PH102: Modern Physics Homework 3 (Due: 2/10/2014, Monday 5 pm)

1. (5 points) Textbook: Harris, Chapter 9 Conceptual Question #5
2. (5 points) Textbook: Harris, Chapter 9 Conceptual Question #8
3. (10 points) For a free electron inside an infinite potential well, energy can be defined as

$$E = \frac{\pi^2\hbar^2}{2mL^2}n^2 \quad \text{(See the textbook p.375)}$$

where L is the size of a one dimensional infinite potential well.

(a) (3 points) Show that the density of states of a free electron in one dimension is $D(E) = \frac{L}{\pi}\sqrt{\frac{2m}{\hbar^2}}\frac{1}{\sqrt{E}}$

(b) (3 points) Show that in two dimensions, the density of states of a free electron is $D(E) = \frac{Am}{\pi\hbar^2}$ where $A = L^2$

(c) (3 points) Show that in three dimensions, the density of states becomes $D(E) = \frac{m^{3/2}V\sqrt{2}}{\pi^2\hbar^3}\sqrt{E}$ where $V = L^3$

(d) (1 point) Sketch the density of states for each of the three dimensions as a function of E.

4. (10 points) Textbook: Harris, Chapter 9 Exercises #60
5. (5 points) According to Planck's hypothesis on the light quanta, the photon energy is quantized as

$$E_n = nh\nu$$

Where n represents the quantum number for the photon where $n = 0,1,2,3 \ldots \infty$, and ν represents frequency. Average Energy $\bar{E}$ can be calculated by using the Botzman expression.

$$\bar{E} = \sum E_n P(E_n) = \frac{\sum_n E_n e^{-\frac{E_n}{k_B T}}}{\sum_n e^{-\frac{E_n}{k_B T}}}$$

Show that $\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{k_B T}}-1}$

6. (10 points) Consider a system of one dimensional harmonic oscillators. The probability that such an oscillator have the energy E at the temperature T is given by the Boltzman factor $e^{-E/k_B T}$, and so its average energy is found by integration $Ee^{-E/k_B T}$over all possible energies and then dividing by the integral of $e^{-E/k_B T}$in order to normalize the result. The total energy of the oscillator at any time is the sum of its instantaneous kinetic and potential energies

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Where v is the particle's speed and x is its displacement from the equilibrium position.

(a) Show that the average energy is $k_B T$, using the following formula:

$$\bar{E} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E e^{-\frac{E}{k_B T}}\, dv dx}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{E}{k_B T}}\, dv dx}$$

(Hint: do the integral separately for *dv* and *dx*.

(b) At high temperature, the average energy of a classical one-dimensional oscillator is $k_B T$ as you proved above, and for an atom in a monatomic ideal gas, it is $\frac{3}{2} k_B T$. Explain the difference, using the equipartition theorem (Chapter 9 conceptual question #18).