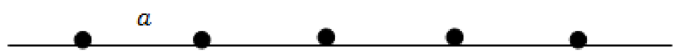
PHY 102 Modern Physics Mid-Term Exam II
Date: February 25, 2014 **Name: ___________________________________**

1. 1 dimensional free particle vs. 1 dimensional free electrons in a crystal (14 points)
During the class, you learned that electrons can move around in a crystal consisting of lattice ions as if they were free. In this problem, you are comparing electrons in a crystal with a free particle. Both of them are in one dimension.

(a) (2 points) Draw two graphs that represent potential energies below: U(x)

For 1 dimensional free particle

For an electron in a 1 dimensional crystal with a lattice spacing of a



(b) (3 points) Explain translational invariance of each case. We are not doing rotational invariance since they are in one dimensional situations.

A free particle

A free electron in a 1 dimensional crystal

(c) (2 points) Write a Schrodinger equation for a 1 dimensional free particle and solve the equation to obtain Energy (E).

(d) (2 points) Draw a graph that shows the relationship between E and $k = \sqrt{\frac{2mE}{\hbar^2}}$ for a 1 dimensional free particle.

(e) (2 points) Draw a graph that shows energies of valence electrons in a single-atom crystal in terms of k.

(f) (3 points) Compare the two energy diagrams and discuss differences in terms of the following:

- energy continuity
- energy band formation
- effective mass

2. Atomic orbital vs. Molecular orbital vs. Energy Band theory (12 points)
A Li atom (atomic number Z = 3) has three electrons.

(a) (1 point) Write the electron configuration for the ground state of a Li atom (such as $1s^2$ etc.).

(b) (2 points) Draw an energy diagram for a Li atom. Label each level with a corresponding atomic orbital and show electron occupancy in each energy level. Use ↑ or ↓ for an electron.

(c) (2 points) Two Li atoms bond to create a Li_2 molecule. Use two ground state atomic orbital energy levels to show a molecular orbital energy level for Li_2. Label each level and show electron occupancy in each energy level. Use ↑ or ↓ for an electron.

(d) (1 point) Assume that a Li_2 molecule consists of two Li atoms connected by a spring. Express a formula that shows quantized energies due to rotational and vibrational motions.

(e) (2 points) Consider a one dimensional Li crystal consisting of 2000 Li atoms. Draw an energy band diagram for a Li crystal. Label the energy band name, the electron occupancy (empty, partially filled or fully filled), and the Fermi Energy Level (E_F) on the energy band diagram.

(f) (1 point) For the 2s energy band, how many energy states are available in the 2000 atom Li crystal? Consider spins.

(g) (1 point) At T=0, how many energy states in the 2s energy band are filled by the 2000 atom Li crystal?

(h) (2 points) Determine whether the Li crystal is a conductor, an insulator, or a semiconductor?

Explain your answer using your energy band diagram.

3. Semiconductors (12 points)

The portion of the periodic table shown on the right may be helpful for this problem.

3	4	5	6
B	C	N	O
Al	Si	P	S
Ga	Ge	As	Se
In	Sn	Sb	Te
Tl	Pb	Bi	Po

(a) (2 points) Si Crystal is a semiconductor with the energy band gap of 1.11 eV. Draw an energy band diagram for Si Crystal. In the diagram, label band name, energy band gap, electron occupancy, and the Fermi energy level.

(b) (2 points) Draw an energy band diagram for the Bi-doped Si crystal. Your diagram should include all of the following: The energy level of the Bi impurity, the position of the valence band, the position of the conduction band, and the position of the Fermi energy level at T=0.

(c) (2 points) In the Bi-doped Si Crystal, what are the majority and minority charge carriers?

Explain your answer.

(d) (2 points) Draw an energy diagram for Al-doped Si crystal. Your diagram should include all of the following: The energy level of the As impurity, the position of the valence band, the position of the conduction band, and the position of the Fermi energy level at T=0.

(e) (2 points) In the Al-doped Si Crystal, what are the majority and minority charge carriers?

Explain your answer.

(e) (2 points) Al-doped and Bi-doped silicon crystals are put together and forward bias is applied. Draw an energy band diagram and explain what would happen to majority and minority charge carriers in this situation.

4. Classical and quantum statistics (10 points)
During the class, you learned three different types of distributions you can apply to a system of particles. They are Boltzmann, Bose-Einstein, and Fermi-Dirac distributions, and they represent occupation numbers as a function of E.

(a) (*3 points*) Write the occupation number, $\mathcal{N}(E)$, for

- Particles that obey the Boltzmann distribution

$$\mathcal{N}(E) =$$

- Particles that obey the Bose-Einstein distribution

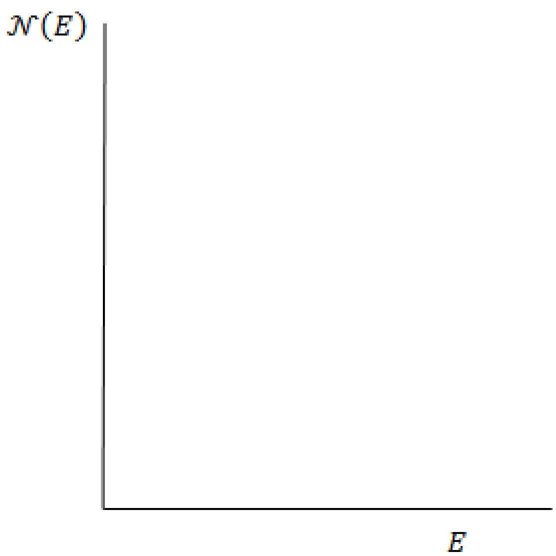
$$\mathcal{N}(E) =$$

- Particles that obey the Fermi-Dirac distribution

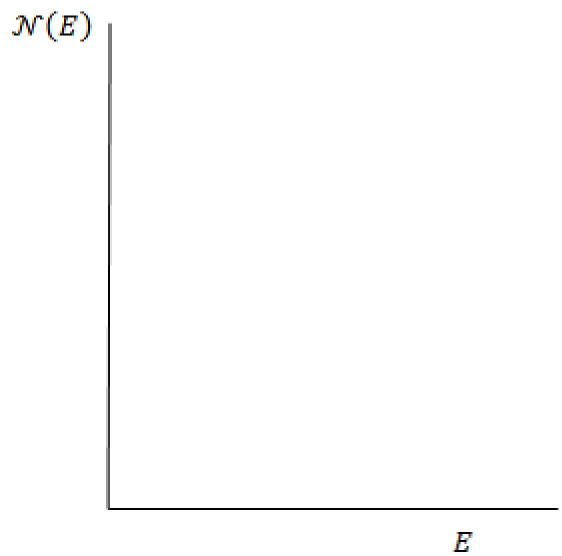
$$\mathcal{N}(E) =$$

(b) (*4 points*) Sketch all three distributions, $\mathcal{N}(E)$, on the following graph

when $k_B T \ll E$



when $k_B T \gg E$



(c) (3 points) At the same low temperature, which of the gas of classical molecules, the gas of bosons (particles that obey Bose-Einstein statistics) or the gas of fermions (particles that obey Fermi-Dirac statistics) exert the greatest pressure? The least pressure? Why?

5. True/False Questions (9 points, 3 points each)

(a) Metastable states are necessary to build a laser. ___ True ___ False

Explain.

(b) Laser could not be explained if photons were ferminons. ___ True ___ False

Explain.

(c) The superfluid state of He atoms can be modeled using Fermi-Dirac distributions.

___ True ___ False

Explain.

(Extra Credit) Thermal Expansion (up to 6 points)
The potential energy U(x) of a pair of atoms in a solid that are displaced by x from their equilibrium separation at 0 K may be written

$$U(x) = ax^2 - bx^3 - cx^4$$

where $-bx^3$ represents the asymmetry introduced by the repulsive forces between the atoms $-cx^4$ represents the leveling off of the attractive forces at large displacements.

At a temperate T the likelihood that a displacement x will occur relative to the likelihood of no displacement is e^{-U/k_BT}, so that the average displacement x at this temperature is

$$\bar{x} = \frac{\int_{-\infty}^{\infty} xe^{-U/k_BT}dx}{\int_{-\infty}^{\infty} e^{-U/k_BT}dx}$$

Show that, for small displacements

$$\bar{x} \approx 3bk_BT/4a^2$$

(This is the reason that the change in length of a solid when its temperature changes is proportional to ΔT.)
Hint:
1. $e^{\delta} \approx 1 + \delta$, if $|\delta| \ll 1$, for example $\delta = (bx^3 + cx^4)/(k_BT)$ is a small parameter in this problem.
2. $\int_{-\infty}^{\infty} x^{2m+1}\, e^{-x^2} dx = 0$ ($m = 0, 1, 2 \ldots$; due to parity).
3. $\int_{-\infty}^{\infty} x^{2m}\, e^{-x^2} dx = \left(m - \frac{1}{2}\right)! = \left(m - \frac{1}{2}\right)\left(m - \frac{3}{2}\right) \ldots \left(-\frac{1}{2}\right)!$ where $\left(-\frac{1}{2}\right)! = \sqrt{\pi}$ and $m = 0, 1, 2 \ldots$