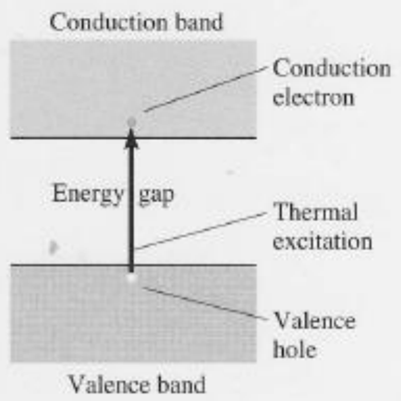
PH102, 2014W, Lecture Notes: February 18 & 20, Tues & Thurs, Class 13 & 14
Solid State: Semiconductor Theory, Devices, and Superconductivity

Semiconductor Theory

Two types of charge carriers:

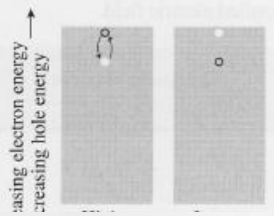
- **Electrons** in the conduction band
- **Holes** in the valence band.
    - Holes are created by electrons in the valence band move to the conduction band when T>0. See the diagram on the right. Holes mean absence of electrons in the band otherwise filled.
    - Holes behave as if they carry positive charges.
    - Holes are free to move around in the valence band while the movement of electrons in the valence band is limited to the emptied state that is available.

Figure 10.35 Thermal excitation creating a pair of charge carriers.



- The lower energy requirement make holes float near the top of the valence band. See Figure 10.36.

Figure 10.36 Holes float.

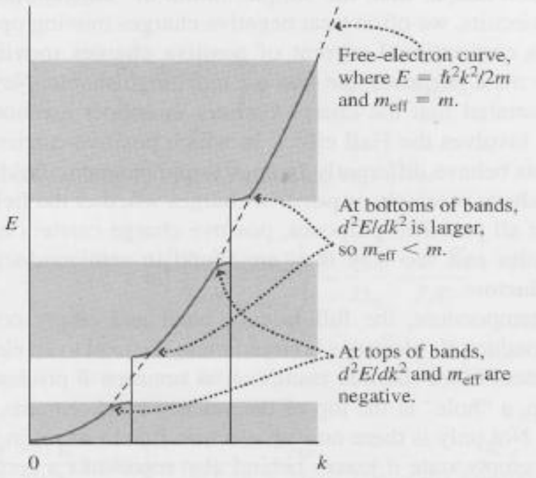


Effective mass: the ratio of the external force that we can control to the acceleration of electron.

$$m_{eff} = \hbar^2 \left(\frac{d^2E}{dk^2}\right)^{-1}$$

- Figure 10.37 shows energy bands over k. The parabolic curve represents the free electron case:
    - $E = \frac{\hbar^2 k^2}{2m}$
- At the bottom of each energy band
    - $\frac{d^2E}{dk^2}\Big|bottom > \frac{d^2E}{dk^2}\Big|free\ electron$
    - $m_{eff} < m$
- At the top of each energy band

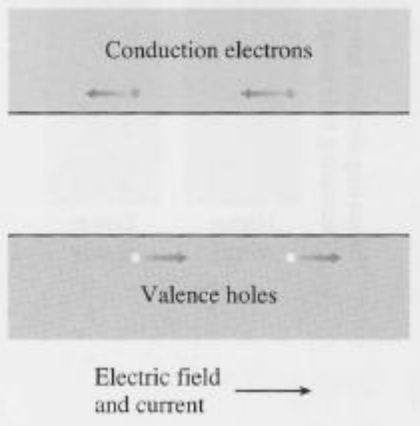
Figure 10.37 Energy versus wave number for electrons in a one-dimensional crystal. An electron's effective mass depends on what state it occupies.



  - $\frac{d^2E}{dk^2} < 0$ thus , $m_{eff} < 0$

- In a conductor, electrons in the conduction band fill energy states around the middle energy band. Therefore, electrons move like free particles of mass m.
- In a semiconductor, electrons fill to the top of the valence band where effective mass is negative. This means that electrons' acceleration is opposite to the external force. If we use holes, instead of electrons, then holes will have a positive effective mass, meaning holes will be accelerating in the same direction as the external force is applied.

Figure 10.38 In a semiconductor at $T > 0$, both holes and electrons contribute to current in the direction of an applied electric field.



Doping

- We can create semiconductors (called extrinsic semiconductors) by doping small amounts of impurities on a crystal. Typical doping would be one of every 10^5 atoms of the intrinsic semiconductors such as Silicon or germanium. This action creates
  - If we use n-type impurities (elements with 5 valence electrons, e.g. phosphorous or arsenic), there will be an extra electron per impurity atom that cannot occupy the valence band. Each impurity atom also has an additional positive charge. These two interact to create new energy levels (called donor states) which will be added just below the conduction band. Typical energy difference between donor states and the conduction band is .05 eV, which is easily executable. Since impurity atoms are sparse, the extra electrons do not form energy bands. Vacated donor states neither form holes nor act as positive charge carriers. In the conduction band, electrons from the donor states are far more abundant than those from electron-hole pairs that require a larger excitation energy. Therefore, in an n-type semiconductor, the majority charge carriers are conduction band electrons and the minority charge carriers are holes left by the excited electrons from the valence band.
  - If we use p-type impurities (elements with 3 valence electrons, e.g. aluminum or gallium), missing electrons create holes in the valence band which interact with the missing positive charges in the crystal. This adds new energy levels (acceptor states) above the valence band. Electrons in the valence band can be easily excited to acceptor levels, making holes in the valence band free to move. The majority charge carriers in a p-type semiconductor are holes in the valence band. The minority charge carriers are electrons in the conduction band that are excited from the valence band.

Figure 10.39 At $T = 0$, extra electrons in an n-type semiconductor occupy donor states. At $T > 0$, they easily become conduction electrons.

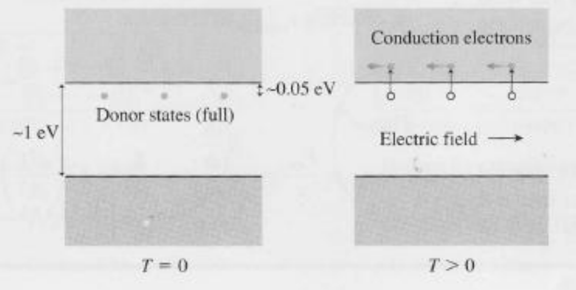
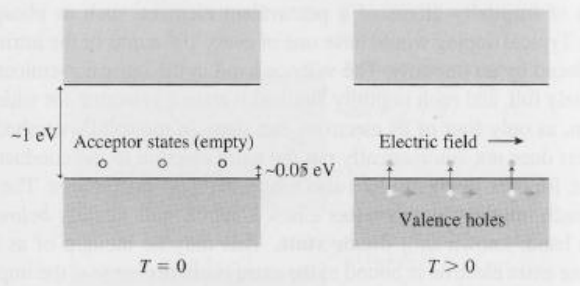


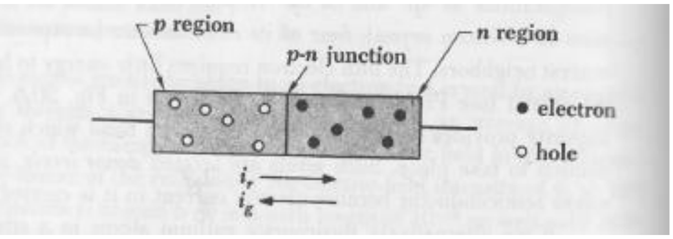
Figure 10.40 At $T = 0$, a p-type semiconductor has empty acceptor states, which at $T > 0$ are filled by electrons from the valence band, freeing holes.

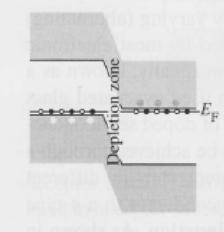


Semiconductor Devices

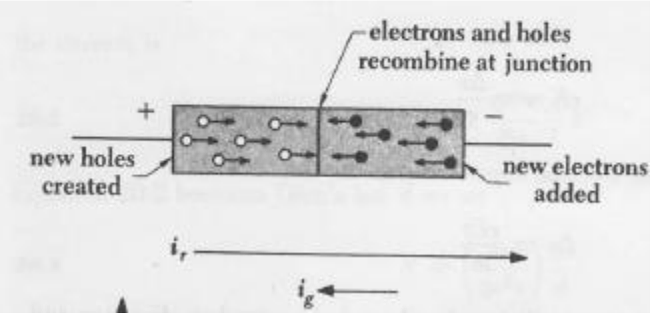
Diode:

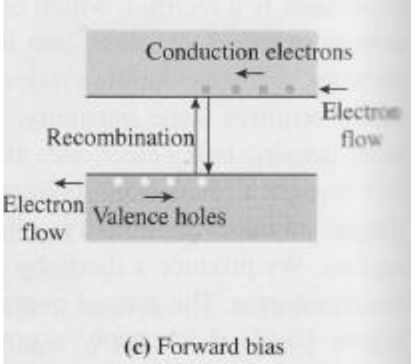
- Diode converts AC (alternating current) supplied by the power plant to DC (direct current) needed to run electronic devices.
- Diode is created by joining an n-type semiconductor to a p-type semiconductor. The area of contact is called a p-n junction.
- At thermal equilibrium (unbiased): a p-type and an n-type semiconductors are joined without an external potential difference. Electrons in the conduction band of the n-type semiconductor diffuse to the p-type side, raising all electron energies by repulsion in the p-type and lowering in the n-type until equilibrium is reached where $E_F$of both sides is the same. Note that electrons fill up to the donor levels and the acceptor levels in both types of semiconductors. As a result, the electron potential energy has changed to look like in the diagram below. Immediately near the p-n junction area called the depletion zone, there are no free charge carriers.





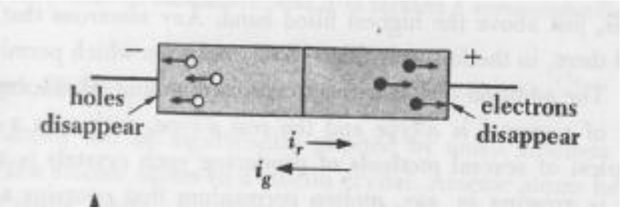
- Forward Bias (Applying potential difference with the p-type at the higher potential): free holes in the p-type and free electrons in the n-type flow toward the junction. When they meet they recombine. Current flows continuously as electrons kept added to the n-type and holes kept added to the p-type (valence electrons re removed). Forward-biasing the diode shifts the n-type conduction band upward, allowing conduction band electrons move to the p-type. So do the holes from the p-type to the n-type. Electrons and holes recombine, meaning the conduction electrons jump down to fill the valence holes. In this process, energy is lost to heat or to light. Therefore, the forward bias needs to shift the band gap for current to flow continuously. A diode of 1eV band needs 1V forward bias before it turns on.

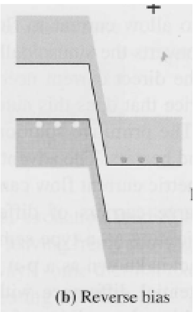




(c) Forward bias

- Reverse Bias (Applying potential difference with the n-type at the higher potential): when the n-type is at the higher potential, both holes and electrons move away from the junction. A region devoid of free charge carriers quickly forms, and current stops almost instantly. Applying a reverse bias makes the potential difference more pronounced. Note that in this condition, still minority charge carriers (electrons in the p-type conduction band and holes in the n-type valence band) would cross the junction. But the numbers for minority carriers that recombine are negligible.





(b) Reverse bias

Transistor

- Joining three extrinsic semiconductors such as npn or pnp.
- For a npn transistor, p the base, one n the emitter and the other n the collector.
    - The emitter-base diode is forward biased, so conduction electrons flow from the emitter into the base. These electrons are discouraged to combine with holes in the base because (1) the base is made small, (2) the base is reverse biased with the collector, and (3) conduction electrons are minority carriers in the base (p-type). So, most electrons move to the collector's conduction band. Only a small fraction of conduction electrons recombine in the base. The fraction of electrons that flows out the collector side is 100 times greater than that which flows out the base returning to the input circuit.

Figure 10.44 Bands and charge flow in a npn transistor.

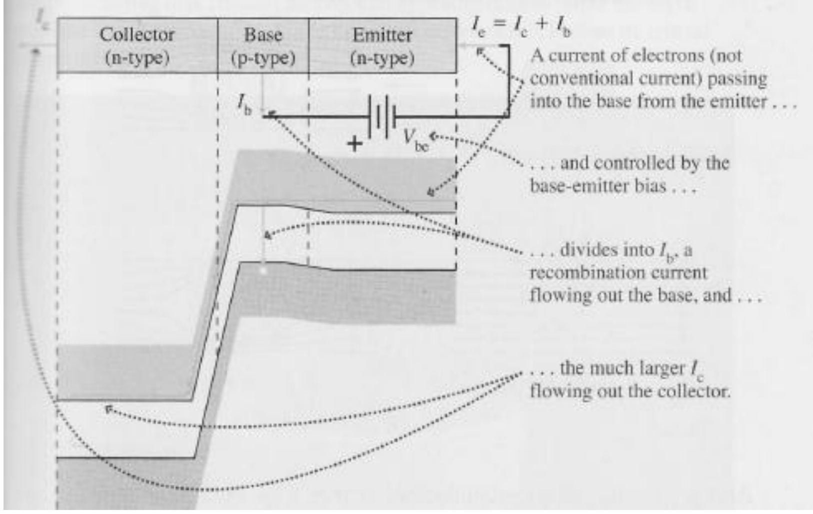
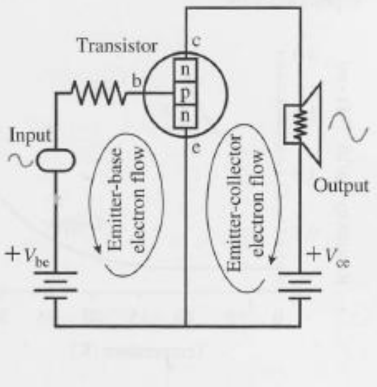


Figure 10.45 The elements of a transistor amplifier.



Superconductivity

Characteristics

- Perfect conductivity (no resistivity): Some material lose electrical resistance below its critical temperature, T_c, making current in the material flows indefinitely without applied voltage. See Figure 10.48. where Tin becomes a superconductor below 3.8 K while Copper maintains certain electrical resistance. 40% of natural elemental meals become superconductors at low temperatures.
- Perfect diamagnetism: At the superconducting stage, the material exclude magnetic field lines shown in Figure 10.49, even in the presence of an external B field. As a result, superconductors repel magnets known as the Meissner effect where a magnet floats above a superconducting material. If the external magnetic field becomes too strong, then the magnetic field can penetrate the material, making it not a superconductor any more. Such external magnetic field is known as the critical field, B_c, which depends on temperature, highest at T=0 and lowest at T_c.

Figure 10.48 Copper always has electrical resistance, while tin becomes a superconductor.

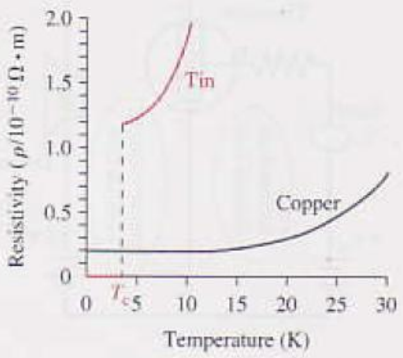
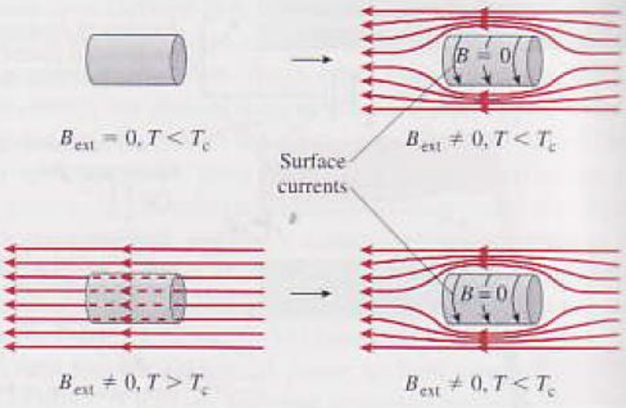


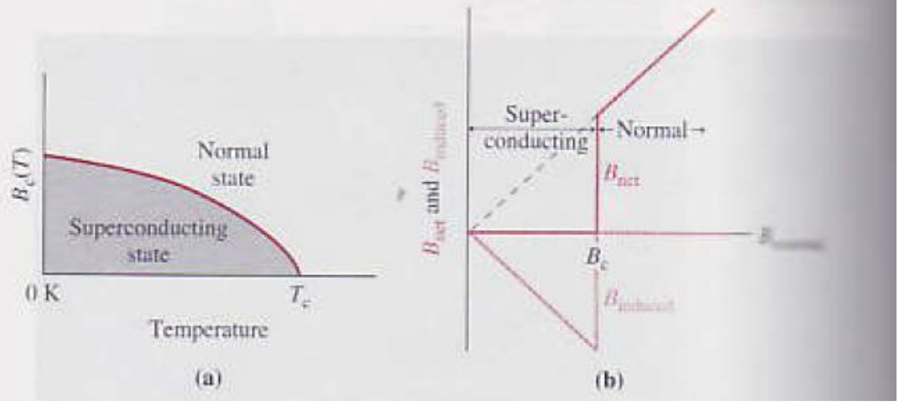
Figure 10.49 When it drops below its critical temperature, a superconductor expels magnetic field lines.



Type 1 and Type 2 Superconductors

- Type 1 Superconductors show a sharp transition of B_{ext}at B_c. Before B_c, there is no B field inside the material because B-external and B-induced fields are cancelled each other. After B_c, there is B field inside the material in a normal state. Superconducting elements tend to be Type 1; Their critical B fields are relatively low, 0.01-0.1 T; Their critical temperatures are relatively low, 1-9K. There is limited applicability.

Figure 10.51 A Type-I superconductor. (a) The critical field B_c decreases as $T \to T_c$. (b) When it is superconducting, the induced field perfectly opposes the external—the net field inside is 0.



- Type 2 Superconductors spontaneously form microscopic vortices shown in Figure 10.52. Vortices are superconducting regions of circulating current surrounding "tubes" known as normal cores that are not in the superconducting state and field lines pass through it. As the external B field increases, vortices become more dense, allowing more field lines to pass through. Figure 10.54 (a) indicates two critical B field lines. When the external B field is below B_{c1}, a type 2 superconductor acts like a type 1 superconductor (compare it with Figure 10.51). When the external B field is between $B_{c1}$and B_{c2}, a type 2 superconductor is in a vortex state when B field lines penetrate the normal cores but surrounding material is still in a superconducting state. As the external B field increases, more and more field lines penetrate the material through normal cores and the induced B field gradually vanishes. At B_{c2}, the material is in a normal state when the external B fields penetrate through the material and no induced fields exist that can weaken the external B field. Metallic compounds and alloys tend to be Type 2 superconductors; their critical B fields are 2 to 3 orders of magnitude higher than Type 1 superconductors; their critical temperatures are twice as much as Type 1 superconductors, around 20K.

Figure 10.52 Magnetic field lines passing through vortices in a type-II superconductor.

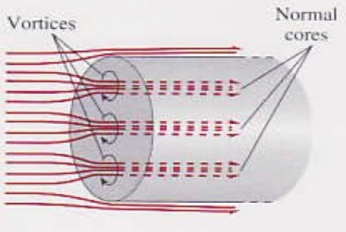


Figure 10.53 Vortices become more dense as field strength increases. (Both represent $B_{c1} < B_{external} < B_{c2}$. See Figure 10.54.)

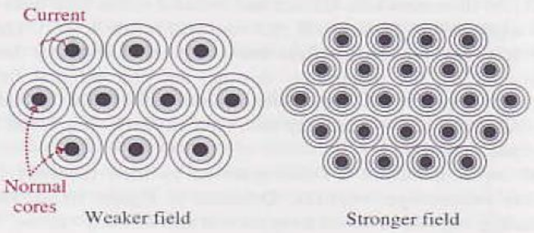
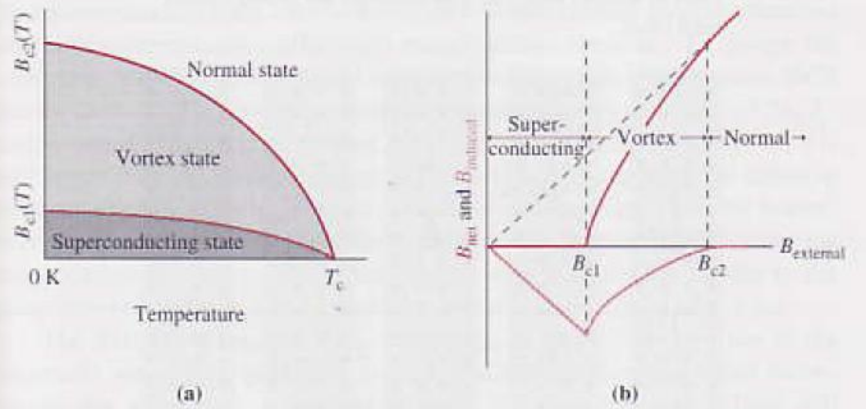


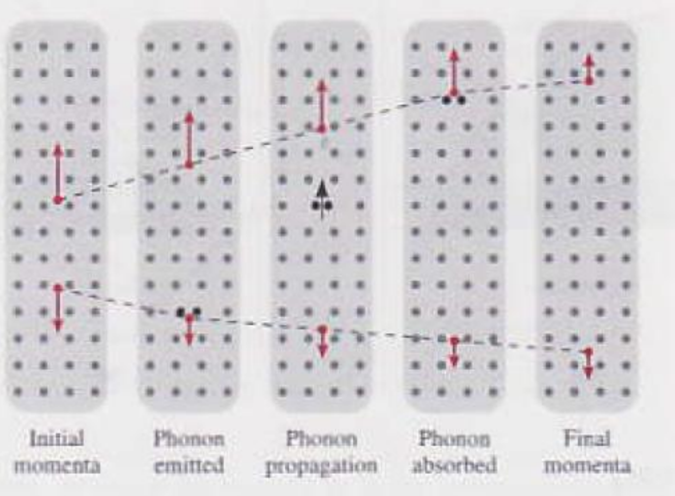
Figure 10.54 In a Type-II superconductor, an external field increasing from B_{c1} would penetrate through an increasing density of vortices, until at B_{c2} the material is normal.



- High Tc Superconductors were first found in ceramics such as $La_{2-x}Ba_xCuO_4$ where x represents the ratio of Ba atoms randomly replacing La atoms in the material. The critical temperatures of superconducting materials have been increasing from 30K to 150K. These High Tc superconductors are referred to as cuprates and possess the Type 2 characteristics. However, mechanisms in High Tc Superconductors cannot be explained with the BCS theory because, for example, they do not exhibit the isotope effect. A comprehensive theory is yet to emerge to explain High Tc superconductivity. Due to their high critical temperatures, they have greater utilities.

BCS Theory

- BCS theory explains superconducting mechanisms in Type 1 and Type 2.
- BCS theory is based on the coherent motion of the electrons in pairs called Cooper pairs that respond to the external magnetic field as one. This makes electrons scatter individually to create resistance.
- Superconductivity via Cooper pairs depends on interactions between the conduction electrons and the lattice of positive ions.
    - Lattice involvement: $T_c \propto (atomic\ mass\ of\ positive\ ions\ in\ the\ lattice)^{-1/2}$
    - elements that make superconductors at low temperatures are usually not good conductors at room temperatures. Thus at the superconducting state, the conduction electrons and the lattice should be doing something together.



- At low temperatures, two electrons in a solid can experience a net attraction. See Figure on the right. An electron moving through a solid can create a local distortion of the lattice, making a relatively higher concentration of positive charge distribution around positive ions. This can pull on the electron, slowing it down. At the same time, a phonon from the lattice is emitted and the phonon/lattice distortion propagates until it meets another electron and pulls on it. As a result the electron absorbs the phonon. The net effect is that both electrons experience equal momentum changes toward one another (opposite directions) and pulled toward each other just like being acted on by an attractive force between the two. This attractive force is small, 10^{-3}, but greater than the electron-electron repulsion and the separation between the two paired electrons is much larger than atomic spacing in a lattice.

- An overall state of lowest energy occurs when the most energetic conduction electrons near E_F form opposite-spin Cooper pairs, with all pairs having the same net momentum. See Figure 10.56. Since the electrons in a Cooper pair have opposite momentum, there is no net current. However, when there is current, the net momentum of Cooper paired electrons is not zero and all pairs have the same momentum, which can amplify.
- Not all electrons can participate in Cooper pairing. Only the electrons near E_F can be paired because at low temperatures the electrons occupy the available energy states without vacancies upto E_F. Raising deeper electrons to free states near the fermi energy increase the energy of the system more than the weak attraction created by Cooper pairs can lower it.
- The order of Cooper pair binding energy is at 3.5 k_BT (0.001 eV), which is greater than typical thermal energy generated by collisions between an electron and a lattice ion at low temperatures. Thus, collisions will not occur.
- Type 1 superconductors have longer mean free paths (the average distance electrons travel without collision) than Type 2 superconductors.

Figure 10.56 All Cooper pairs have the same momentum: zero when current is zero; nonzero otherwise.