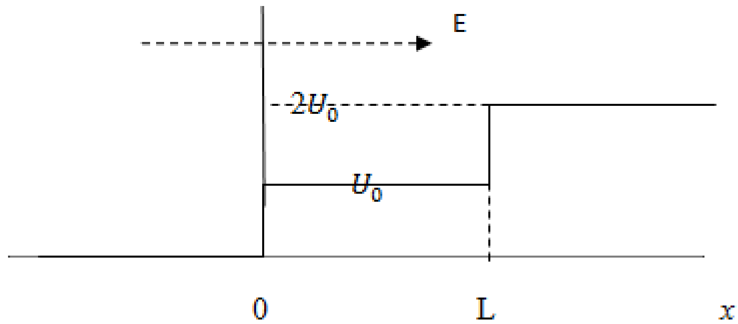
PH102: Modern Physics Homework 1 (Due: 1/17/2014)

1. **(10 points)** Consider the following potential steps:

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \le x < L \\ 2U_0 & x \ge L \end{cases}$$



(a) Write the time-independent form of the Schrodinger Equation in three regions: $x < 0$, $0 \le x < L$, $and\ x \ge L$.

(b) The electron is impinging from the left with energy $E > 2\,U_0$. Write down the wave function $\psi(x)$ in the three regions.

(c) What conditions should $\psi(x)$ meet at the boundaries, $= 0\ and\ x = L$? What do these boundary conditions tell about the relationships among coefficients of wave function components in $\psi(x)$?

(d) Define R (reflection probability) and T (transmission probability) using the coefficients of wave function components.

2. **(10 points) A particle of mass m in a 2 dimensional infinite well L long and W wide.**

(a) Write the time-independent Schrodinger Equation for a particle in the two dimensional infinite well.

(b) Solve the Schrodinger Equation and obtain allowed energy values and associated wave functions.

(c) Obtain the ground state energy level and the associated wave function. Obtain the probability density related to the ground state. Sketch the probability density.

(d) Consider the special case $L = W$. Find the five lowest energy levels of a particle in this square well. Which of these levels is degenerate?

3. **(15 points) Angular momentum**

The angular momentum **L** of a particle at the position **r** whose linear momentum is **p** is defined by the vector formula

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

(a) From the above expression, explain how three Cartesian components of **L** can be written as:

$$\begin{cases} L_x = yp_z - zp_y \\ L_y = zp_x - xp_z \\ L_z = xp_y - yp_x \end{cases}$$

(b) Using the momentum operators such as $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$, etc. express three Cartesian angular momentum operators L_x, L_y, L_z.

(c) Prove that the three Cartesian angular momentum operators in spherical polar coordinates can be written as:

$$\boldsymbol{L_x = \frac{\hbar}{i}\left(-sin\phi\frac{\partial}{\partial\theta} - cot\theta cos\phi\frac{\partial}{\partial\phi}\right)}$$

$$\boldsymbol{L_y = \frac{\hbar}{i}\left(cos\phi\frac{\partial}{\partial\theta} - cot\theta sin\phi\frac{\partial}{\partial\phi}\right)}$$

$$\boldsymbol{L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}}$$

(d) $\mathbf{L^2}$ operator is defined by**:**

$$\mathbf{L^2 = L_xL_x + L_yL_y + L_zL_z}$$

Prove that

$$\mathbf{L^2} = -\hbar^2\left[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

(e) Using $\boldsymbol{L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}}$ and : $\Phi(\phi) = A\,e^{im_l\phi}$ **prove that**

$$\boldsymbol{L_z\psi = m_l\hbar\psi}$$

(f) Using $\mathbf{L^2} = -\hbar^2\left[\frac{1}{sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$

$\Phi(\phi) = A\,e^{im_l\phi}$

$sin\theta\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\Theta}{\partial\theta}\right) + \left[l(l+1)sin^2\theta - m_l^2\right]\Theta = 0$

Prove that

$$\boldsymbol{L^2\,\psi = l(l+1)\hbar^2\psi}$$

4. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #8
5. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #12
6. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #39
7. (5 points) Textbook: Harris, Chapter 7 Exercises Question #70