Announcements

- Homework #1 Question 2
- Homework #1 Question 8 from the textbook
- Homework #2 will be handed out on Tuesday (22nd) and due Monday noon (28th)
- Schedule a review time before the first exam on 29th

PH102: Interactive Lecture 4

- Topics
  - 3d Schrodinger Equation for Hydrogen atom
  - $(x, y, z) \leftrightarrow (r, \theta, \phi)$
  - Separation of variables $R\Theta\Phi$
  - Three equations
  - Three quantum numbers
  - Wave functions
  - Quantization of angular momentum (L)
  - Degeneracies
  - Normalization
  - Electron whereabouts

Schrodinger Equation

$$\begin{cases} \dfrac{\partial^2 \Phi}{\partial \phi^2} = -m_l^2 \Phi & \text{Azimuthal Equation} \\ sin\theta \dfrac{\partial}{\partial \theta}\left(sin\theta \dfrac{\partial \Theta}{\partial \theta}\right) + [l(l+1)sin^2\theta - m_l^2]\Theta = 0 & \text{Polar Equation} \\ \dfrac{\partial}{\partial r}\left(r^2 \dfrac{\partial R}{\partial r}\right) + \dfrac{2mr^2}{\hbar^2}\left(E - U(r)\right)R - l(l+1)R = 0 & \text{Radial Equation} \end{cases}$$

Schrodinger Equation: Hydrogen Atom

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{x}) + U(\vec{x})\psi(\vec{x}) = E\,\psi(\vec{x}) \qquad U(r) = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + U(\vec{x})\right]\psi(\vec{x}) = E\,\psi(\vec{x})$$

$$\mathbf{H}\psi_{n,l,m_l} = E_n\psi_{n,l,m_l}$$

Hamiltonian (H) = T (kinetic) + U (Potential)

Kinetic energy w.r.t. r + Kinetic energy w.r.t. rotation

$$\begin{cases} -\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) - \frac{2mr^2}{\hbar^2}\left(E - U(r)\right) = C = -l(l+1) \\ \frac{1}{\Theta}\csc\theta\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Theta}{\partial\theta}\right) + \frac{1}{\Phi}\csc^2\theta\,\frac{\partial^2\Phi}{\partial\phi^2} = C = -l(l+1) \end{cases}$$

$$\mathbf{L}^2\psi_{n,l,m_l} = l(l+1)\hbar^2\psi_{n,l,m_l}$$

$$\frac{\partial^2\Phi}{\partial\phi^2} = -m_l^2\Phi$$

$$\mathbf{L_z}\psi_{n,l,m_l} = m_l\hbar\psi_{n,l,m_l}$$

n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi\,(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0{}^{3/2}}\, e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\, a_0{}^{3/2}}\, e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\, a_0{}^{3/2}}\left(2-\frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0{}^{3/2}}\left(2-\frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{1}{2\sqrt{6}\, a_0{}^{3/2}}\, \frac{r}{a_0}\, e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0{}^{3/2}}\, \frac{r}{a_0}\, e^{-r/2a_0}\cos\theta$
2	1	± 1	$\frac{1}{\sqrt{2\pi}}\, e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{1}{2\sqrt{6}\, a_0{}^{3/2}}\, \frac{r}{a_0}\, e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\, a_0{}^{3/2}}\, \frac{r}{a_0}\, e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\, a_0{}^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_0{}^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\, a_0{}^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_0{}^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$	$\frac{4}{81\sqrt{6}\, a_0{}^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}\, e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\, a_0{}^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}\, e^{-r/3a_0}\cos\theta$
3	1	± 1	$\frac{1}{\sqrt{2\pi}}\, e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$	$\frac{4}{81\sqrt{6}\, a_0{}^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}\, e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0{}^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}\, e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta-1)$	$\frac{4}{81\sqrt{30}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}(3\cos^2\theta-1)$
3	2	± 1	$\frac{1}{\sqrt{2\pi}}\, e^{\pm i\phi}$	$\frac{\sqrt{15}}{2}\sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	± 2	$\frac{1}{\sqrt{2\pi}}\, e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4}\sin^2\theta$	$\frac{4}{81\sqrt{30}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\, a_0{}^{3/2}}\, \frac{r^2}{a_0{}^2}\, e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$

Three quantum numbers

- Principal quantum number: *n*

$$E_n = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{n^2}\right) = -\left(\frac{e^2}{8\pi\varepsilon_0}\right)\left(\frac{me^2}{4\pi\varepsilon_0\hbar^2}\right)\left(\frac{1}{n^2}\right) =$$

$$-\left(\frac{e^2}{8\pi\varepsilon_0 a_0}\right)\left(\frac{1}{n^2}\right) = -13.6\ eV\left(\frac{1}{n^2}\right)$$

$$L^2 = l(l+1)\hbar^2$$

$$L_z = m_l\hbar$$

- Orbital quantum number: $l = 0, 1, 2, \ldots(n\text{-}1)$
- Magnetic quantum number: $m_l = 0, \pm 1, \pm 2, \ldots \pm l$

Degeneracies

$$\psi(r,\theta,\phi) = \psi_{n,l,m_l} = R(r)\Theta(\theta)\Phi(\phi) = R_{n,l}\Theta_{l,m_l}\Phi_{m_l} = R_{n,l}\mathbf{Y}_l^{m_l}$$

where $\Theta_{l,m_l}\Phi_{m_l} = \mathbf{Y}_l^{m_l}$ (Spherical harmonics)

n	l	m_l	E_n(eV)	$\lvert L \rvert$	L_z	ψ_{n,l,m_l} =	$R_{n,l}\mathbf{Y}_l^{m_l}$	degeneracies	Orbital name
1	0	0	-13.6	0	0	ψ_{100}	$R_{10}\mathbf{Y}_0^0$	Non-degenerate	1s
2	0	0	-3.40	0	0	ψ_{200}	$R_{20}\mathbf{Y}_0^0$	4 (=2^2)	2s
	1	-1		$\sqrt{2}\hbar$	$-\hbar$	ψ_{21-1}	$R_{21}\mathbf{Y}_1^{-1}$		2p
		0			0	ψ_{210}	$R_{21}\mathbf{Y}_1^0$		
		1			$+\hbar$	ψ_{211}	$R_{21}\mathbf{Y}_1^{+1}$		

n	l	m_l	E_n(eV)	$\|L\|$	L_z	ψ_{n,l,m_l} =	$R_{n,l}\mathbf{Y}_l^{m_l}$	degeneracies	Orbital name
1	0	0	-13.6	0	0	ψ_{100}	$R_{10}\mathbf{Y}_0^0$	Non-degenerate	1s
2	0	0	-3.40	0	0	ψ_{200}	$R_{20}\mathbf{Y}_0^0$	4 (=2²)	2s
	1	-1		$\sqrt{2}\hbar$	$-\hbar$	ψ_{21-1}	$R_{21}\mathbf{Y}_1^{-1}$		2p
		0			0	ψ_{210}	$R_{21}\mathbf{Y}_1^0$		
		1			$+\hbar$	ψ_{211}	$R_{21}\mathbf{Y}_1^{+1}$		
3	0	0	-1.51	0	0	ψ_{300}	$R_{30}\mathbf{Y}_0^0$	9 (=3²)	3s
	1	-1		$\sqrt{2}\hbar$	$-\hbar$	ψ_{31-1}	$R_{31}\mathbf{Y}_1^{-1}$		3p
		0			0	ψ_{310}	$R_{31}\mathbf{Y}_1^0$		
		1			$+\hbar$	ψ_{311}	$R_{31}\mathbf{Y}_1^1$		
	2	-2		$\sqrt{6}\hbar$	$-2\hbar$	ψ_{32-2}	$R_{32}\mathbf{Y}_2^{-2}$		3d
		--1			$-\hbar$	ψ_{32-1}	$R_{32}\mathbf{Y}_2^{-1}$		
		0			0	ψ_{320}	$R_{32}\mathbf{Y}_2^0$		
		1			$+\hbar$	ψ_{321}	$R_{32}\mathbf{Y}_2^1$		
		2			$+2\hbar$	ψ_{322}	$R_{32}\mathbf{Y}_2^2$		

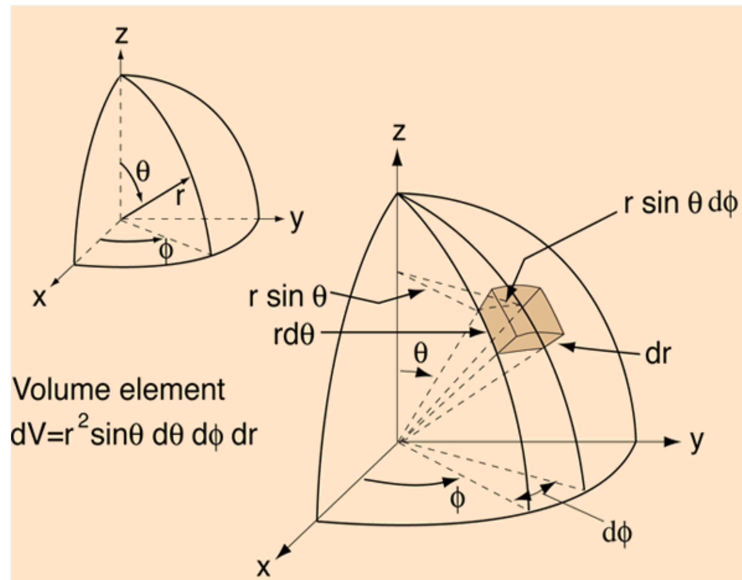
Normalization

$$|\psi_{n,l,m_l}|^2 = R(r)^2\Theta(\theta)^2\Phi(\phi)^2$$

$$(x, y, z) \quad \leftrightarrow \quad (r, \theta, \phi)$$

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = \tan^{-1}\dfrac{y}{x} \\ \theta = \cos^{-1}\dfrac{z}{r} \end{cases}$$

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$



$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \csc\theta\,\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) + \csc^2\theta\,\frac{\partial}{\partial\phi^2}\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi^2}$$

$$dV = r^2\sin\theta\; dr\,d\theta\,d\phi$$

Normalization

$$|\psi_{n,l,m_l}|^2 = R(r)^2\Theta(\theta)^2\Phi(\phi)^2$$

$$\int |\psi_{n,l,m_l}|^2 \, dV = \int |\psi_{n,l,m_l}|^2 \, r^2 sin\theta \; dr d\theta d\phi$$

$$= \int_0^{\infty} R(r)^2 r^2 \, dr \int_0^{\pi} \Theta(\theta)^2 sin\theta \; d\theta \int_0^{2\pi} \Phi(\phi)^2 d\phi$$

Normalization

$$|\psi_{n,l,m_l}|^2 = R(r)^2\Theta(\theta)^2\Phi(\phi)^2$$

$$\int |\psi_{n,l,m_l}|^2 \, dV = \int |\psi_{n,l,m_l}|^2 \, r^2 sin\theta \; dr d\theta d\phi$$

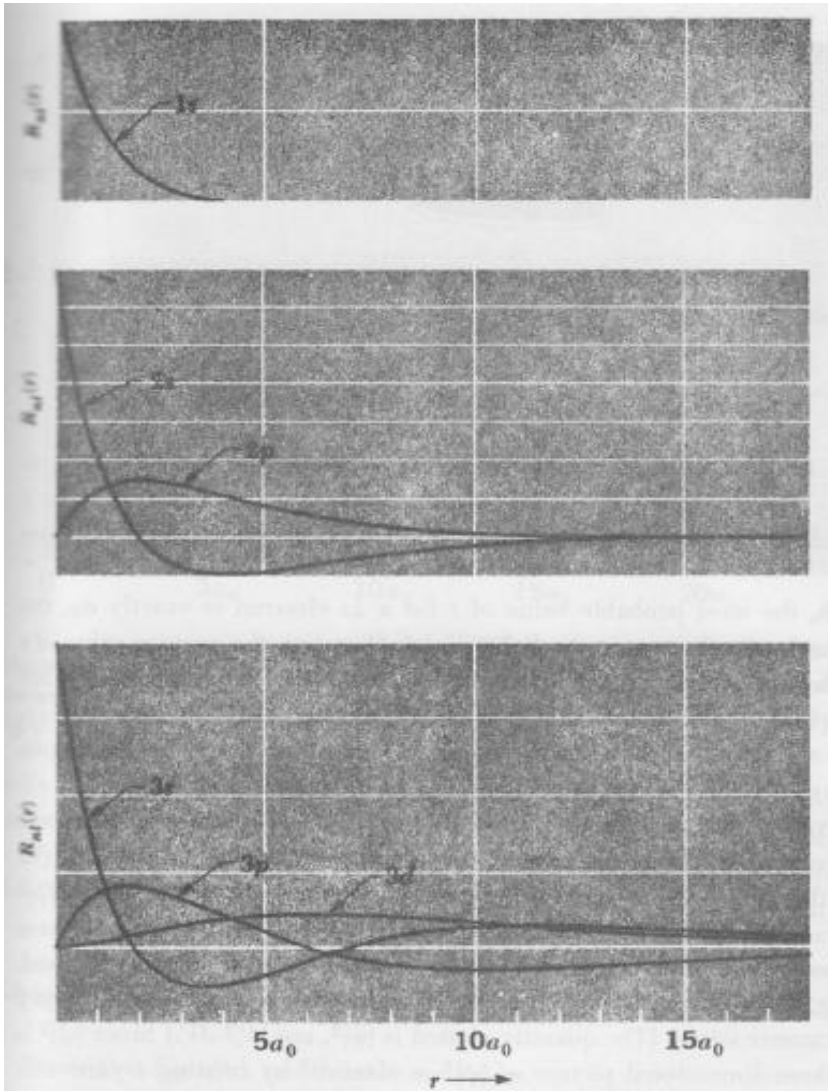
$$= \int_0^\infty R(r)^2 r^2 \, dr \int_0^\pi \Theta(\theta)^2 sin\theta \; d\theta \int_0^{2\pi} \Phi(\phi)^2 d\phi$$

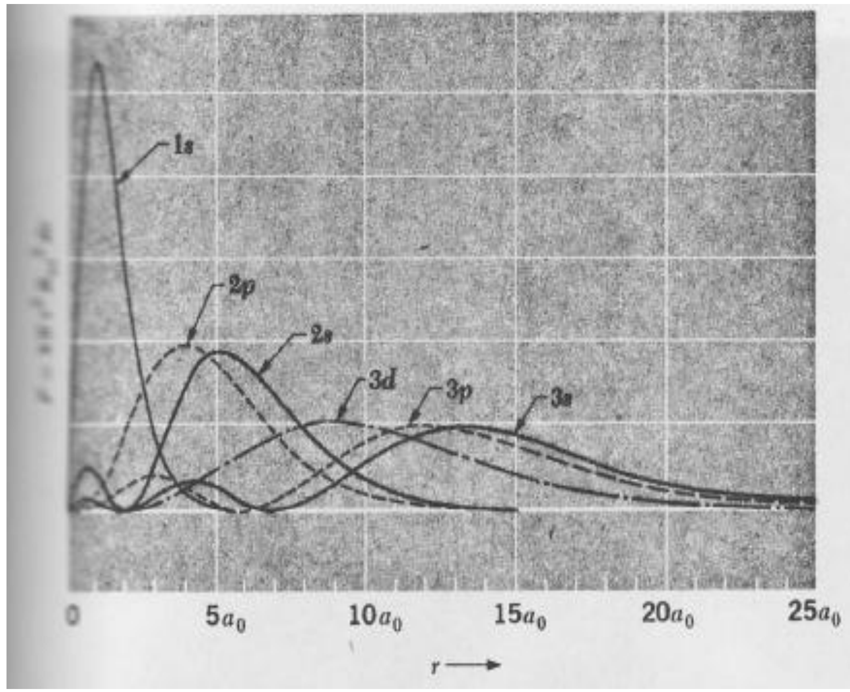
$$\begin{cases} \int_0^\infty R(r)^2 r^2 \, dr = 1 \\ \int_0^\pi \Theta(\theta)^2 sin\theta \; d\theta \int_0^{2\pi} \Phi(\phi)^2 d\phi = 2\pi \int_0^\pi \Theta(\theta)^2 sin\theta \; d\theta = 1 \end{cases}$$

R(r) vs. $r^2 R^2(r)$

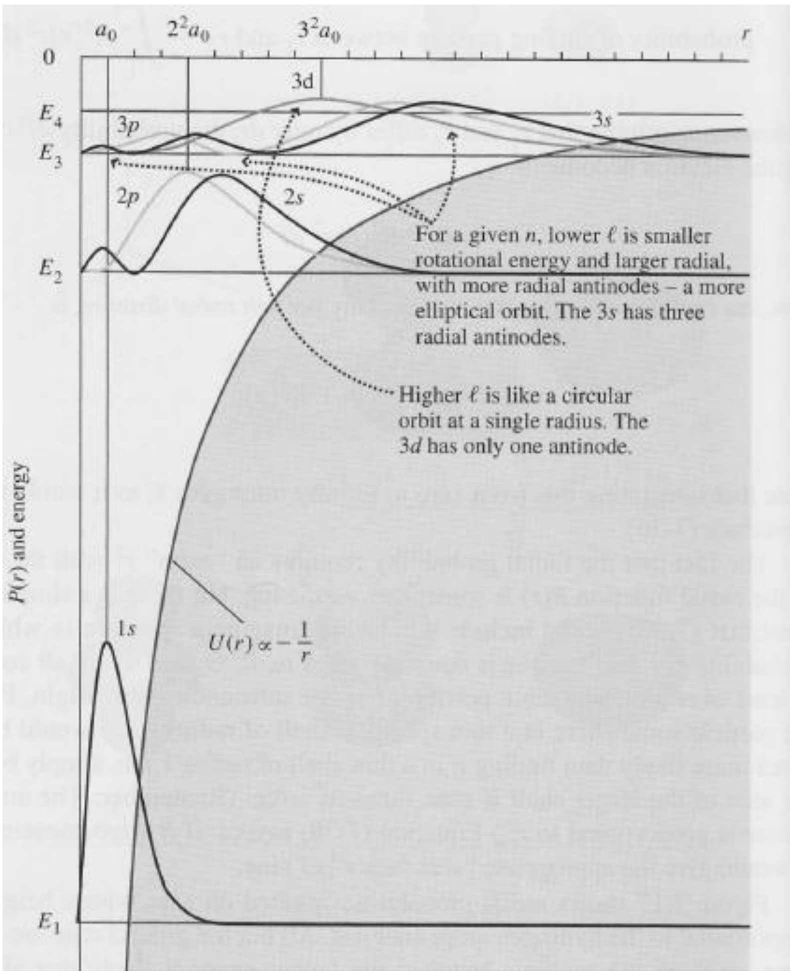
n	l	m_l	$\Phi(\phi)$	$\Theta(\theta)$	$R(r)$	$\psi(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{a_0^{3/2}} e^{-r/a_0}$	$\frac{1}{\sqrt{\pi}\, a_0^{3/2}} e^{-r/a_0}$
2	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2\sqrt{2}\, a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-r/2a_0}$
2	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{4\sqrt{2\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta$
2	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{1}{2\sqrt{6}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$	$\frac{1}{8\sqrt{\pi}\, a_0^{3/2}} \frac{r}{a_0} e^{-r/2a_0} \sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$	$\frac{2}{81\sqrt{3}\, a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$	$\frac{1}{81\sqrt{3\pi}\, a_0^{3/2}} \left(27 - 18\frac{r}{a_0} + 2\frac{r^2}{a_0^2}\right) e^{-r/3a_0}$
3	1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2} \cos\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{\sqrt{2}}{81\sqrt{\pi}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \cos\theta$
3	1	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{3}}{2} \sin\theta$	$\frac{4}{81\sqrt{6}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}} \left(6 - \frac{r}{a_0}\right) \frac{r}{a_0} e^{-r/3a_0} \sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4} (3\cos^2\theta - 1)$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{6\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} (3\cos^2\theta - 1)$
3	2	± 1	$\frac{1}{\sqrt{2\pi}} e^{\pm i\phi}$	$\frac{\sqrt{15}}{2} \sin\theta\cos\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{81\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	± 2	$\frac{1}{\sqrt{2\pi}} e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4} \sin^2\theta$	$\frac{4}{81\sqrt{30}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0}$	$\frac{1}{162\sqrt{\pi}\, a_0^{3/2}} \frac{r^2}{a_0^2} e^{-r/3a_0} \sin^2\theta\, e^{\pm 2i\phi}$

$R(r)$ vs. $r^2 R^2(r)$

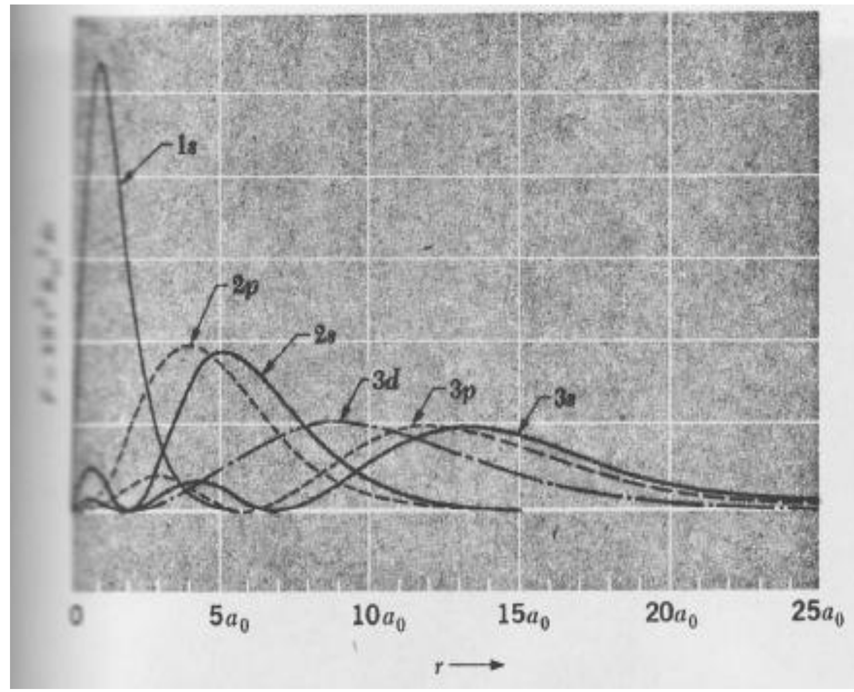

1s
2s
2p
3s
3p
3d
5a0
10a0
15a0
r


1s
2p
2s
3d
3p
3s
0
5a0
10a0
15a0
20a0
25a0
r

$P(r) = r^2 R^2(r)$



a_0
$2^2 a_0$
$3^2 a_0$
r
0
3d
E_4
3p
3s
E_3
2p
2s
For a given n, lower ℓ is smaller rotational energy and larger radial, with more radial antinodes – a more elliptical orbit. The 3s has three radial antinodes.
E_2
Higher ℓ is like a circular orbit at a single radius. The 3d has only one antinode.
P(r) and energy
1s
$U(r) \propto -\frac{1}{r}$
E_1



1s
2p
2s
3d
3p
3s
0
$5a_0$
$10a_0$
$15a_0$
$20a_0$
$25a_0$
$r \longrightarrow$

Most probable vs. expectation value

the expected r value is obtained when <r> = $\int_0^\infty r \cdot r^2 R_{n,l}(r)^2 dr$

The most probable r value is obtained when $\frac{dP(r)}{dr} = \frac{d\, r^2 R_{n,l}(r)^2}{dr} = 0$

$r_{n,\, l=n-1}$ (most probable) $= n^2 a_0$

That is,

r_{10} (1s, most probable) $= a_0$

r_{21} (2p, most probable) $= 4a_0$

r_{32} (3d, most probable) $= 9a_0$

Most probable vs. expectation value

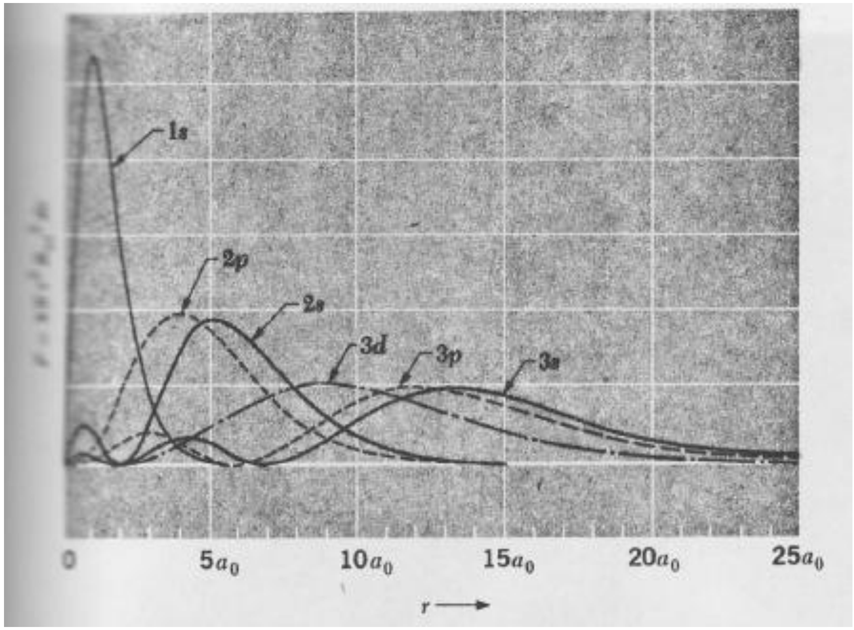
Expectation Value r $$\langle r \rangle = \int_0^\infty r \cdot r^2 R_{n,l}(r)^2 dr$$

Most probable r Value $$\frac{dP(r)}{dr} = \frac{d r^2 R_{n,l}(r)^2}{dr} = 0$$

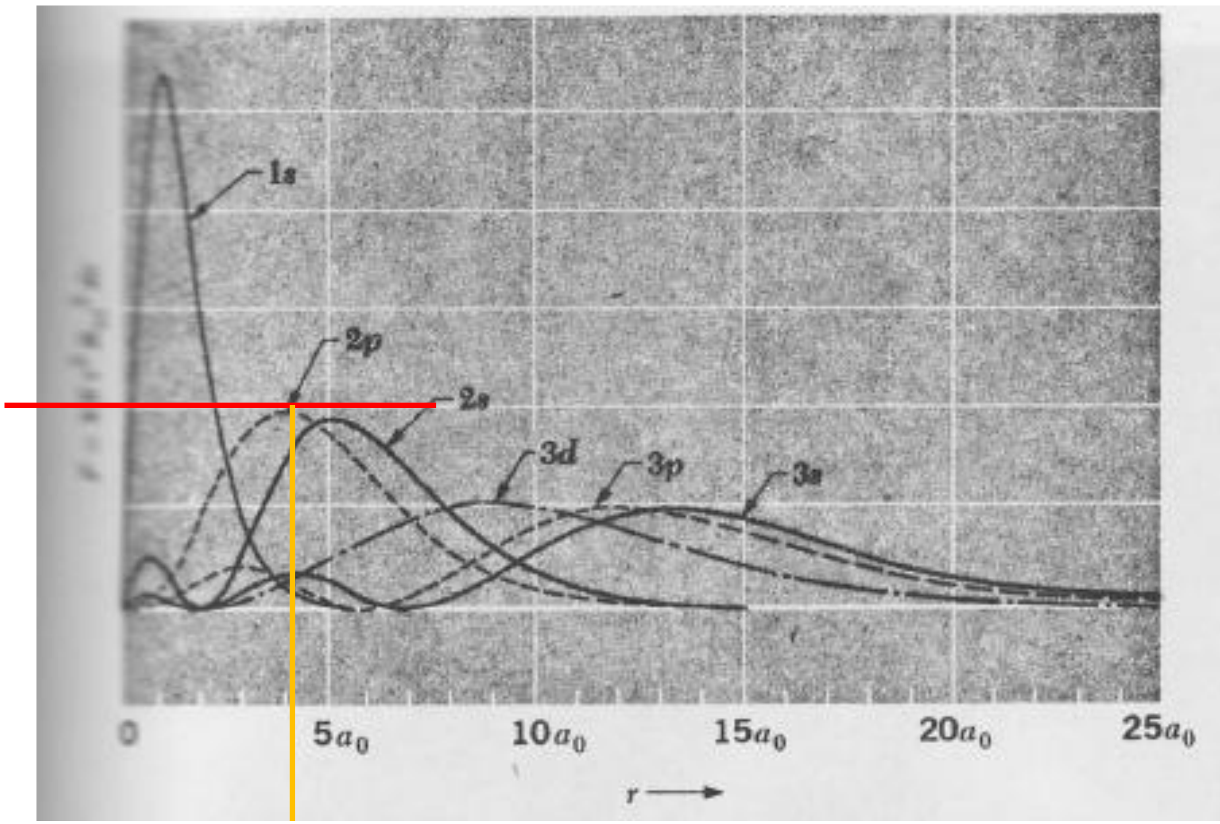
Most probable vs. expectation value

Expectation Value r $\langle r \rangle = \int_0^\infty r \cdot r^2 R_{n,l}(r)^2 dr$

Most probable r Value $\frac{dP(r)}{dr} = \frac{d\, r^2 R_{n,l}(r)^2}{dr} = 0$

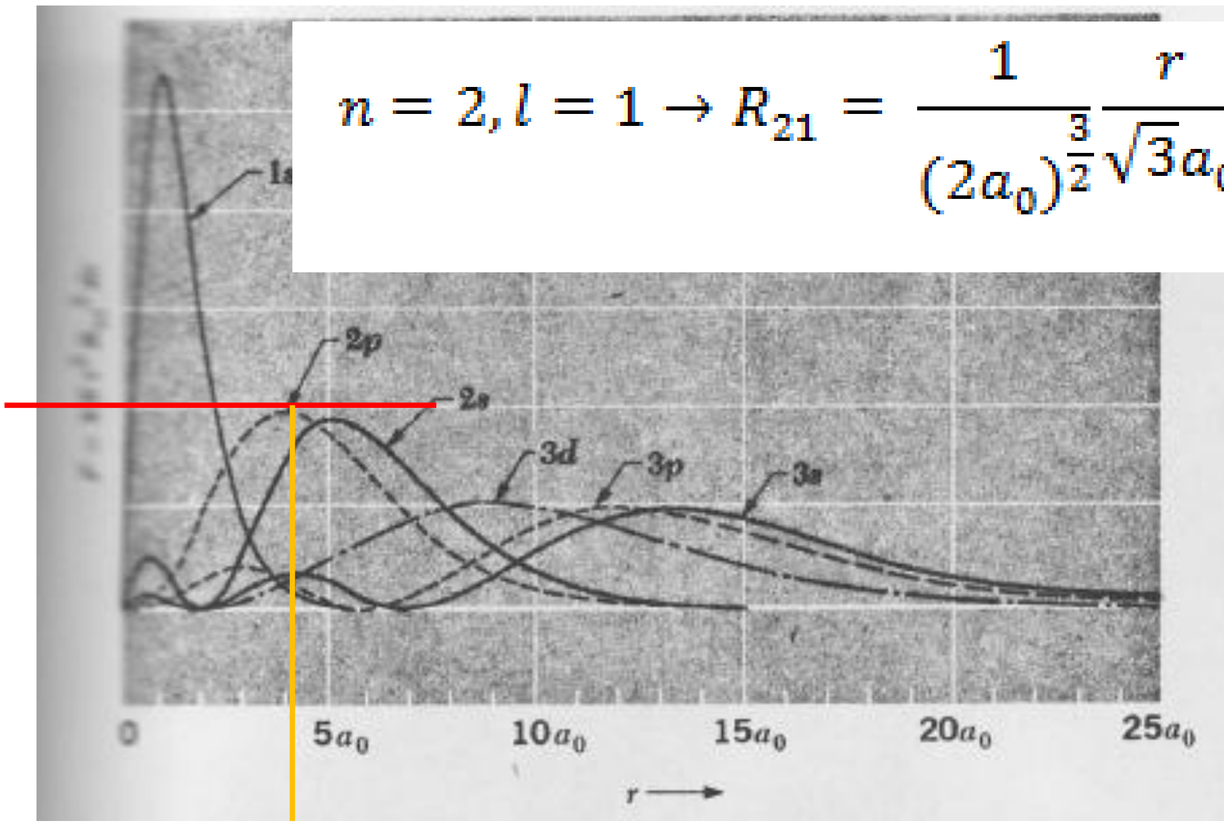


P(r) ~ r² R²(r)



$P(r) \sim r^2 R^2(r)$

$$n = 2, l = 1 \rightarrow R_{21} = \frac{1}{(2a_0)^{\frac{3}{2}}} \frac{r}{\sqrt{3}a_0} e^{-\frac{r}{2a_0}}$$



Expected value

$$\langle r \rangle = \int_0^{\infty} r \cdot r^2 R_{n,l}(r)^2 dr$$

$$\int_0^{\infty} r \cdot r^2 \cdot \frac{1}{(2a_0)^3} \frac{r^2}{3a_0^2} e^{-\frac{r}{a_0}} \, dr$$

$$\int_0^{\infty} x^m e^{-bx} dr = \frac{m!}{b^{m+1}}$$

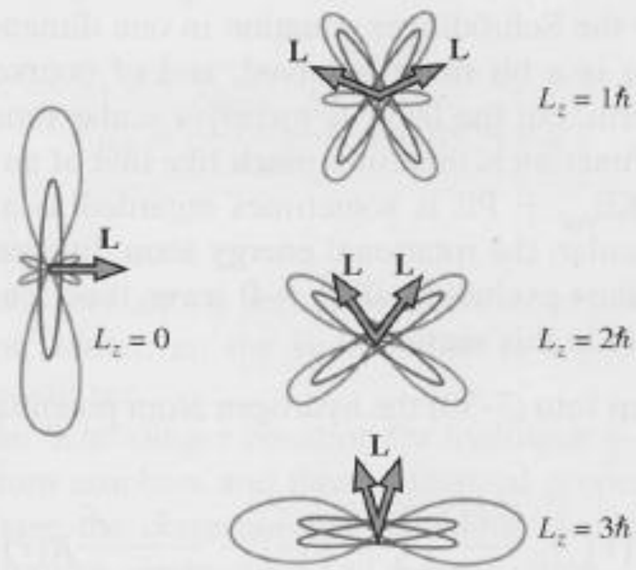
$$\frac{1}{3 \cdot 2^3 a_0^5} \int_0^{\infty} r^5 e^{-\frac{r}{a_0}} dr = \frac{1}{3 \cdot 2^3 a_0^5} 5! \, a_0^6 = 5 \, a_0$$

Angular Probability Density

$$\Theta(\theta)^2\Phi(\phi)^2 \equiv \mathbf{Y}_l^{m_l *}\mathbf{Y}_l^{m_l}$$

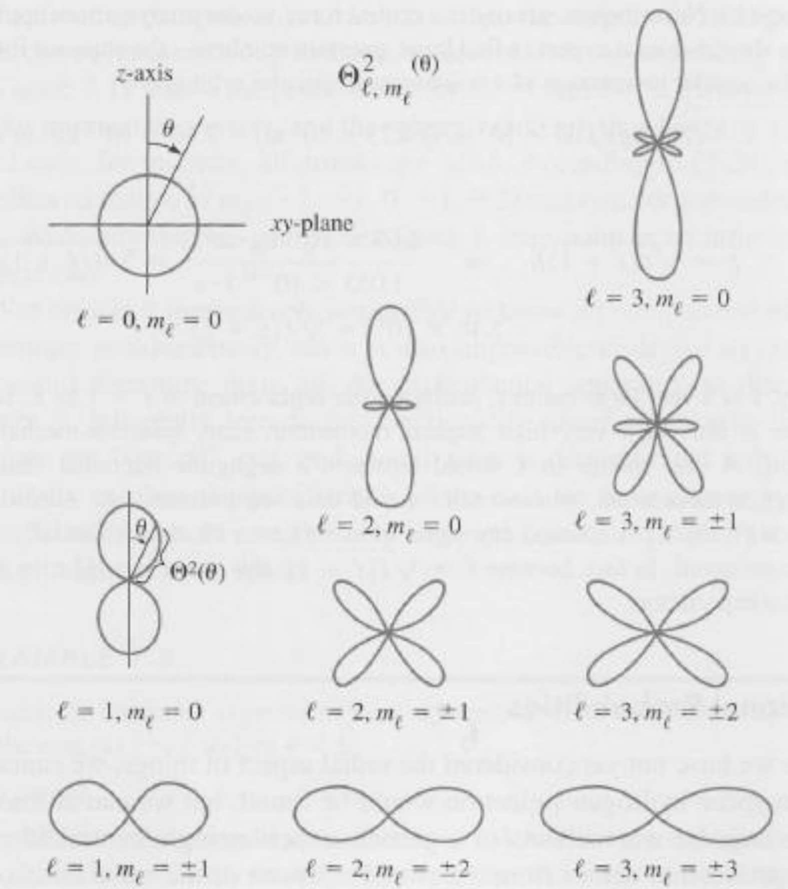
l	m_l	$\Phi(\phi)$	$\Theta(\theta)$
0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$
0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$
1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$
1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$
0	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{1}{\sqrt{2}}$
1	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{6}}{2}\cos\theta$
1	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{3}}{2}\sin\theta$
2	0	$\frac{1}{\sqrt{2\pi}}$	$\frac{\sqrt{10}}{4}(3\cos^2\theta - 1)$
2	±1	$\frac{1}{\sqrt{2\pi}}e^{\pm i\phi}$	$\frac{\sqrt{15}}{2}\sin\theta\cos\theta$
2	±2	$\frac{1}{\sqrt{2\pi}}e^{\pm 2i\phi}$	$\frac{\sqrt{15}}{4}\sin^2\theta$

Figure 7.14 A crude correspondence to orbital motion.



Angular Probability Density

Figure 7.13 Angular probability densities for a central force.



Angular Probability Density

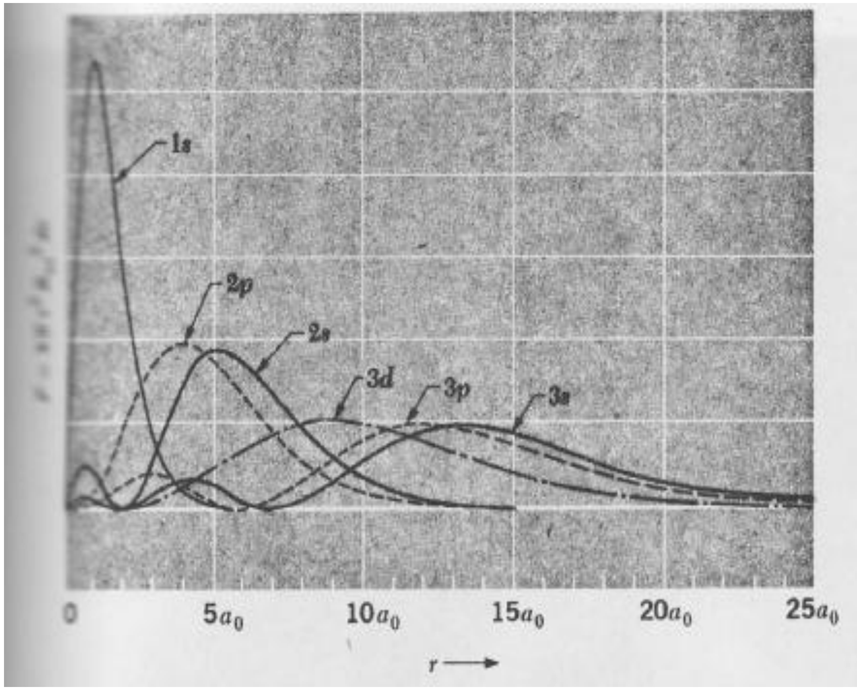
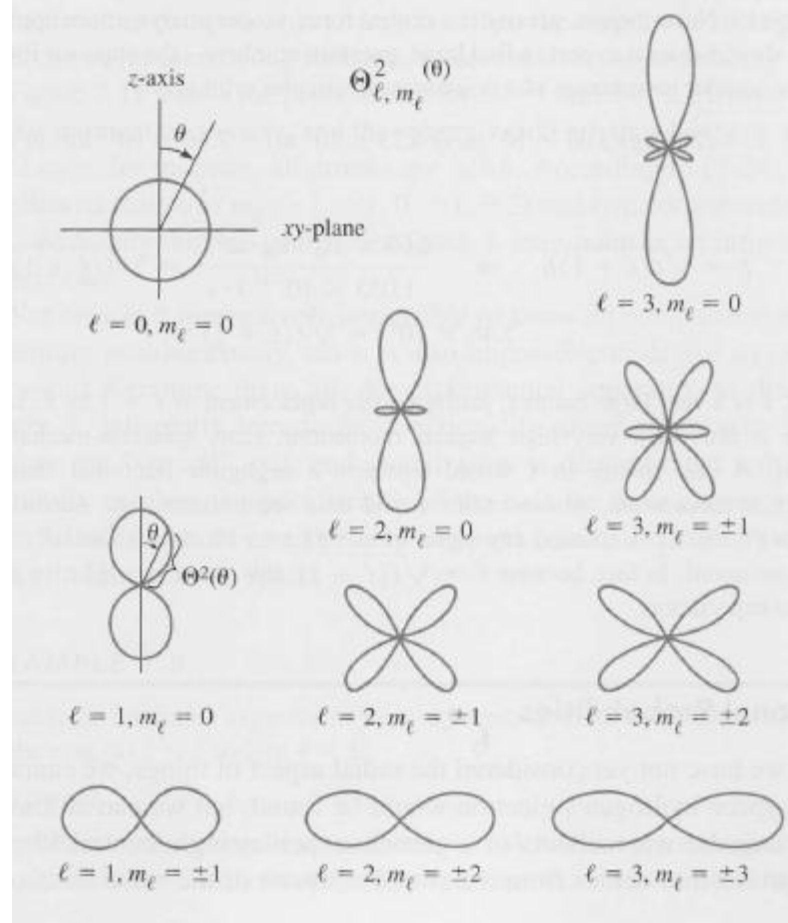
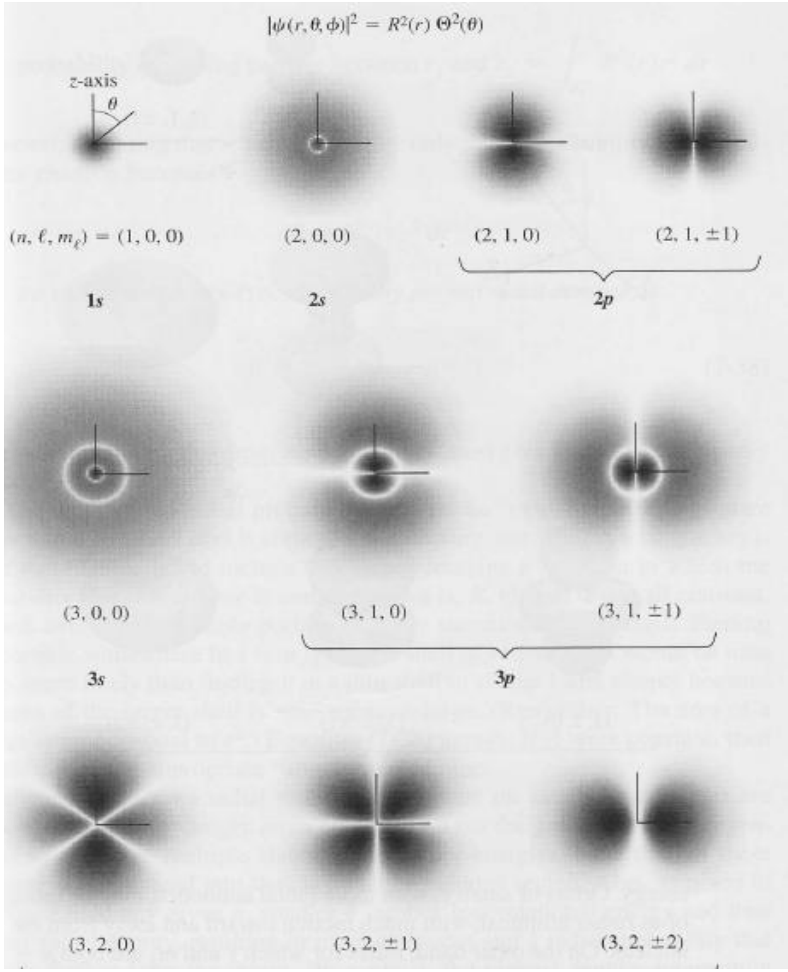
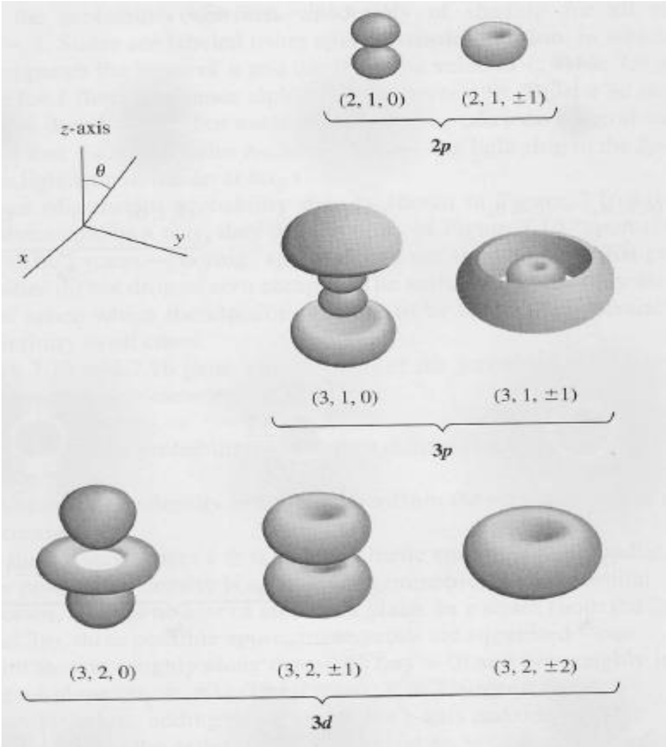


Figure 7.13 Angular probability densities for a central force.

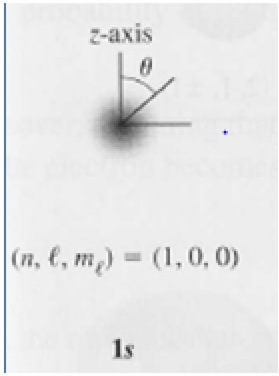


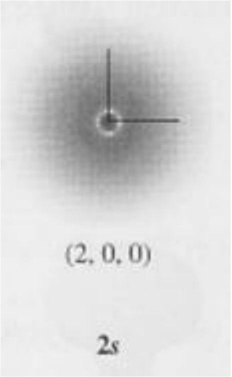
Angular Probability Density

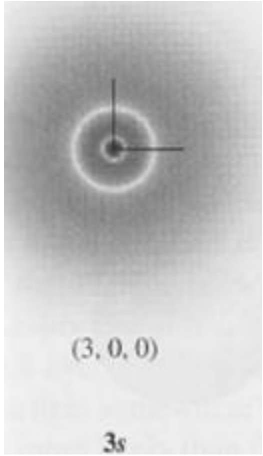




1s v. 2s vs. 3s



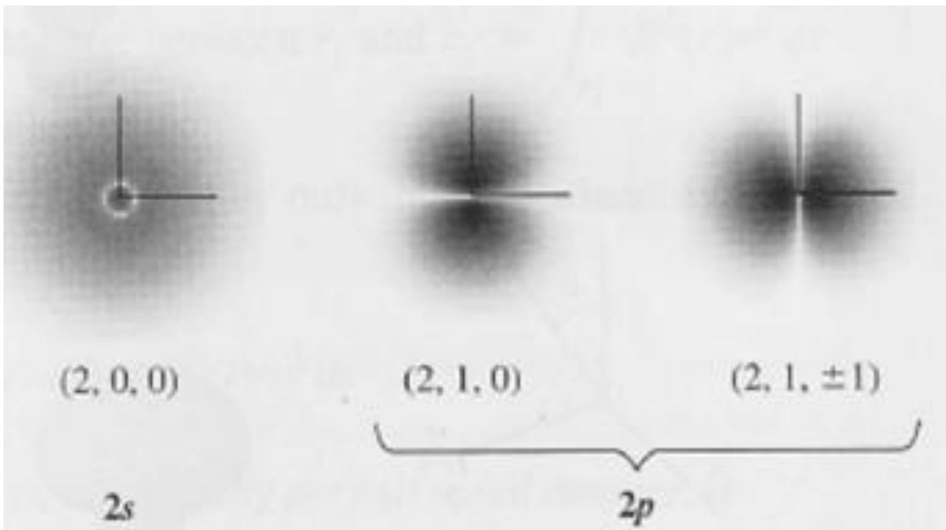


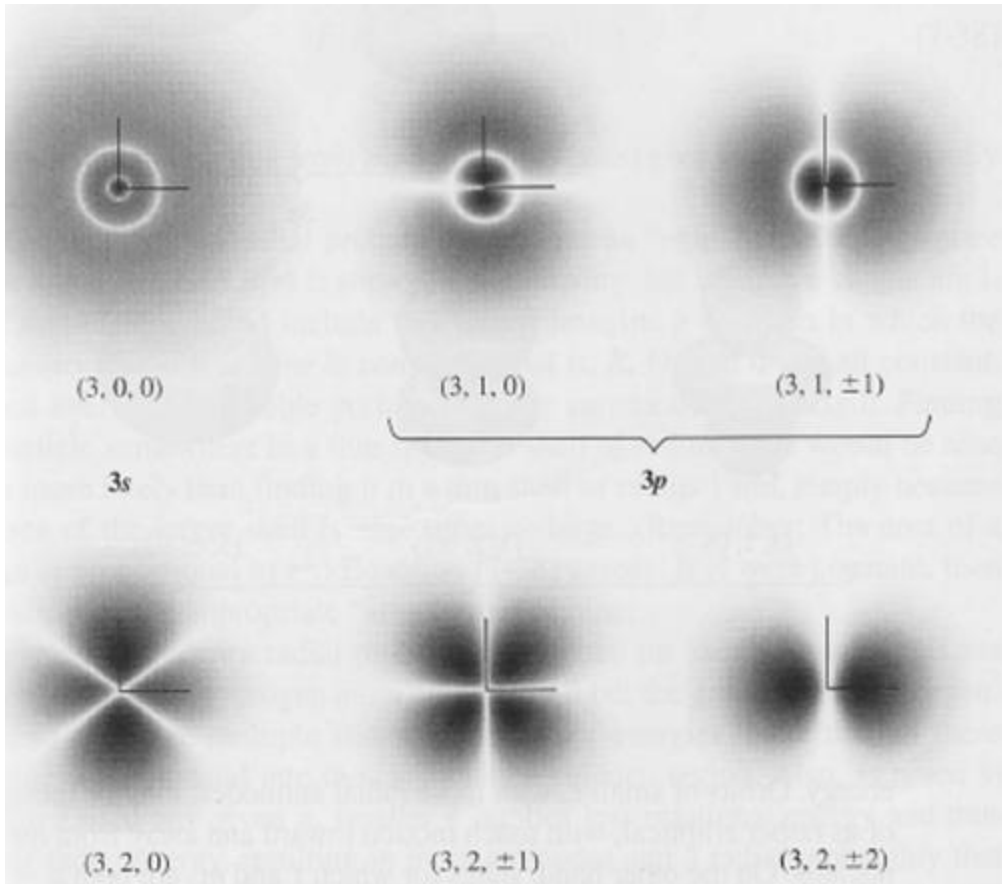


$$l = 0,\ m_l = 0, \quad \Theta_{00} = \frac{1}{\sqrt{2}}, \Phi_0 = \frac{1}{\sqrt{2\pi}}$$

$$l = 1, m_l = 0, \quad \Theta_{10} = \frac{\sqrt{6}}{2}\cos\theta, \Phi_0 = \frac{1}{\sqrt{2\pi}}$$

$$l = 2, m_l = 0, \quad \Theta_{20} = \frac{\sqrt{10}}{4}(3\cos^2\theta - 1), \Phi_0 = \frac{1}{\sqrt{2\pi}}$$


(2, 0, 0)
(2, 1, 0)
(2, 1, ±1)
2s
2p


(3, 0, 0)
(3, 1, 0)
(3, 1, ±1)
3s
3p
(3, 2, 0)
(3, 2, ±1)
(3, 2, ±2)