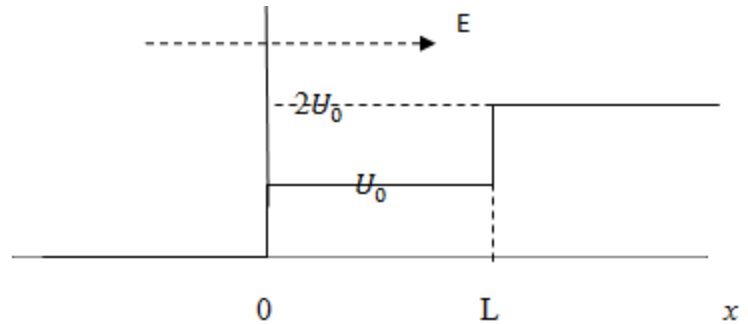


PH102: Modern Physics Homework 1 (Due: 1/17/2014)

1. (10 points) Consider the following potential steps:

$$U(x) = \begin{cases} 0 & x < 0 \\ U_0 & 0 \leq x < L \\ 2U_0 & x \geq L \end{cases}$$



- Write the time-independent form of the Schrodinger Equation in three regions: $x < 0$, $0 \leq x < L$, and $x \geq L$.
 - The electron is impinging from the left with energy $E > 2U_0$. Write down the wave function $\psi(x)$ in the three regions.
 - What conditions should $\psi(x)$ meet at the boundaries, $x = 0$ and $x = L$? What do these boundary conditions tell about the relationships among coefficients of wave function components in $\psi(x)$?
 - Define R (reflection probability) and T (transmission probability) using the coefficients of wave function components.
2. (10 points) A particle of mass m in a 2 dimensional infinite well L long and W wide.
- Write the time-independent Schrodinger Equation for a particle in the two dimensional infinite well.
 - Solve the Schrodinger Equation and obtain allowed energy values and associated wave functions.
 - Obtain the ground state energy level and the associated wave function. Obtain the probability density related to the ground state. Sketch the probability density.
 - Consider the special case $L = W$. Find the five lowest energy levels of a particle in this square well. Which of these levels is degenerate?

3. (15 points) **Angular momentum**

The angular momentum \mathbf{L} of a particle at the position \mathbf{r} whose linear momentum is \mathbf{p} is defined by the vector formula

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

- From the above expression, explain how three Cartesian components of \mathbf{L} can be written as:

$$\begin{cases} L_x = yp_z - zp_y \\ L_y = zp_x - xp_z \\ L_z = xp_y - yp_x \end{cases}$$

(b) Using the momentum operators such as $p_x = \frac{\hbar}{i} \frac{\partial}{\partial x}$, etc. express three Cartesian angular momentum operators L_x, L_y, L_z .

(c) Prove that the three Cartesian angular momentum operators in spherical polar coordinates can be written as:

$$L_x = \frac{\hbar}{i} \left(-\sin\phi \frac{\partial}{\partial \theta} - \cot\theta \cos\phi \frac{\partial}{\partial \phi} \right)$$

$$L_y = \frac{\hbar}{i} \left(\cos\phi \frac{\partial}{\partial \theta} - \cot\theta \sin\phi \frac{\partial}{\partial \phi} \right)$$

$$L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$$

(d) L^2 operator is defined by:

$$L^2 = L_x L_x + L_y L_y + L_z L_z$$

Prove that

$$L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \phi^2} \right]$$

$$L^2 = L_x L_x + L_y L_y + L_z L_z$$

(e) Using $L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$ and $\Phi(\phi) = A e^{im_l \phi}$ prove that

$$L_z \psi = m_l \hbar \psi$$

(f) Using $L^2 = -\hbar^2 \left[\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \phi^2} \right]$

$$\Phi(\phi) = A e^{im_l \phi}$$

$$\sin\theta \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial \Theta}{\partial \theta} \right) + [l(l+1)\sin^2\theta - m_l^2] \Theta = 0$$

Prove that

$$L^2 \psi = l(l+1)\hbar^2 \psi$$

4. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #8
5. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #12
6. (5 points) Textbook: Harris, Chapter 7 Conceptual Question #39
7. (5 points) Textbook: Harris, Chapter 7 Exercises Question #70