velocity (=drift velocity) will emerge.

$$v_{drift} = \frac{eE}{m_e}\tau$$

Current density (j)

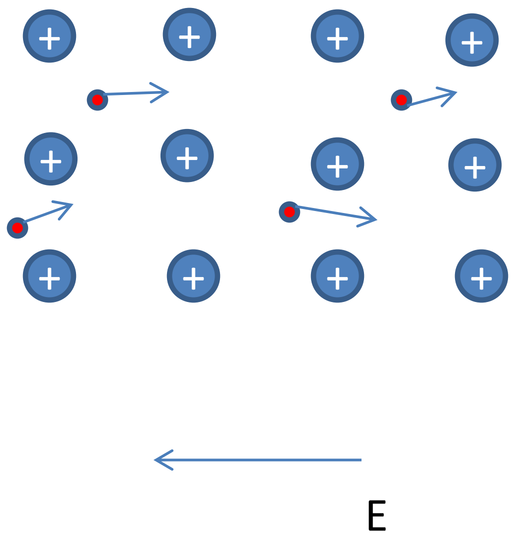
$$j \equiv \frac{charge}{time \text{ x } area} = \frac{charge}{distance \text{ x } area}\frac{distance}{time}$$

$$= \frac{e \text{ x } No.of\ electrons}{volume}v_{drift} = eNv_{drift}$$

$$j = eN\frac{eE}{m_e}\tau = e^2\frac{N\tau}{m_e}E$$

Drift Velocity

Yes electric field: Net velocity (=drift velocity) will emerge.



$$v_{drift} = \frac{eE}{m_e}\tau$$

Current density (j)

$$j \equiv \frac{charge}{time\ x\ area} = \frac{charge}{distance\ x\ area}\frac{distance}{time}$$

$$= \frac{e\ x\ No.of\ electrons}{volume} v_{drift} = eNv_{drift}$$

$$j = eN\frac{eE}{m_e}\tau = e^2\frac{N\tau}{m_e}E$$

Conductivity

Silver Example

Density of Silver: $10.5 \text{ x } 10^3 \ kg/m^3$

$$density = \frac{mass\ (= N \text{ x } mass\ of\ one\ Silver\ atom)}{volume\ (= a^3)}$$

$$a = \left(\frac{N \text{ x } mass\ of\ one\ Silver\ atom}{density}\right)^{\frac{1}{3}} = 2.57 \text{ x } 10^{-10}\ m$$

Silver Example

Density of Silver: $10.5 \text{ x } 10^3 \ kg/m^3$

$$density = \frac{mass\ (= N \text{ x } mass\ of\ one\ Silver\ atom)}{volume\ (= a^3)}$$

$$a = \left(\frac{N \text{ x } mass\ of\ one\ Silver\ atom}{density}\right)^{\frac{1}{3}} = 2.57 \text{ x } 10^{-10}\ m$$

Velocity of electrons associated with E_F

Fermi energy in silver = 5.5 eV

$$E_F = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2E_F}{m}} = 1.39\ x\ 10^6\ m/s$$

Silver Example

Density of Silver: $10.5 \text{ x } 10^3 \; kg/m^3$

$$density = \frac{mass\ (= N \text{ x } mass\ of\ one\ Silver\ atom)}{volume\ (= a^3)}$$

$$a = \left(\frac{N \text{ x } mass\ of\ one\ Silver\ atom}{density}\right)^{\frac{1}{3}} = 2.57 \text{ x } 10^{-10}\ m$$

Velocity of electrons associated with E_F

Fermi energy in silver = 5.5 eV

$$E_F = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2E_F}{m}} = 1.39\ x\ 10^6\ m/s$$

Collision time

$$\tau = \frac{a}{v} = 1.85 \text{ x } 10^{-16}\ sec$$

Silver Example

Density of Silver: $10.5 \times 10^3 \ kg/m^3$

$$density = \frac{mass \ (= N \times mass \ of \ one \ Silver \ atom)}{volume \ (= a^3)}$$

$$a = \left(\frac{N \times mass \ of \ one \ Silver \ atom}{density}\right)^{\frac{1}{3}} = 2.57 \times 10^{-10} \ m$$

Velocity of electrons associated with E_F

Fermi energy in silver = 5.5 eV

$$E_F = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2E_F}{m}} = 1.39 \times 10^6 \ m/s$$

Collision time

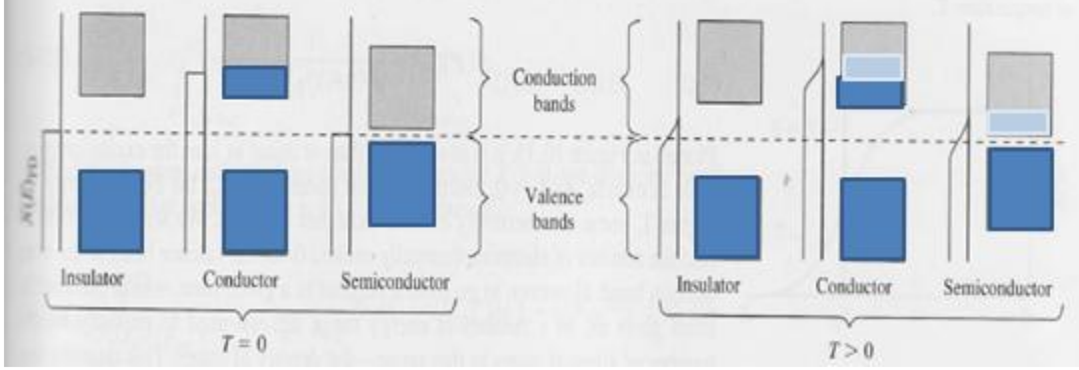
$$\tau = \frac{a}{v} = 1.85 \times 10^{-16} \ sec$$

Conductivity

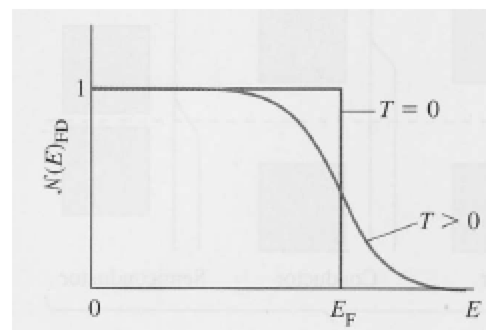
$$\sigma = e^2 \frac{N\tau}{m_e} = 3 \times 10^5 \ \Omega^{-1} m^{-1}$$

This is much smaller!

Figure 10.32 Band filling for an insulator, a conductor, and a semiconductor at zero and nonzero temperature. When $T > 0$, a semiconductor will have some electrons in the conduction band.



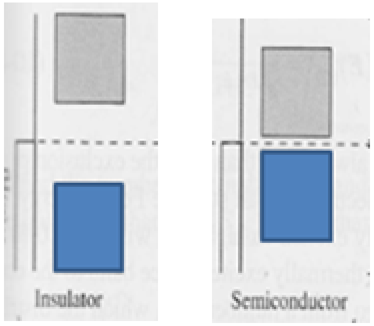
$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1}$$



$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \qquad D(E) = \mathrm{D}$$

$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E) D(E) dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot D dE = D \Delta E_{Valence}$$



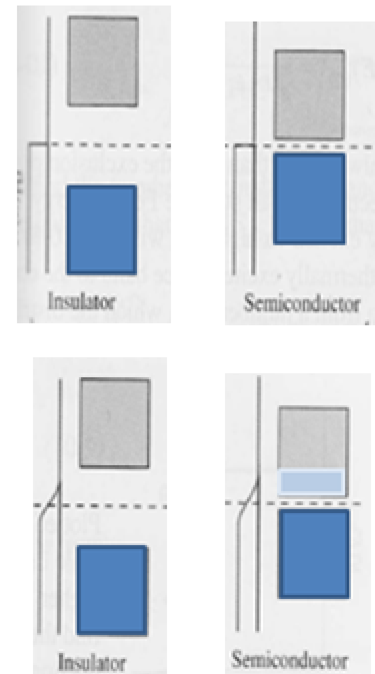
$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \qquad D(E) = \mathrm{D}$$

$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot DdE = D\Delta E_{Valence}$$

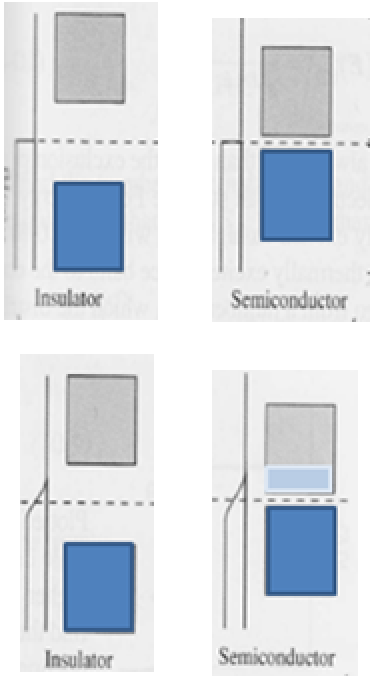
$$N_{Excited} = \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT} + 1} dE = Dk_BT\ln(1 + e^{-E_{gap}/2k_BT}) \sim Dk_BTe^{-E_{gap}/2k_BT}$$



$\ln(1+x) \sim x$ *when* x *is small*

$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT}+1} \qquad D(E) = \mathrm{D}$$



$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E)D(E)dE$$

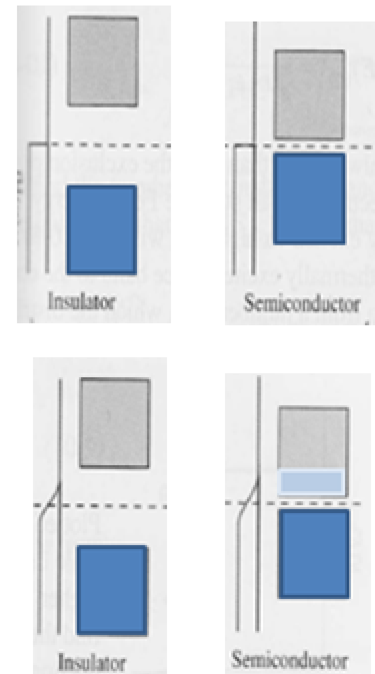
$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot D dE = D\Delta E_{Valence}$$

$$N_{Excited} = \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT}+1} dE = Dk_BT\ln(1+e^{-E_{gap}/2k_BT}) \sim Dk_BTe^{-E_{gap}/2k_BT}$$

$$\frac{N_{Excited}}{N_{Valence}} = \frac{Dk_BTe^{-E_{gap}/2k_BT}}{D\Delta E_{Valence}} = \frac{k_BT}{\Delta E_{Valence}} e^{-E_{gap}/2k_BT}$$

$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \qquad D(E) = \mathrm{D}$$



$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E) D(E) dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot D dE = D \Delta E_{Valence}$$

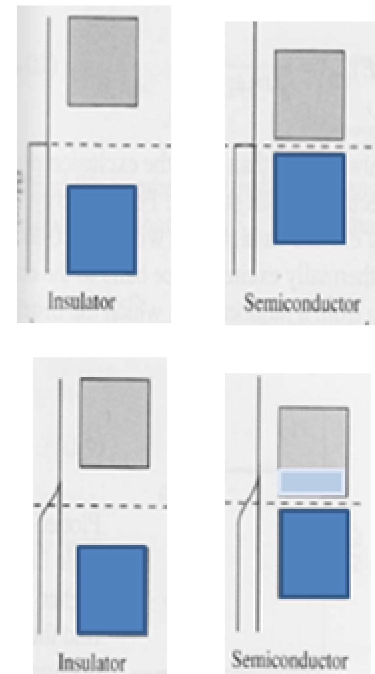
$$N_{Excited} = \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E) D(E) dE$$

$$= \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT} + 1} dE = D k_B T \ln(1 + e^{-E_{gap}/2k_BT}) \sim D k_B T e^{-E_{gap}/2k_BT}$$

$$\frac{N_{Excited}}{N_{Valence}} = \frac{D k_B T e^{-E_{gap}/2k_BT}}{D \Delta E_{Valence}} = \underbrace{\frac{k_B T}{\Delta E_{Valence}}}_{0.0026} e^{-E_{gap}/2k_BT}$$

0.0026

$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \qquad D(E) = \mathrm{D}$$



$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot DdE = D\Delta E_{Valence}$$

$$N_{Excited} = \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT} + 1} dE = Dk_BT\ln(1 + e^{-E_{gap}/2k_BT}) \sim Dk_BTe^{-E_{gap}/2k_BT}$$

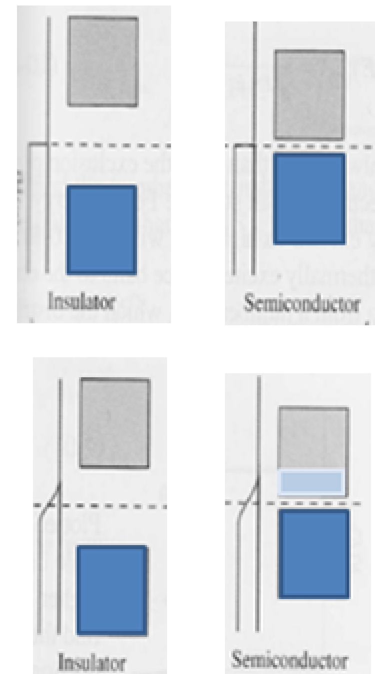
$$\frac{N_{Excited}}{N_{Valence}} = \frac{Dk_BTe^{-E_{gap}/2k_BT}}{D\Delta E_{Valence}} = \frac{k_BT}{\Delta E_{Valence}} e^{-E_{gap}/2k_BT}$$

Insulator band gap: 5 eV

Semiconductor band gap: 1 eV

Energy band size~10 eV, k_BT at 300K = 0.026eV

$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \qquad D(E) = \mathrm{D}$$



$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot DdE = D\Delta E_{Valence}$$

$$N_{Excited} = \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E)D(E)dE$$

$$= \int_{E_F+\frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_BT} + 1} dE = Dk_BT\ln(1 + e^{-E_{gap}/2k_BT}) \sim Dk_BTe^{-E_{gap}/2k_BT}$$

$$\frac{N_{Excited}}{N_{Valence}} = \frac{Dk_BTe^{-E_{gap}/2k_BT}}{D\Delta E_{Valence}} = \frac{k_BT}{\Delta E_{Valence}} e^{-E_{gap}/2k_BT}$$

0.0026

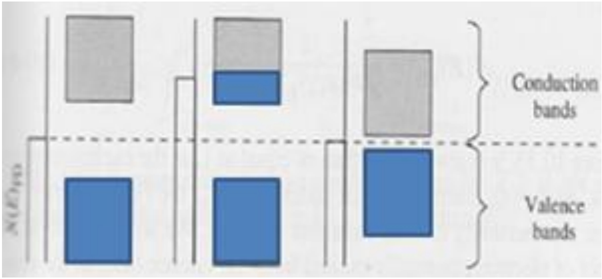
Insulator band gap: 5 eV

~ 10^{-42}

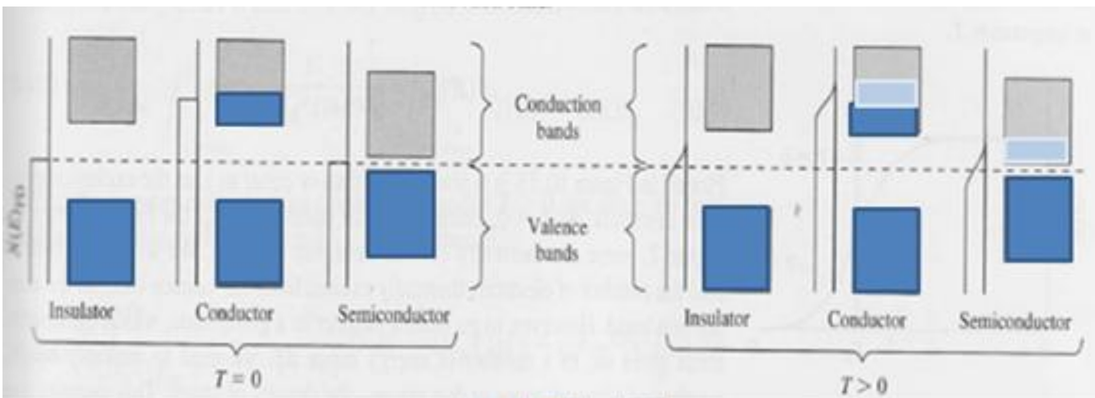
Semiconductor band gap: 1 eV

~ 10^{-8}

Energy band size~10 eV, k_BT at 300K = 0.026eV



At T = 0


Conduction bands
Valence bands
$N(E)_{FD}$
Insulator
Conductor
Semiconductor
$T = 0$
Insulator
Conductor
Semiconductor
$T > 0$

At T = 0

T > 0

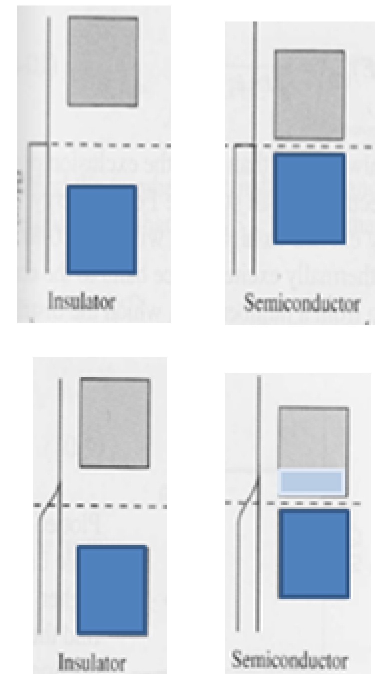
$$\mathcal{N}(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \qquad D(E) = \mathrm{D}$$

$$N_{Valence} = \int_{E_{valence-bottom}}^{E_{valence-top}} \mathcal{N}(E) D(E) dE$$

$$= \int_{E_{valence-bottom}}^{E_{valence-top}} 1 \cdot D dE = D \Delta E_{Valence}$$

$$N_{Excited} = \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \mathcal{N}(E) D(E) dE$$

$$= \int_{E_F + \frac{1}{2}E_{gap}}^{\infty} \frac{D}{e^{(E-E_F)/k_B T} + 1} dE = D k_B T \ln(1 + e^{-E_{gap}/2k_B T}) \sim D k_B T e^{-E_{gap}/2k_B T}$$



T=0

T > 0

$$\frac{N_{Excited}}{N_{Valence}} = \frac{D k_B T e^{-E_{gap}/2k_B T}}{D \Delta E_{Valence}} = \frac{k_B T}{\Delta E_{Valence}} e^{-E_{gap}/2k_B T}$$

0.0026 eV

Insulator: 5 eV

$\sim 10^{-42}$

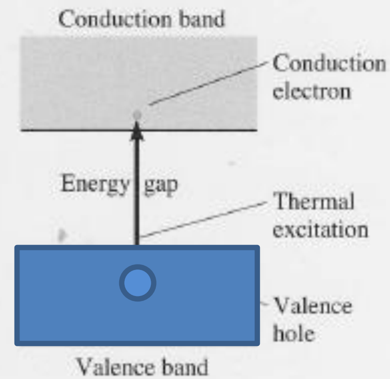
Semiconductor: 1 eV

$\sim 10^{-8}$

Two types of charge carriers

Conduction band electrons

Figure 10.35 Thermal excitation creating a pair of charge carriers.

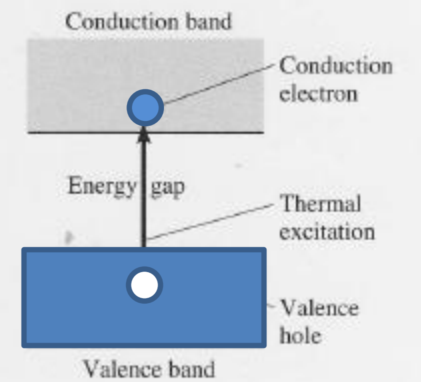


Two types of charge carriers

Conduction band electrons

Valence band holes

Figure 10.35 Thermal excitation creating a pair of charge carriers.



Two types of charge carriers

Conduction band electrons

Valence band holes

- created by electrons left the valence band at T > 0
-Behave like positive charge carriers
-Are free to move in the valence band
-Are always found near the top of the valence band

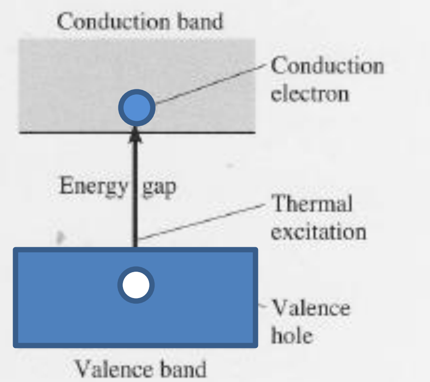


Figure 10.35 Thermal excitation creating a pair of charge carriers.

Two types of charge carriers

Conduction band electrons

Valence band holes

- created by electrons left the valence band at T > 0
-Behave like positive charge carriers
-Are free to move in the valence band
-Are always found near the top of the valence band

Figure 10.35 Thermal excitation creating a pair of charge carriers.

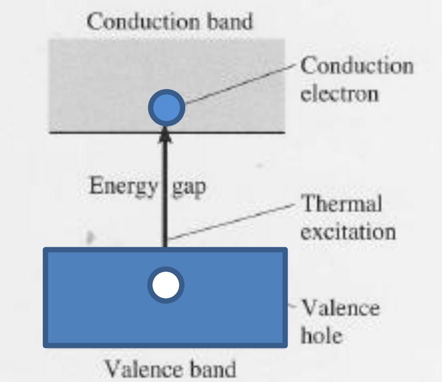
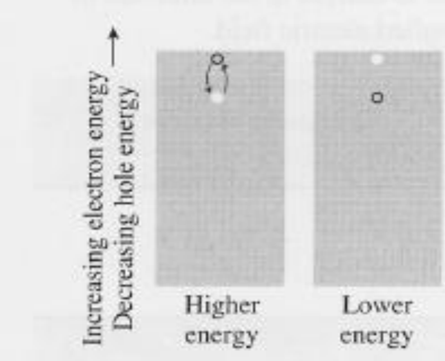


Figure 10.36 Holes float.



Effective Mass

m_{eff} : the ratio between external force and acceleration

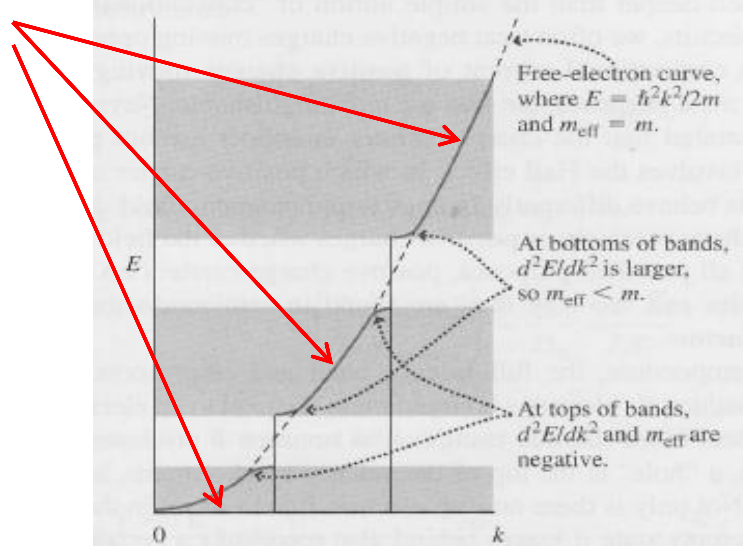
$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

For free electrons, $E = \frac{\hbar^2 k^2}{2m}$

Electrons in the middle of the band

m_{eff}=

Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.



Effective Mass

m_{eff} : the ratio between external force and acceleration

$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

For free electrons, $E = \frac{\hbar^2 k^2}{2m}$

Electrons in the middle of the band

$m_{eff}=$ m

At the top of each band

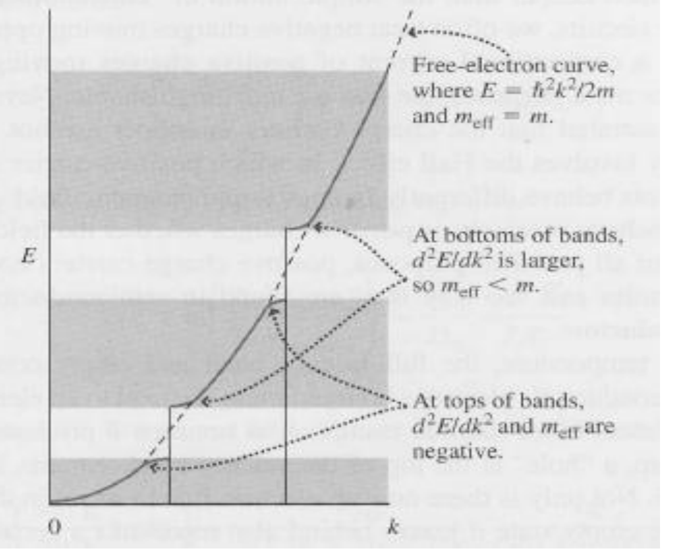


Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.

Effective Mass

$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

m_{eff} : the ratio between external force and acceleration

For free electrons, $$E = \frac{\hbar^2 k^2}{2m}$$

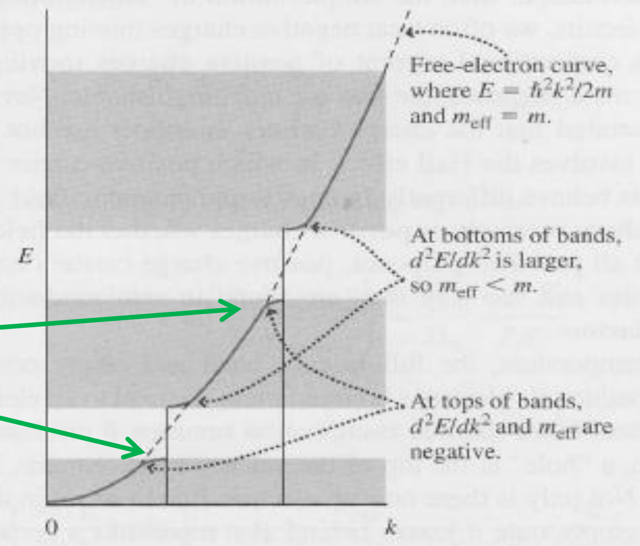
Electrons in the middle of the band

m_{eff}= m

At the top of each band

$$\frac{d^2E}{dk^2} < 0 \quad \text{thus}, \; m_{eff} < 0$$

Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.



Effective Mass

$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

m_{eff} : the ratio between external force and acceleration

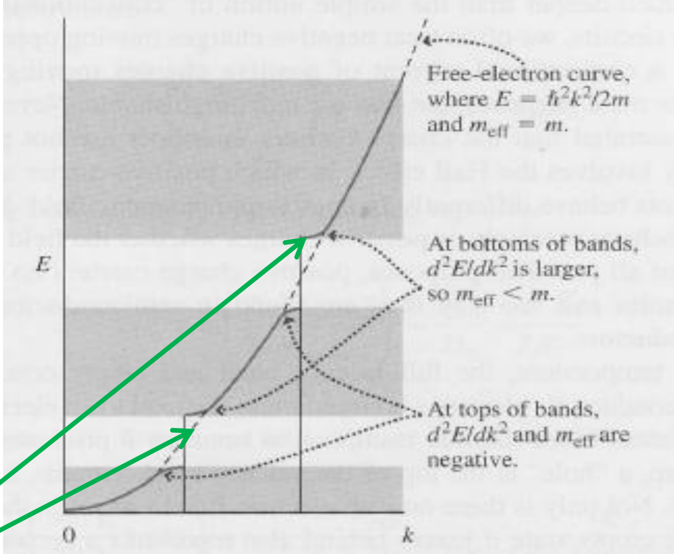


Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.

For free electrons, $E = \frac{\hbar^2 k^2}{2m}$

Electrons in the middle of the band

$m_{eff} = m$

At the top of each band

$$\frac{d^2E}{dk^2} < 0 \quad \text{thus}, \; m_{eff} < 0$$

At the bottom of each band

$$\frac{d^2E}{dk^2}\Big|_{bottom} > \frac{d^2E}{dk^2}\Big|_{free\ electron} \qquad m_{eff} < m$$

Effective Mass

m_{eff} : the ratio between external force and acceleration $$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

For free electrons, $$E = \frac{\hbar^2 k^2}{2m}$$

$m_{eff} = m$

At the top of each band

$$\frac{d^2E}{dk^2} < 0 \quad \text{thus}, \; m_{eff} < 0$$

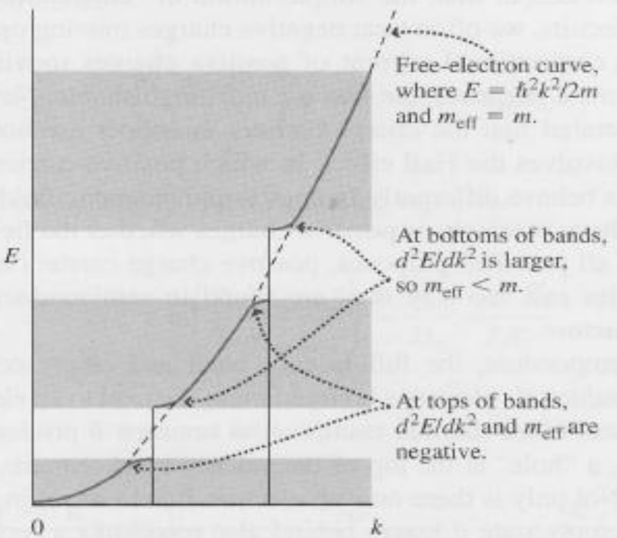


Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.

Use Holes to get the positive effective mass!!

At the bottom of each band

$$\frac{d^2E}{dk^2}\Big|bottom > \frac{d^2E}{dk^2}\Big|free\ electron \qquad m_{eff} < m$$

Current in an E field

Figure 10.38 In a semiconductor at $T > 0$, both holes and electrons contribute to current in the direction of an applied electric field.

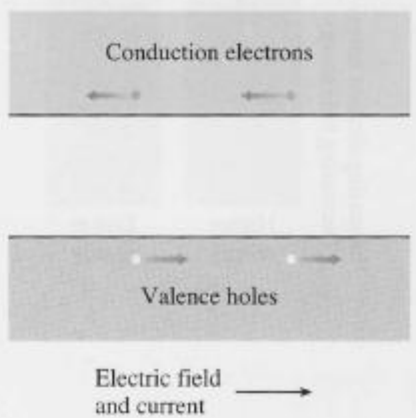


Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.